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UNIVERSITI SAINS MALAYSIA

First Semester Examination  
2012/2013 Academic Session

January 2013

**MAT 203 – Vector Calculus**  
**[Kalkulus Vektor]**

Duration : 3 hours  
[Masa : 3 jam]

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Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions:** Answer **all ten** [10] questions.

**Arahan:** Jawab **semua sepuluh** [10] soalan.]

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].*

1. Given the points  $A(2,0,6)$  and  $B(2,-8,5)$   
Find
- (a) a vector  $\mathbf{v}$  representing the directed line segment  $AM$ .
  - (b) the midpoint  $M$  of the line segment  $AB$ , and the magnitude of  $\overline{AB}$ .
  - (c) a vector of length 8 in the direction apposite to that of  $\overline{AB}$ .
- [8 marks]

1. Diberi titik-titik  $A(2,0,6)$  dan  $B(2,-8,5)$   
Dapatkan
- (i) Suatu vector  $\mathbf{v}$  mewakili segmen garis berarah  $AB$ .
  - (ii) titik tengah  $M$  pada segmen garis  $AB$ , dan magnitud  $\overline{AM}$ .
  - (iii) suatu vektor yang panjang 8 pada arah bertentangan  $\overline{AB}$ .
- [8 markah]

2. Find the volume of the parallelepiped determine by the vectors,  $\mathbf{u} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$ ,  
 $\mathbf{v} = -\mathbf{i} - \mathbf{j}$  and  $\mathbf{w} = \mathbf{j} + \mathbf{k}$
- [5 marks]

- 2 Dapatkan isipadu parallelepiped yang ditentukan oleh vector-vektor  $\mathbf{u} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$ ,  
 $\mathbf{v} = -\mathbf{i} - \mathbf{j}$  and  $\mathbf{w} = \mathbf{j} + \mathbf{k}$
- [5 markah]

3. Find an equation of the plane that contains the points  $(3,0,1)$ ,  $(0,4,-2)$  and  
 $(2,-3,0)$ .
- [7 marks]

3. Dapatkan suatu persamaan satah yang mempunyai titik-titik  $(3,0,1)$ ,  $(0,4,-2)$  and  
 $(2,-3,0)$ .
- [7 markah]

4. (a) Let  $P$  be a point not on the line  $L$  that passes through the point  $Q$  and  $R$ . Show that the distance  $d$  from the point  $P$  to the line  $L$  is

$$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|}$$

where  $\mathbf{a} = \overrightarrow{QR}$  and  $\mathbf{b} = \overrightarrow{QP}$ .

- (b) Use the formula in part (a) to find the distance from the point  $P(1,1,1)$  to the line through  $Q(0,6,8)$  and  $R(-1,4,7)$ .

[10 marks]

4. (a) *Biarkan  $P$  suatu titik bukan di atas garis  $L$  yang melalui titik  $Q$  dan  $R$ . Tunjukkan bahawa jarak  $d$  daripada titik  $P$  kepada garis  $L$  ialah*

$$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|}$$

yang mana  $\mathbf{a} = \overrightarrow{QR}$  dan  $\mathbf{b} = \overrightarrow{QP}$ .

- (b) *Guna formula di bahagian (a) dapatkan jarak daripada titik  $P(1,1,1)$  kepada garis  $L$  yang melalui  $Q(0,6,8)$  and  $R(-1,4,7)$ .*

[10 markah]

5. The position vector of a moving object is given by

$$\mathbf{r}(t) = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + 10 \mathbf{k}, \quad 0 \leq t \leq 2\pi$$

Find the normal and tangential components of acceleration of the object.

[10 marks]

5. *Vektor kedudukan bagi suatu objek yang sedang bergerak diberi oleh*

$$\mathbf{r}(t) = 2 \cos t \mathbf{i} + 2 \sin t \mathbf{j} + 10 \mathbf{k}, \quad 0 \leq t \leq 2\pi$$

*Dapatkan komponen normal dan tangen kecepatan objek tersebut.*

[10 markah]

6. Consider the function  $f(x, y, z) = 4 - x^2 + 3y^2 + \frac{z^2}{2}$  and the point  $P(0, 2, -1)$ .

Find at the point  $P$

- (a) the gradient of  $f$ .
- (b) the unit vector in the direction of maximum increase of  $f$ .
- (c) the rate of change of  $f$  in the direction of maximum increase.
- (d) the directional derivative of  $f$  in the direction of the vector  $\mathbf{v} = \mathbf{j} - \mathbf{k}$ .

