UNIVERSITI SAINS MALAYSIA

First Semester Examination
2012/2013 Academic Session

January 2013

MSG 388 – Mathematical Algorithms for Computer Graphics
[Algoritma Matematik untuk Grafik Komputer]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all three** [3] questions.

*[**Arahan:** Jawab **semua tiga** [3] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Find a Bézier function that coincides with a polynomial $y=(x-1)(x-3)$, for $1 \le x \le 3$.

(b) Figure 1 shows the control polygon of a quadratic Bézier curve $\boldsymbol{P}$ defined with 3 control points $\boldsymbol{C}_0$, $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ as

$$\boldsymbol{P}(t)=(1-t)^2\boldsymbol{C}_0 + 2t(1-t)\boldsymbol{C}_1 + t^2\boldsymbol{C}_2, \qquad t\in[0, 1].$$

Suppose the points $\boldsymbol{K}$ and $\boldsymbol{L}$ are defined by

$$\boldsymbol{K}=(1-t)\boldsymbol{C}_0+t\boldsymbol{C}_1,$$

$$\boldsymbol{L}=(1-t)\boldsymbol{C}_1+t\boldsymbol{C}_2.$$

Derive the first order derivative of $\boldsymbol{P}$ in terms of $\boldsymbol{K}$ and $\boldsymbol{L}$.

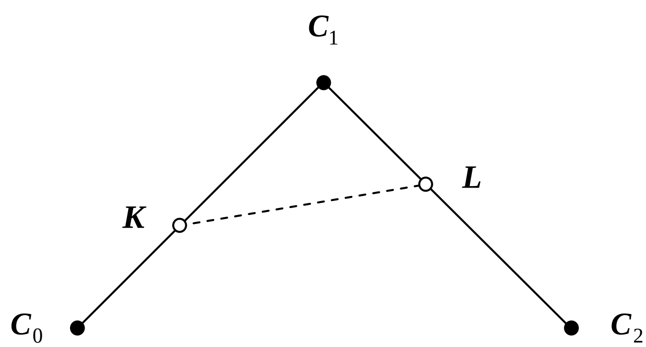


Figure 1

(c) Given a quadratic rational Bézier curve

$$\boldsymbol{R}(t)=\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}=\frac{\boldsymbol{C}_0B_0^2(t) + w\boldsymbol{C}_1B_1^2(t) + \boldsymbol{C}_2B_2^2(t)}{B_0^2(t) + wB_1^2(t) + B_2^2(t)}, \qquad t\in[0, 1]$$

where $w\ge 0$, $\boldsymbol{C}_0=(1, 1)$, $\boldsymbol{C}_1=(2, -2)$, $\boldsymbol{C}_2=(3, 2)$ and

$$B_i^2(t)=\frac{2!}{i!(2-i)!}(1-t)^{2-i}t^i, \qquad i=0, 1, 2.$$

(i) Evaluate the weight w such that the curve $\boldsymbol{R}$ touches the x-axis at a point.

(ii) Suppose the point $\boldsymbol{C}_2$ is shifted to position $(3, 1)$, evaluate the w such that the curve $\boldsymbol{R}$ is a circular arc.

[100 marks]

1. (a) *Cari fungi Bézier yang sama dengan polinomial* $y = (x-1)(x-3)$, *bagi* $1 \le x \le 3$.

(b) *Rajah 1 menunjukkan poligon kawalan bagi lengkung Bézier kuadratik* $\boldsymbol{P}$ *yang ditakrif dengan* 3 *titik kawalan* $\boldsymbol{C}_0$, $\boldsymbol{C}_1$ *dan* $\boldsymbol{C}_2$ *sebagai*

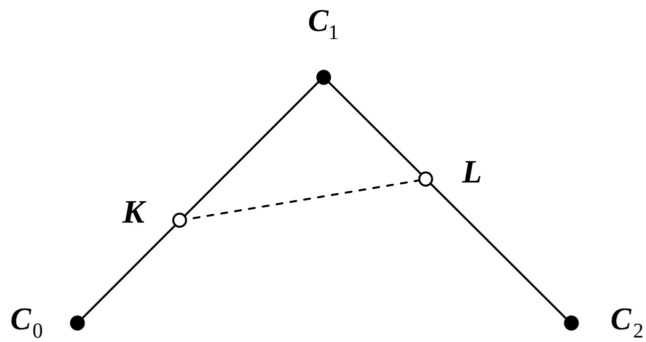
$$\boldsymbol{P}(t) = (1-t)^2\boldsymbol{C}_0 + 2t(1-t)\boldsymbol{C}_1 + t^2\boldsymbol{C}_2, \qquad t \in [0, 1].$$

Andaikan titik $\boldsymbol{K}$ *dan* $\boldsymbol{L}$ *ditakrif oleh*

$$\boldsymbol{K} = (1-t)\boldsymbol{C}_0 + t\boldsymbol{C}_1,$$

$$\boldsymbol{L} = (1-t)\boldsymbol{C}_1 + t\boldsymbol{C}_2.$$

Terbitkan terbitan pertama $\boldsymbol{P}$ *dalam sebutan* $\boldsymbol{K}$ *dan* $\boldsymbol{L}$.



