UNIVERSITI SAINS MALAYSIA

First Semester Examination 2012/2013
Academic Session
January 2013

MAA111 Algebra for Science Students
[Aljabar Untuk Pelajar Sains]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of 4 pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi 4 muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer all five [5] questions.

[Arahan: Jawab semua lima [5] soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

(1) (a) Prove that the product of two invertible matrices A and B of the same order n is invertible. (6 marks)

(b) Prove that

$$AB = AC \Rightarrow B = C$$

for any invertible matrix A. (4 marks)

(c) Solve by Cramer's rule:

$$x + y = 2,\ y + z = 2 \text{ and } x + z = 2.$$ (6 marks)

(d) Evaluate the determinant:

$$\begin{vmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{vmatrix}$$ (4 marks)

(1) (a) *Buktikan bahawa hasildarab dua matriks disongsangkan A dan B perintah yang sama n disongsangkan.* *(6 markah)*

(b) *Buktikan bahawa*

$$AB = AC \Rightarrow B = C$$

untuk sebarang matriks tersongsang A. *(4 markah)*

(c) *Menggunakan petua Cramer, selesaikan:*

$$x + y = 2,\ y + z = 2 \text{ dan } x + z = 2.$$ *(6 markah)*

(d) *Nilaikan penentu:*

$$\begin{vmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{vmatrix}$$ *(4 markah)*

(2) (a) Solve by Gauss-Jordan elimination method:
$x_1 + x_2 - 3x_3 = 0,\ x_2 - x_3 + x_4 = 0,\ x_1 + x_2 + x_3 + 4x_4 = 0.$ (8 marks)

(b) Solve by LU decomposition method:
$x_1 + x_2 = 1,\ x_1 + 2x_2 + x_3 = 0,\ 2x_1 + x_2 + x_3 = 1.$ (12 marks)

(2) (a) *Selesaikan dengan kaedah penghapusan Gauss-Jordan:*
$x_1 + x_2 - 3x_3 = 0,\ x_2 - x_3 + x_4 = 0,\ x_1 + x_2 + x_3 + 4x_4 = 0.$ *(8 markah)*

(b) *Selesaikan dengan kaedah penguraian LU:*
$x_1 + x_2 = 1,\ x_1 + 2x_2 + x_3 = 0,\ 2x_1 + x_2 + x_3 = 1.$ *(12 markah)*

(3) (a) (i) Define norm of a vector $\mathbf{u} \in \mathbb{R}^n$.

(ii) Show that $\|\lambda\mathbf{u}\| = |\lambda|\|\mathbf{u}\|$ for any $\mathbf{u} \in \mathbb{R}^n$ and any scalar λ.

(iii) Show that $\|\mathbf{u}+\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 + 2(\mathbf{u}\cdot\mathbf{v})$.
(iv) Assuming the Cauchy-Schwarz inequality $|\mathbf{u}\cdot\mathbf{v}| \le \|\mathbf{u}\|\|\mathbf{v}\|$, prove the triangle inequality $\|\mathbf{u}+\mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$.

(1+3+3+3 marks)

(b) Show that

$$0\mathbf{u} = \mathbf{0} \quad \text{and} \quad (-1)\mathbf{u} = -\mathbf{u}$$

for every element $\mathbf{u}$ of a vector space V. (5 marks)

(c) Show that the set of all solutions of the homogeneous linear system

$$x+y+z=0, \quad y-z=0$$

is a subspace of $\mathbb{R}^3$. (5 marks)

(3) (a) (i) *Takrifkan norma vektor* $\mathbf{u} \in \mathbb{R}^n$.
(ii) *Tunjukkan bahawa* $\|\lambda\mathbf{u}\| = |\lambda|\|\mathbf{u}\|$ *untuk sebarang* $\mathbf{u} \in \mathbb{R}^n$ *dan sebarang skalar* λ.
(iii) *Tunjukkan bahawa* $\|\mathbf{u}+\mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 + 2(\mathbf{u}\cdot\mathbf{v})$.
(iv) *Andaikan ketidaksamaan Cauchy-Schwarz* $|\mathbf{u}\cdot\mathbf{v}| \le \|\mathbf{u}\|\|\mathbf{v}\|$, *buktikan ketidaksamaan segitiga* $\|\mathbf{u}+\mathbf{v}\| \le \|\mathbf{u}\| + \|\mathbf{v}\|$.

(1+3+3+3 markah)

(b) *Tunjukkan bahawa*

$$0\mathbf{u} = \mathbf{0} \quad \textit{dan} \quad (-1)\mathbf{u} = -\mathbf{u}$$

bagi setiap elemen $\mathbf{u}$ *dari ruang vektor* V. *(4 markah)*

(c) *Tunjukkan bahawa set semua penyelesaian sistem linear homogen*

$$x+y+z=0, \quad y-z=0$$

adalah subruang $\mathbb{R}^3$. *(6 markah)*

(4) (a) (i) Define span of a subset S of a vector space V.
(ii) Show that $\{(1,0,0),(0,1,1),(1,0,1)\}$ spans $\mathbb{R}^3$.
(iii) Show that $\{(1,0,0),(0,1,0)\}$ does not span $\mathbb{R}^3$.
(iv) Define linear independence of a subset S of a vector space V.
(v) Show that $\{(1,0,0),(0,1,1),(1,0,1)\}$ is linearly independent subset of $\mathbb{R}^3$. (2+2+2+2+2 marks)

(b) (i) Prove the following: A subset $S = \{v_1, v_2, \cdots, v_k\}$ with two or more vectors of vector space V is linearly dependent if and only if at least one of the vector in S is a linear combination of other vectors in S.
(ii) Prove the following: A linear system $A\mathbf{x} = \mathbf{0}$ has only trivial solution if and only if the column vectors of A are linearly independent.

(5+5 marks)

(4) (a) (i) *Takrif rentangan bagi* S *pada ruang vektor* V.

(ii) *Tunjukkan bahawa* $\{(1,0,0),(0,1,1),(1,0,1)\}$ *merentang* $\mathbb{R}^3$.
(iii) *Tunjukkan bahawa* $\{(1,0,0),(0,1,0)\}$ *tidak merentang* $\mathbb{R}^3$.
(iv) *Takrifkan ketakbersandaran linear subset* S *pada ruang vektor* V.
(v) *Tunjukkan bahawa* $\{(1,0,0),(0,1,1),(1,0,1)\}$ *subset tak bersandar linear* $\mathbb{R}^3$.

(2+2+2+2+2 markah)

(b) (i) *Buktikan berikut: Subset* $S = \{v_1, v_2, \cdots, v_k\}$ *dengan dua atau lebih vektor dari ruang vektor* V *bersandar linear jika dan hanya jika sekurang-kurangnya satu vektor dalam* S *adalah gabungan linear vektor lain dalam* S.
(ii) *Buktikan berikut: Suatu sistem linear* $A\mathbf{x} = \mathbf{0}$ *hanya mempunyai penyelesaian remeh jika dan hanya jika vektor lajur* A *tak bersandar linear.*

(5+5 markah)

(5) (a) Diagonalize the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}.$$

(10 marks)

(b) Verify the Cayley-Hamilton theorem for the matrix

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

and find its inverse. (10 marks)

(5) (a) *Pepenjurukan matriks*

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}.$$

(10 markah)

(b) *Tentusahkan teorem Cayley-Hamilton untuk matriks*

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

dan cari songsangannya. *(10 markah)*