
UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 2012/2013

Januari 2013

MAT 101 Calculus
[Kalkulus]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all eight** [8] questions.

*[Arahan: Jawab **semua lapan** [8] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

- 1 (a) Let $f(x) = -x^2$ and $g(x) = \frac{1}{\sqrt{x}}$.
- (i) Find the formulas for $f \circ g$ and $g \circ f$.
 - (ii) State the domain of each composition in part (i).
- (b) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be two functions.
- (i) If $g \circ f$ is one to one, show that f is one to one.
 - (ii) If f is one to one, then is $g \circ f$ also one to one? If yes, prove it; if no, give a counterexample.

[13 marks]

- 1 (a) *Andaikan $f(x) = -x^2$ dan $g(x) = \frac{1}{\sqrt{x}}$.*
- (i) Cari formula untuk $f \circ g$ dan $g \circ f$.*
 - (ii) Nyatakan domain untuk setiap gubahan di bahagian (i).*
- (b) *Andaikan $f : A \rightarrow B$ dan $g : B \rightarrow C$ dua fungsi.*
- (i) Jika $g \circ f$ satu ke satu, tunjukkan bahawa f satu ke satu.*
 - (ii) Jika f satu ke satu, adakah $g \circ f$ juga satu ke satu? Jika ya, buktikan; jika bukan, beri satu contoh lawan.*

[13 markah]

- 2 (a) Given that

$$\lim_{x \rightarrow a} f(x) = 2, \quad \lim_{x \rightarrow a} g(x) = -4, \quad \lim_{x \rightarrow a} h(x) = 0,$$

find the limit

$$\lim_{x \rightarrow a} \frac{h(x)[f(x) - 3g(x) + 1] - [g(x) + 4]}{\sqrt[3]{6 + h(x)}}.$$

- (b) If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ exists and $\lim_{x \rightarrow a} g(x) = 0$, show that $\lim_{x \rightarrow a} f(x) = 0$.
- (c) Find all values of a such that $\lim_{x \rightarrow 1} \left[\frac{a}{x-1} - \frac{2}{x^2-1} \right]$ exists and is finite.

[12 marks]

2 (a) Diberi

$$\lim_{x \rightarrow a} f(x) = 2, \quad \lim_{x \rightarrow a} g(x) = -4, \quad \lim_{x \rightarrow a} h(x) = 0,$$

cari had

$$\lim_{x \rightarrow a} \frac{h(x)[f(x) - 3g(x) + 1] - [g(x) + 4]}{\sqrt[3]{6 + h(x)}}.$$

(b) Jika $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ wujud dan $\lim_{x \rightarrow a} g(x) = 0$, tunjukkan bahawa $\lim_{x \rightarrow a} f(x) = 0$.

(c) Cari semua nilai a supaya $\lim_{x \rightarrow 1} \left[\frac{a}{x-1} - \frac{2}{x^2-1} \right]$ wujud dan terhingga.

[12 markah]

3 (a) Determine whether the following functions are continuous at $x = 2$. Give your reason.

(i)

$$f(x) = \frac{x^2 - 4}{x - 2}.$$

(ii)

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2; \\ 4, & x = 2. \end{cases}$$

(iii)

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2; \\ 3, & x = 2. \end{cases}$$

(b) (i) Give the statement of the Intermediate Value Theorem for a continuous function f on an interval $[a, b]$.

(ii) Give an example to show that the conclusion of the Intermediate Value Theorem may be false if f is not continuous on the interval $[a, b]$.

[16 marks]

3 (a) Tentukan sama ada fungsi berikut selanjar pada $x = 2$. Beri alasan anda.

(i)

$$f(x) = \frac{x^2 - 4}{x - 2}.$$

(ii)

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2; \\ 4, & x = 2. \end{cases}$$

(iii)

$$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2; \\ 3, & x = 2. \end{cases}$$

(b) (i) Beri pernyataan untuk Teorem Nilai Pertengahan untuk fungsi f yang selanjar pada selang $[a, b]$.

(ii) Beri satu contoh yang menunjukkan kesimpulan untuk Teorem Nilai Pertengahan tidak benar jika f tak selanjar pada selang $[a, b]$.

[16 markah]

4 (a) Suppose that

$$f(x) = \begin{cases} x^2 - 1, & x \leq 1; \\ k(x - 1), & x > 1. \end{cases}$$

For what values of k is f

(i) continuous at $x = 1$?

(ii) differentiable at $x = 1$?

(b) Find all the asymptotes (vertical, horizontal or slant) for the graph of $f(x) = x - \frac{1}{x} - \frac{1}{x^2}$.