Rajah 1

(c) *Diberi suatu lengkung Bézier nisbah kuadratik*

$$\boldsymbol{R}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \frac{\boldsymbol{C}_0 B_0^2(t) + w\boldsymbol{C}_1 B_1^2(t) + \boldsymbol{C}_2 B_2^2(t)}{B_0^2(t) + wB_1^2(t) + B_2^2(t)}, \qquad t \in [0, 1]$$

di mana $w \ge 0$, $\boldsymbol{C}_0 = (1, 1)$, $\boldsymbol{C}_1 = (2, -2)$, $\boldsymbol{C}_2 = (3, 2)$ *dan*

$$B_i^2(t) = \frac{2!}{i!(2-i)!}(1-t)^{2-i}t^i, \qquad i = 0, 1, 2.$$

(i) *Nilaikan pemberat* w *supaya lengkung* $\boldsymbol{R}$ *menyentuh paksi-*x *pada satu titik.*

(ii) *Andaikan titik* $\boldsymbol{C}_2$ *dialih ke kedudukan* $(3, 1)$, *nilaikan* w *supaya lengkung* $\boldsymbol{R}$ *ialah satu lengkok bulatan.*

[100 markah]

2. (a) Figure 2 shows a pair of triangles with vertices $\boldsymbol{A}(1, 1)$, $\boldsymbol{B}(2, 0)$, $\boldsymbol{C}(2, 3)$ and $\boldsymbol{D}(4, 1)$. Let $\boldsymbol{P}$ be the centroid of the triangle $\boldsymbol{BCD}$. Find the barycentric coordinates of point $\boldsymbol{P}$ with respect to the triangle $\boldsymbol{ABC}$.

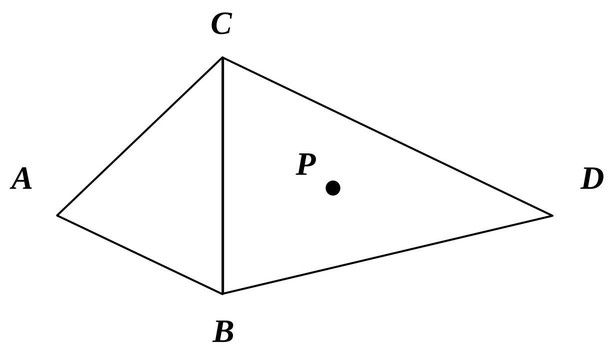


Figure 2

(b) Given a cubic Bézier triangular patch

$$\boldsymbol{S}(u, v, w) = \sum_{\substack{i,j,k \geq 0 \\ i+j+k=3}} \boldsymbol{C}_{i,j,k} B^3_{i,j,k}(u, v, w), \quad 0 \leq u, v, w \leq 1, \quad u+v+w=1$$

where $\boldsymbol{C}_{i,j,k} \in \mathbb{R}^3$ and

$$B^3_{i,j,k}(u, v, w) = \frac{3!}{i!\,j!\,k!} u^i v^j w^k .$$

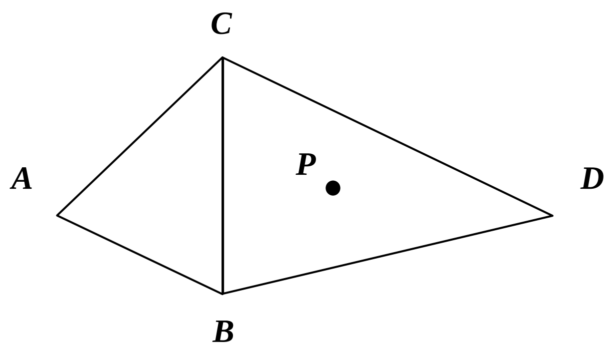
Let $\boldsymbol{C}_{3,0,0} = (1, 1, 1)$, $\boldsymbol{C}_{2,1,0} = (2, 1, 2)$, $\boldsymbol{C}_{2,0,1} = (2, 2, 2)$, $\boldsymbol{C}_{1,1,1} = (3, 2, 3)$, $\boldsymbol{C}_{0,3,0} = (4, 1, 1)$ and $\boldsymbol{C}_{0,0,3} = (4, 4, 1)$.

