UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2012/2013

January 2013

MAT 514 - Mathematical Modelling
[Pemodelan Matematik]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of TEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEPULUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Discuss briefly on the classifications of fluid flow.

   (b) Explain about the no-slip condition in a fluid flow.

[15 marks]

1. (a) Bincangkan secara ringkas mengenai pengelasan-pengelasan aliran bendalir.

(b) Terangkan tentang syarat tak gelincir dalam aliran bendalir.

[15 markah]

2. Consider a steady flow along a semi-infinite two dimensional surface of small curvature with a free-stream velocity u_∞. Let x be measured along the surface and y normal to the surface. Cut out an infinitesimal stationary control volume of unit depth within the boundary layer and consider the external forces acting on this control volume in the x direction and the momentum fluxes crossing the control surface as shown in Figure 1.

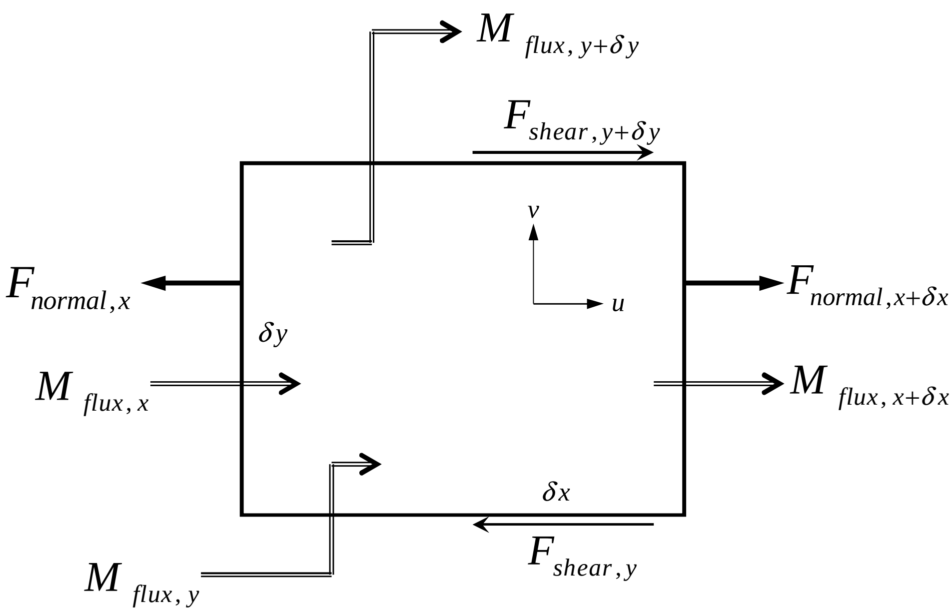


Figure 1: Control volume for development of the momentum equation.

Table 1: Basic rates of momentum transfer.

$$F_{normal,x} = \sigma_x \delta y,$$

$$F_{shear,y} = \tau_{yx} \delta x,$$

$$M_{flux,x+\delta x} = \left[G_x u + \frac{\partial}{\partial x}(G_x u)\delta x \right] \delta y.$$

(a) What is the momentum theorem?

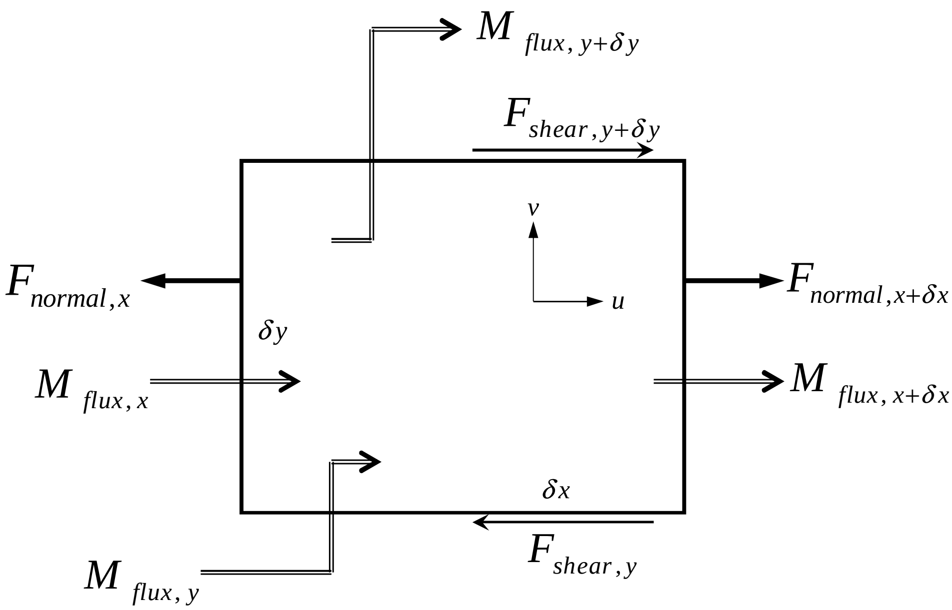
(b) Based on Figure 1, Table 1 and applying 2(a), the two-dimensional boundary layer continuity equation ($\nabla \cdot G = 0$) and the boundary layer approximation, show that the equation (1) is the momentum equation of the boundary layer for two dimensional flow.