[18 marks]

4 (a) Andaikan bahawa

$$f(x) = \begin{cases} x^2 - 1, & x \leq 1; \\ k(x - 1), & x > 1. \end{cases}$$

Apakah nilai k supaya f

(i) selanjar pada $x = 1$?

(ii) terbezakan pada $x = 1$?

(b) Cari semua asimptot (mencancang, mengufuk atau sendeng) untuk graf untuk $f(x) = x - \frac{1}{x} - \frac{1}{x^2}$.

[18 markah]

5 (a) Determine, by using the second derivative f'' , the concavity of the general cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$. Then show that there is exactly one inflection point of f . State the value of x of that inflection point.

(b) Suppose that the general cubic polynomial f has three x -intercepts: x_1 , x_2 and x_3 . Then f can be written in the form $f(x) = a(x - x_1)(x - x_2)(x - x_3)$. Show that the inflection point of f occurs at $x = \frac{x_1 + x_2 + x_3}{3}$.

(c) By using the result in part (b), find the inflection point of the cubic polynomial $f(x) = x^3 - 3x^2 + 2x$.

[10 marks]

5 (a) Tentukan, dengan menggunakan terbitan kedua f'' , kecembungan untuk polinomial kubik am $f(x) = ax^3 + bx^2 + cx + d$. Kemudian tunjukkan bahawa terdapat hanya satu titik lengkok balas untuk f . Nyatakan nilai x untuk titik lengkok balas tersebut.

(b) Andaikan polinomial kubik am tersebut mempunyai 3 pintasan x : x_1 , x_2 dan x_3 . Maka f boleh ditulis dalam bentuk $f(x) = a(x - x_1)(x - x_2)(x - x_3)$. Tunjukkan bahawa titik lengkok balas untuk f berlaku pada $x = \frac{x_1 + x_2 + x_3}{3}$.

(c) Dengan menggunakan keputusan di bahagian (b), cari titik lengkok balas untuk polinomial kubik $f(x) = x^3 - 3x^2 + 2x$.

[10 markah]

- 6 (a) The right endpoint approximation for the area between a curve $y = f(x)$ and an interval $[a, b]$ is given by

$$\sum_{k=1}^n \left[9 - \frac{4k^2}{n^2} \right] \left(\frac{2}{n} \right).$$

Determine the actual function f and the interval $[a, b]$.

- (b) In each part, evaluate the integral, given that

$$f(x) = \begin{cases} |x - 2|, & x \geq 0; \\ x + 2, & x < 0. \end{cases}$$

(i) $\int_{-2}^0 f(x) dx$

(ii) $\int_{-2}^6 f(x) dx$

[13 marks]

- 6 (a) *Penghampiran titik hujung kanan untuk luas di antara suatu lengkung $y = f(x)$ dan selang $[a, b]$ diberikan oleh*

$$\sum_{k=1}^n \left[9 - \frac{4k^2}{n^2} \right] \left(\frac{2}{n} \right).$$

Tentukan fungsi hakiki f dan selang $[a, b]$.

- (b) *Untuk setiap bahagian, nilaikan kamiran, dengan diberi*

$$f(x) = \begin{cases} |x - 2|, & x \geq 0; \\ x + 2, & x < 0. \end{cases}$$

(i) $\int_{-2}^0 f(x) dx$

(ii) $\int_{-2}^6 f(x) dx$

[13 markah]

7 Let $F(x) = \int_0^x \frac{t-3}{t^2+7} dt$, $-\infty < x < \infty$.

- (a) Find the value of x where F attains its minimum value.
- (b) Find the intervals over which F is only increasing or only decreasing.

[10 marks]

7 *Andaikan* $F(x) = \int_0^x \frac{t-3}{t^2+7} dt$, $-\infty < x < \infty$.

- (a) *Cari nilai untuk x sedemikian F mencapai nilai minimum.*
- (b) *Cari selang(atau selang-selang) supaya F hanya menaik atau hanya menyusut.*

[10 markah]

- 8 (a) Setup, without evaluating, the integral for the arc length of the curve $y = x^3$ from $x = 1$ to $x = 2$.
- (b) Setup, without evaluating, the integral for the volume of the solid generated by rotating about $y = 1$ the region bounded by $y = x^3$, $y = 1$ and $x = 0$.

[9 marks]

- 8 (a) *Bentukkan, tanpa menilai, kamiran untuk panjang lengkok bagi lengkung $y = x^3$ dari $x = 1$ ke $x = 2$.*
- (b) *Bentukkan, tanpa menilai, kamiran untuk isipadu bongkah yang terjana dengan memutar sekitar $y = 1$ rantau yang dibatasi oleh $y = x^3$, $y = 1$ dan $x = 0$.*

[9 markah]