(i) Determine the coefficients $\boldsymbol{C}_{1,2,0}$, $\boldsymbol{C}_{1,0,2}$, $\boldsymbol{C}_{0,2,1}$ and $\boldsymbol{C}_{0,1,2}$ such that the patch $\boldsymbol{S}$ can be reduced to a quadratic Bézier patch.

(ii) Suppose $\boldsymbol{C}_{1,2,0} = (3, 1, 2)$, $\boldsymbol{C}_{1,0,2} = (3, 3, 2)$, $\boldsymbol{C}_{0,2,1} = (4, 2, 2)$ and $\boldsymbol{C}_{0,1,2} = (4, 3, 2)$. Use the de Casteljau algorithm to evaluate the surface point $\boldsymbol{S}$ at $(u, v, w) = (1/3, 1/6, 1/2)$.

[100 marks]

2\. (a) *Rajah 2 menunjukkan sepasang segi tiga dengan bucu* $\boldsymbol{A}(1,\ 1)$, $\boldsymbol{B}(2,\ 0)$, $\boldsymbol{C}(2,\ 3)$ *dan* $\boldsymbol{D}(4,\ 1)$. *Katakan* $\boldsymbol{P}$ *ialah sentroid segi tiga* $\boldsymbol{BCD}$. *Cari koordinat baripusat titik* $\boldsymbol{P}$ *terhadap segi tiga* $\boldsymbol{ABC}$.



Rajah 2

(b) *Diberi suatu tampalan segi tiga Bézier kubik*

$$\boldsymbol{S}(u, v, w) = \sum_{\substack{i,\, j,k \geq 0 \\ i+j+k=3}} \boldsymbol{C}_{i,\,j,\,k}\, B^3_{i,\,j,\,k}(u, v, w), \qquad 0 \leq u, v, w \leq 1, \quad u+v+w=1$$

di mana $\boldsymbol{C}_{i,j,k} \in \mathbb{R}^3$ *dan*

$$B^3_{i,\,j,\,k}(u, v, w) = \frac{3!}{i!\,j!\,k!} u^i v^j w^k .$$

Katakan $\boldsymbol{C}_{3,0,0} = (1,\ 1,\ 1)$, $\boldsymbol{C}_{2,1,0} = (2,\ 1,\ 2)$, $\boldsymbol{C}_{2,0,1} = (2,\ 2,\ 2)$, $\boldsymbol{C}_{1,1,1} = (3,\ 2,\ 3)$, $\boldsymbol{C}_{0,3,0} = (4,\ 1,\ 1)$ *dan* $\boldsymbol{C}_{0,0,3} = (4,\ 4,\ 1)$.

(i) *Tentukan koefisien* $\boldsymbol{C}_{1,2,0}$, $\boldsymbol{C}_{1,0,2}$, $\boldsymbol{C}_{0,2,1}$ *dan* $\boldsymbol{C}_{0,1,2}$ *supaya tampalan* $\boldsymbol{S}$ *boleh dikurangkan kepada tampalan Bézier kuadratik.*

(ii) *Andaikan* $\boldsymbol{C}_{1,2,0} = (3,\ 1,\ 2)$, $\boldsymbol{C}_{1,0,2} = (3,\ 3,\ 2)$, $\boldsymbol{C}_{0,2,1} = (4,\ 2,\ 2)$ *dan* $\boldsymbol{C}_{0,1,2} = (4,\ 3,\ 2)$. *Gunakan algoritma de Casteljau untuk menilai titik permukaan* $\boldsymbol{S}$ *pada* $(u, v, w) = (1/3,\ 1/6,\ 1/2)$.

[100 markah]

3. (a) Given a polygon of 10 points in Figure 3. Find the number of points produced after the polygon is refined twice by Chaikin's subdivision scheme.

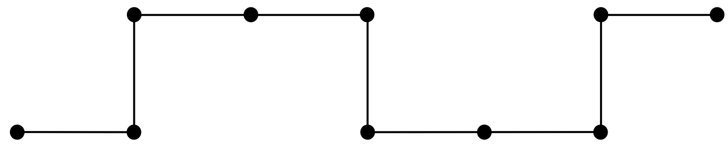


Figure 3

(b) Given a cubic B-spline curve defined on a non-decreasing knot vector $\boldsymbol{u} = (u_0, u_1, \ldots, u_7)$ as

$$\boldsymbol{P}(u) = \sum_{i=0}^{3} \boldsymbol{D}_i N_i^4(u), \qquad u_3 \le u \le u_4$$

where $\boldsymbol{D}_i \in \mathbb{R}^2$ are distinct de Boor points. The functions $N_i^4(u)$ can be formulated recursively by

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad k > 1,$$

and

$$N_i^1(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0, & \text{otherwise}. \end{cases}$$

(i) Describe the conditions on the knot vector $\boldsymbol{u}$ such that the curve $\boldsymbol{P}$ interpolates the points $\boldsymbol{D}_0$ and $\boldsymbol{D}_3$.