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y} \right). \tag{1}$$

[Note: (u, v) = velocity components along (x, y) axes; $(G_x, G_y) = (\rho u, \rho v)$ where G represent mass flux and ρ = density of the fluid.]

[20 marks]

2. *Pertimbangkan suatu aliran mantap terhadap permukaan berkelengkungan kecil separuh tak terhingga dengan halaju strim bebas u_∞. x diukur di sepanjang permukaan dan y serenjang terhadap permukaan. Satu unit kedalaman unsur isipadu kawalan pegun dalam lapisan sempadan dikeluarkan dan daya-daya luar yang bertindak ke atas unsur isipadu kawalan ini dalam arah x serta fluks-fluks momentum merentasi permukaan kawalan seperti yang ditunjukkan dalam Rajah 1 dipertimbangkan.*



Rajah 1: Isipadu kawalan untuk menerbitkan persamaan momentum.

Jadual 1: Kadar-kadar asas pemindahan momentum.

$$F_{normal,x} = \sigma_x \delta y,$$

$$F_{shear,y} = \tau_{yx} \delta x,$$

$$M_{flux,x+\delta x} = \left[G_x u + \frac{\partial}{\partial x}(G_x u)\delta x \right] \delta y.$$

(a) *Apakah teorem momentum?*

(b) *Berdasarkan Rajah 1, Jadual 1 dan aplikasikan 2(a), persamaan keselanjaran lapisan sempadan dua dimensi* ($\nabla \cdot G = 0$) *dan penghampiran lapisan sempadan, tunjukkan bahawa persamaan (1) adalah persamaan momentum lapisan sempadan bagi aliran dua dimensi.*

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dP}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y} \right). \tag{1}$$

[Nota: (u, v) = *komponen-komponen halaju sepanjang paksi* (x, y)*;* $(G_x, G_y) = (\rho u, \rho v)$ *dengan* G *adalah fluks jisim dan* ρ = *ketumpatan bendalir.]*

[20 markah]

3. Suppose that the energy equation of constant heat rate for circular tube of radius r_s with fully developed velocity and temperature profiles is given by the following equation

$$u\left(\frac{dT_m}{dx}\right) = \frac{\kappa}{\rho c_p}\frac{1}{r^n}\frac{\partial^2 T}{\partial r^2},$$

where x and r are the axial and radial coordinates of circular tube, respectively, T_m is the mass-averaged fluid temperature, κ is the constant thermal conductivity, ρ is the constant fluid density, c_p is the specific heat at constant pressure and n is a power index.

(a) Derive the fully developed temperature profile T by applying the following velocity profile

$$u = 2V\left(1 - \frac{r^n}{r_s^n}\right),$$

and boundary condition

$$\frac{\partial T}{\partial r} = -2r \quad \text{at} \quad r = 0,$$
$$T = T_s \quad \text{at} \quad r = r_s.$$

[T_s is the fluid temperature at the surface of circular tube and V is the mean velocity.]

(b) What is the temperature profile at the centerline of the tube?

[25 marks]

3. Andaikan bahawa persamaan tenaga dengan kadar haba malar bagi tiub bulat berjejari r_s yang profil halaju dan suhu terbangun penuh diberikan oleh persamaan berikut

$$u\left(\frac{dT_m}{dx}\right) = \frac{\kappa}{\rho c_p}\frac{1}{r^n}\frac{\partial^2 T}{\partial r^2},$$

dengan x dan r masing-masing adalah koordinat paksian dan jejarian tiub bulat, T_m adalah suhu jisim-purata bendalir, κ adalah kekonduksian terma malar, ρ adalah ketumpatan bendalir malar, c_p adalah haba spesifik pada tekanan malar dan n adalah indeks kuasa.