[12 marks]

6. *Pertimbangkan fungsi  $f(x, y, z) = 4 - x^2 + 3y^2 + \frac{z^2}{2}$  dan titik  $P(0, 2, -1)$ .*

*Dapatkan pada titik  $P$*

- (a) *gradien  $f$ .*
- (b) *unit vektor dalam arah menokok maksimum.*
- (c) *kadar perubahan bagi  $f$  dalam arah menokok maksimum.*
- (d) *pembezaan terarah bagi  $f$  dalam arah vektor  $\mathbf{v} = \mathbf{j} - \mathbf{k}$ .*

[12 markah]

7. Given a surface equation

$$2x^2 + y^2 + z^2 = 10$$

Find at the point  $A(3, 3, 5)$  the equation of

- (a) the tangent plane.
- (b) the normal line to the surface.

[11 marks]

7. *Diberi suatu persamaan permukaan*

$$2x^2 + y^2 + z^2 = 10$$

*Dapatkan pada titik  $A(3, 3, 5)$ , persamaan*

- (a) *satah tangen.*
- (b) *garis normal kepada permukaan tersebut.*

[11 markah]

- 8 A particle moves along the line segment from the point  $R(1,0,0)$  to the point  $S(3,4,2)$  by the vector field

$$\mathbf{F}(x, y, z) = (y + z) \mathbf{i} + (x + z) \mathbf{j} + (x + y) \mathbf{k}$$

Find

- (a) the vector function for the line segment  $RS$ .
- (b) the work done in moving the object.

[12 marks]

8. *Suatu benda bergerak sepanjang segmen garis daripada titik  $R(1,0,0)$  kepada titik  $S(3,4,2)$  oleh suatu vektor.*

$$\mathbf{F}(x, y, z) = (y + z) \mathbf{i} + (x + z) \mathbf{j} + (x + y) \mathbf{k}$$

*Dapatkan*

- (a) *fungsi vektor bagi segmen garis  $RS$ .*
- (b) *kerja yang dilakukan untuk menggerakkan benda tersebut.*

[12 markah]

9. (a) Show that  $\nabla \cdot \frac{\langle x, y, z \rangle}{|\mathbf{r}|^p} = \frac{3-p}{|\mathbf{r}|^p}$   
where  $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$  and  $p$  is a real number.
- (b) Compute the divergence and curl of the vector fields

$$\mathbf{F}(x, y, z) = \langle 2xy + z^4, x^2, 4xz^3 \rangle$$

[12 marks]

- 9 (a) *Tunjukkan bahawa  $\nabla \cdot \frac{\langle x, y, z \rangle}{|\mathbf{r}|^p} = \frac{3-p}{|\mathbf{r}|^p}$   
yang mana  $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$  and  $p$  ialah suatu nombor nyata.*
- (b) *Kirakan divergen dan kurl suatu medan vektor.*

$$\mathbf{F}(x, y, z) = \langle 2xy + z^4, x^2, 4xz^3 \rangle$$

[12 markah]

10. (a) Find the flux of the vector field

$$\mathbf{F} = \frac{\mathbf{r}}{|\mathbf{r}|}$$

across the sphere of radius  $a$  and centered at the origin, where  $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$ .

- (c) Given  $\mathbf{F}(x, y, z) = -z \mathbf{i} + x \mathbf{j} + y \mathbf{k}$ , and  $S$  is the hyperboloid  $z = 10 - \sqrt{1 + x^2 + y^2}$  for  $z \geq 0$ . Use Stoke's theorem to evaluate the surface integral. Assume that  $\bar{\mathbf{n}}$  is the outward normal vector.

[13 marks]

10. (a) Dapatkan fluk medan vektor

$$\mathbf{F} = \frac{\mathbf{r}}{|\mathbf{r}|}$$

merentasi suatu sfera berjejari  $a$  dan berpusat pada asalan, yang mana  $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$

- (c) Diberi  $\mathbf{F}(x, y, z) = -z \mathbf{i} + x \mathbf{j} + y \mathbf{k}$ , dan  $S$  ialah hiperboloid  $z = 10 - \sqrt{1 + x^2 + y^2}$  untuk  $z \geq 0$ . Gunakan teorem Stoke untuk menilai kamiran permukaan. Anggapkan  $\bar{\mathbf{n}}$  ialah vektor normal kepada permukaan.

[13 markah]