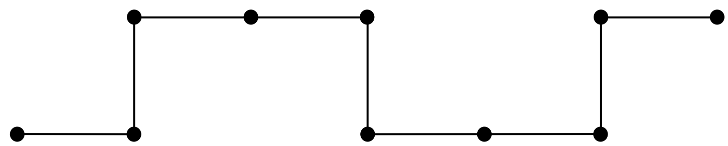
(ii) Suppose the curve $\boldsymbol{P}$ is defined by $\boldsymbol{D}_0 = (1, 1)$, $\boldsymbol{D}_1 = (1, 2)$, $\boldsymbol{D}_2 = (2, 2)$ and $\boldsymbol{D}_3 = (2, 1)$ over $\boldsymbol{u} = (-3, -2, -1, 0, 1, 2, 3, 4)$. A set of new de Boor points is gained when a knot value $u = 0.75$ is inserted twice into $\boldsymbol{u}$ without changing the shape of $\boldsymbol{P}$. Find the positions of these new de Boor points.

(iii) Suppose $\boldsymbol{u} = (-3, -2, -0.5, 0, 1, 3, 4, 5)$, find the de Boor points $\boldsymbol{D}_i$, $i = 0, 1, 2, 3$, such that the curve

$$\boldsymbol{P}(u) = \begin{pmatrix} 1 \\ 3 \end{pmatrix} u^3 + \begin{pmatrix} 6 \\ 3 \end{pmatrix} (1-u) u^2 + \begin{pmatrix} 9 \\ 3 \end{pmatrix} (1-u)^2 u + \begin{pmatrix} 4 \\ 3 \end{pmatrix} (1-u)^3.$$

[100 marks]

3. (a) *Diberi suatu poligon* 10 *titik dalam Rajah 3. Cari bilangan titik yang dihasilkan selepas poligon diperhalus dua kali oleh skema subdivisi Chaikin.*



Rajah 3

(b) *Diberi suatu lengkung splin-B kubik yang ditakrif pada vektor simpulan tak menyusut* $\boldsymbol{u} = (u_0,\ u_1,\ \ldots,\ u_7)$ *sebagai*

$$\boldsymbol{P}(u) = \sum_{i=0}^{3} \boldsymbol{D}_i N_i^4(u)\,, \qquad\qquad u_3 \le u \le u_4$$

di mana $\boldsymbol{D}_i \in \mathbb{R}^2$ *adalah titik-titik de Boor yang berbeza. Fungsi* $N_i^4(u)$ *boleh dirumuskan secara rekursi oleh*

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u)\,, \qquad k > 1\,,$$

dan

$$N_i^1(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0, & \textit{di tempat lain}\,. \end{cases}$$

(i) *Nyatakan syarat pada vektor simpulan* $\boldsymbol{u}$ *supaya lengkung* $\boldsymbol{P}$ *menginterpolasi titik* $\boldsymbol{D}_0$ *dan titik* $\boldsymbol{D}_3$.

(ii) *Andaikan lengkung* $\boldsymbol{P}$ *ditakrif dengan* $\boldsymbol{D}_0 = (1,\ 1)$, $\boldsymbol{D}_1 = (1,\ 2)$, $\boldsymbol{D}_2 = (2,\ 2)$ *dan* $\boldsymbol{D}_3 = (2,\ 1)$ *pada* $\boldsymbol{u} = (-3\,,\ -2\,,\ -1,\ 0\,,\ 1\,,\ 2\,,\ 3\,,\ 4)$. *Satu set titik de Boor baru diperolehi apabila nilai simpulan* $u = 0.75$ *dimasukkan dua kali ke dalam* $\boldsymbol{u}$ *tanpa mengubah bentuk* $\boldsymbol{P}$. *Cari kedudukan titik de Boor baru ini.*

(iii) *Andaikan* $\boldsymbol{u} = (-3\,,\ -2\,,\ -0.5\,,\ 0\,,\ 1,\ 3,\ 4\,,\ 5)$, *cari titik-titik de Boor* $\boldsymbol{D}_i$, $i = 0\,,\ 1,\ 2\,,\ 3$, *supaya lengkung*

$$\boldsymbol{P}(u) = \begin{pmatrix} 1 \\ 3 \end{pmatrix} u^3 + \begin{pmatrix} 6 \\ 3 \end{pmatrix} (1-u)\, u^2 + \begin{pmatrix} 9 \\ 3 \end{pmatrix} (1-u)^2\, u + \begin{pmatrix} 4 \\ 3 \end{pmatrix} (1-u)^3\,.$$

[100 markah]

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