(a) *Terbitkan profil suhu terbangun penuh T dengan mengaplikasikan profil halaju berikut*

$$u = 2V\left(1 - \frac{r^n}{r_s^n}\right),$$

dan syarat sempadan

$$\frac{\partial T}{\partial r} = -2r \quad at \quad r = 0,$$
$$T = T_s \quad at \quad r = r_s.$$

[T_s adalah suhu bendalir pada permukaan tiub bulat dan V adalah halaju purata.]

(b) *Apakah profil suhu pada garis tengah tiub?*

[25 markah]

4. Consider a steady boundary layer flow and heat transfer near a vertical cone (with half angle a) embedded in a saturated porous medium filled with a nanofluid. The origin O of the coordinate system is placed at the vertex of the cone, where x is the coordinate measured from the origin O along the surface of the cone and y is the coordinate normal to the surface of the cone. It is assumed that the surface of the cone is held at a constant heat flux q_w while a temperature of the ambient nanofluid is T_∞. Under these assumptions, the basic governing equations are

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0, \tag{2}$$

$$\frac{\mu_{nf}}{\mu_f}u = \frac{gK\cos a[\varphi\rho_s\beta_s + (1-\varphi)\rho_f\beta_f]}{\mu_f}(T - T_\infty), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

subject to the boundary conditions

$$\begin{aligned} &v = 0, \quad -k_{nf}\frac{\partial T}{\partial y} = q_w \quad \text{at} \quad y = 0, \\ &u = 0, \quad T = T_\infty \quad \text{as} \quad y \to \infty. \end{aligned} \tag{5}$$

Here u and v are the velocity components along x and y axes, respectively, T is the temperature of the nanofluid, g is the acceleration due to gravity, K is the permeability of the porous medium, $r = x\sin a$, φ is the nanoparticle volume fraction, β_f and β_s are the coefficients of the thermal expansion of the fluid and of the solid, respectively, μ_{nf} is the viscosity of the nanofluid, α_{nf} is the thermal diffusivity of the nanofluid and k_{nf} is the thermal conductivity of the nanofluid, which are given by

$$\begin{aligned}\mu_{nf} &= \frac{\mu_f}{(1-\varphi)^{2.5}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}},\\ \alpha_f &= \frac{k_f}{(\rho C_p)_f}, \quad (\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s,\\ \frac{k_{nf}}{k_f} &= \frac{(k_s+2k_f)-2\varphi(k_f-k_s)}{(k_s+2k_f)+\varphi(k_f-k_s)},\end{aligned}$$

where μ_f is the dynamic viscosity of the base fluid, α_f is the thermal diffusivity of the base fluid, k_f and k_s are the thermal conductivities of the base fluid and of the solid, respectively, and, $(\rho C_p)_{nf}$ and $(\rho C_p)_f$ are the heat capacitance of the nanofluid and base fluid, respectively.

$\left[\text{Note: } \psi \text{ is a stream function which is defined as } u = \frac{1}{r}\frac{\partial \psi}{\partial y} \text{ and } v = -\frac{1}{r}\frac{\partial \psi}{\partial x}.\right]$

(a) By using similarity variables of the following form

$$\psi = \alpha_f \, r Ra^{1/3} \, f(\eta), \quad T - T_\infty = \frac{q_w}{k_f} \, x Ra^{-1/3} \, \theta(\eta),$$
$$\eta = Ra^{1/3} \, \frac{y}{x}, \quad Ra = gK\beta_f(q_w/k_f)x^2 \cos a/(\alpha_f \upsilon_f),$$

where $\upsilon_f = \mu_f/\rho_f$, show that the governing equations (2) – (4) can be reduced to the following system of differential equations

$$\frac{1}{(1-\varphi)^{2.5}}\frac{\partial f}{\partial \eta} - \left[1-\varphi+\varphi\left(\frac{\rho_s}{\rho_f}\right)\left(\frac{\beta_s}{\beta_f}\right)\right]\theta = 0, \tag{6}$$

$$\frac{k_{nf}/k_f}{(1-\varphi)+\varphi(\rho C_p)_s/(\rho C_p)_f}\frac{\partial^2 \theta}{\partial \eta^2} + \frac{5}{3}f\frac{\partial \theta}{\partial \eta} - \frac{1}{3}\theta\frac{\partial f}{\partial \eta} = 0, \tag{7}$$

with the boundary conditions (5) become

$$f(0) = 0, \quad \frac{k_{nf}}{k_f}\frac{\partial \theta}{\partial \eta}(0) = -1, \quad \theta(\infty) = 0. \tag{8}$$

(b) Figure 2 shows the temperature profiles of the system of equations (6) and (7) subject to the boundary conditions (8) for Cu-water nanofluid with various values of nanoparticle volume fraction φ. Interpret the obtained results based on this figure.

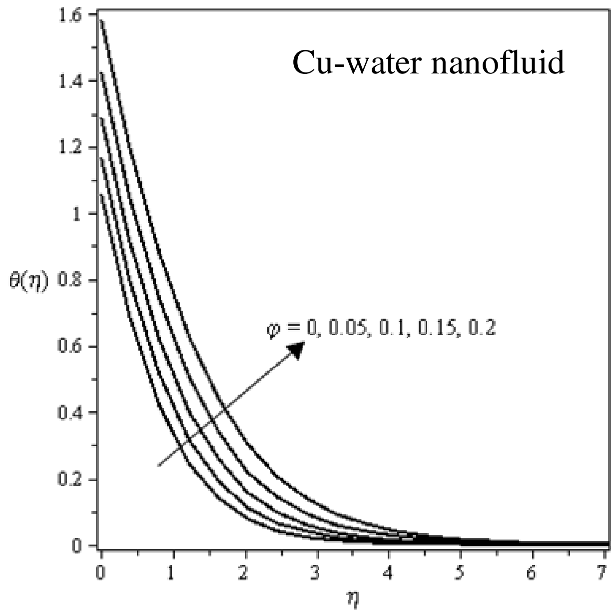


Figure 2: Temperature profiles $\theta(\eta)$ for Cu-water nanofluid with various values of nanoparticle volume fraction φ.

[40 marks]

4. *Pertimbangkan suatu aliran lapisan sempadan dan pemindahan haba yang mantap menghampiri sebuah kon tegak (dengan sudut separuh a) dalam bahantara berliang tepu yang dipenuhi bendalir nano. Asalan O pada sistem koordinat ditandakan pada puncak kon, dengan x adalah koordinat yang diukur di sepanjang permukaan kon bermula dari asalan O dan y adalah koordinat berserenjang dengan permukaan kon. Andaikan bahawa permukaan kon ditetapkan pada fluks haba q_w malar dan T_∞ adalah suhu persekitaran. Berdasarkan andaian-andaian tersebut, persamaan-persamaan menakluk asas adalah*

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0, \tag{2}$$

$$\frac{\mu_{nf}}{\mu_f}u = \frac{gK \text{ kos } a\ [\varphi\rho_s\beta_s + (1-\varphi)\rho_f\beta_f]}{\mu_f}(T - T_\infty), \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2}, \tag{4}$$

tertakluk kepada syarat-syarat sempadan

$$v = 0, \quad -k_{nf}\frac{\partial T}{\partial y} = q_w \quad at \quad y = 0,$$
$$u = 0, \quad T = T_\infty \quad as \quad y \to \infty. \tag{5}$$

Di sini u dan v masing-masing adalah komponen-komponen halaju pada paksi x dan y, T adalah suhu bendalir nano, g adalah pecutan graviti, K adalah kebolehtelapan bahantara berliang, $r = x \sin a$, φ adalah pecahan isipadu zarah nano, β_f dan β_s masing-masing adalah pekali-pekali pengembangan terma bendalir dan pepejal (zarah), μ_{nf} adalah kelikatan dinamik bendalir nano, α_{nf} adalah resapan terma bendalir nano dan k_{nf} adalah kekonduksian terma bendalir nano, yang diberikan oleh

$$\begin{aligned}
\mu_{nf} &= \frac{\mu_f}{(1-\varphi)^{2.5}}, \quad \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \\
\alpha_f &= \frac{k_f}{(\rho C_p)_f}, \quad (\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s, \\
\frac{k_{nf}}{k_f} &= \frac{(k_s + 2k_f) - 2\varphi(k_f - k_s)}{(k_s + 2k_f) + \varphi(k_f - k_s)},
\end{aligned}$$

dengan μ_f adalah kelikatan dinamik bendalir asas, α_f adalah resapan terma bendalir asas, k_f dan k_s masing-masing adalah kekonduksian terma bendalir asas dan pepejal, dan, $(\rho C_p)_{nf}$ dan $(\rho C_p)_f$ masing-masing adalah muatan haba bendalir nano dan bendalir asas.

[*Nota: ψ adalah fungsi strim yang ditakrifkan sebagai $u = \dfrac{1}{r}\dfrac{\partial \psi}{\partial y}$ dan $v = -\dfrac{1}{r}\dfrac{\partial \psi}{\partial x}$.*]

(a) *Dengan menggunakan pemboleh-pemboleh ubah keserupaan berbentuk berikut*

$$\begin{aligned}
&\psi = \alpha_f \, r Ra^{1/3} \, f(\eta), \quad T - T_\infty = \frac{q_w}{k_f} \, x Ra^{-1/3} \, \theta(\eta), \\
&\eta = Ra^{1/3} \, \frac{y}{x}, \quad Ra = gK\beta_f (q_w/k_f) x^2 \text{ kos } a \,/(\alpha_f \upsilon_f),
\end{aligned}$$

yang $\upsilon_f = \mu_f/\rho_f$, tunjukkan bahawa persamaan-persamaan menakluk (2) – (4) boleh diturunkan kepada sistem persamaan pembezaan berikut

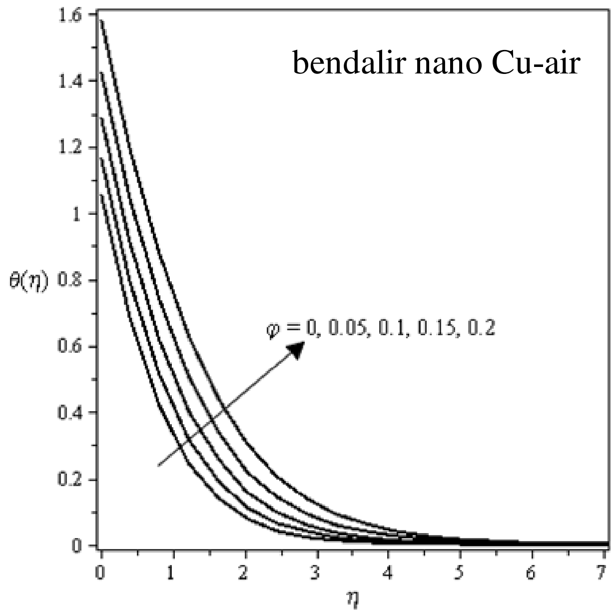
$$\frac{1}{(1-\varphi)^{2.5}}\frac{\partial f}{\partial \eta} - \left[1 - \varphi + \varphi\left(\frac{\rho_s}{\rho_f}\right)\left(\frac{\beta_s}{\beta_f}\right)\right]\theta = 0, \tag{6}$$

$$\frac{k_{nf}/k_f}{(1-\varphi) + \varphi(\rho C_p)_s/(\rho C_p)_f}\frac{\partial^2 \theta}{\partial \eta^2} + \frac{5}{3} f \frac{\partial \theta}{\partial \eta} - \frac{1}{3}\theta\frac{\partial f}{\partial \eta} = 0, \tag{7}$$

dengan syarat-syarat sempadan (5) menjadi

$$f(0) = 0, \quad \frac{k_{nf}}{k_f}\frac{\partial \theta}{\partial \eta}(0) = -1, \quad \theta(\infty) = 0. \tag{8}$$

(b) *Rajah 2 menunjukkan profil-profil suhu bagi sistem persamaan (6) dan (7) tertakluk kepada syarat-syarat sempadan (8) untuk bendalir nano Cu-air dengan beberapa nilai pecahan isipadu zarah nano φ. Tafsirkan keputusan yang diperoleh berdasarkan rajah tersebut.*



Rajah 2: Profil suhu $\theta(\eta)$ bagi bendalir nano Cu-air dengan beberapa nilai pecahan isipadu zarah nano φ.

[40 markah]

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