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UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang  
Sidang Akademik 2012/2013

Ogos 2013

**MAT 102 – Advanced Calculus**  
***[Kalkulus Lanjutan]***

Duration : 3 hours  
*[Masa : 3 jam]*

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Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

*[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]*

**Instructions:** Answer **all ten** [10] questions.

**Arahan:** Jawab **semua sepuluh** [10] soalan.]

In the event of any discrepancies, the English version shall be used.

*[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].*

1. Find the rate of change of  $f(x, y) = x^2y^3 - y^4$  at  $P(2, 1)$  from the direction of

$$\theta = \frac{\pi}{4}.$$

[6 marks]

1. Dapatkan kadar perubahan  $f(x, y) = x^2y^3 - y^4$  pada  $P(2, 1)$  dari arah  $\theta = \frac{\pi}{4}$ .

[6 markah]

2. Evaluate the integral  $\int_{-1}^1 \frac{dx}{2x+1}^{\frac{1}{3}}$ .

[6 marks]

2. Nilaikan kamiran  $\int_{-1}^1 \frac{dx}{2x+1}^{\frac{1}{3}}$ .

[6 markah]

3. Show that  $\lim_{x \rightarrow 0} e^x + x^{\frac{1}{x}} = e^2$ .

[7 marks]

3. Tunjukkan bahawa  $\lim_{x \rightarrow 0} e^x + x^{\frac{1}{x}} = e^2$ .

[7 markah]

4. Discuss the continuity of  $f$  at  $(0, 0)$ .

$$f(x, y) = \begin{cases} \frac{2x^2y}{x^4 + y^2} & , \quad x, y \neq 0, 0 \\ 0 & , \quad x, y = 0, 0 \end{cases}$$

[7 marks]

4. Bincangkan keselanjutan bagi  $f$  pada  $(0,0)$ .

$$f(x, y) = \begin{cases} \frac{2x^2y}{x^4 + y^2} & , \quad x, y \neq 0,0 \\ 0 & , \quad x, y = 0,0 \end{cases}$$

[7 markah]

5. Use a power series to approximate  $\int_0^{0.3} \frac{x^2}{1+x^4} dx$  to six decimal places.

[9 marks]

5. Gunakan siri kuasa untuk menganggarkan  $\int_0^{0.3} \frac{x^2}{1+x^4} dx$  kepada enam tempat perpuluhan.

[9 markah]

6. Determine whether the following series converges absolutely, converges conditionally or diverges?

(a)  $\sum_{k=1}^{\infty} \frac{\sin k + \cos k}{k^3}$                       (b)  $\sum_{k=1}^{\infty} \frac{k \ k+1}{k+2 \ k+3}$

[11 marks]

6. Tentukan samada siri berikut menumpu secara mutlak, menumpu secara bersyarat atau mencapah.

(a)  $\sum_{k=1}^{\infty} \frac{\sin k + \cos k}{k^3}$                       (b)  $\sum_{k=1}^{\infty} \frac{k \ k+1}{k+2 \ k+3}$

[11 markah]

7. Find the points on the sphere  $x^2 + y^2 + z^2 = 25$  where  $f(x, y, z) = x + 2y - 3z$  has its maximum and minimum values.

[12 marks]

7. Dapatkan titik-titik di sfera  $x^2 + y^2 + z^2 = 25$  yang mana  $f(x, y, z) = x + 2y - 3z$  mempunyai nilai maksimum dan minimum.

[12 markah]

8. Find all values of  $x$  for which  $\sum_{k=1}^{\infty} \frac{x+4^k}{k3^k}$  converges absolutely.

[13 marks]

8. Dapatkan semua nilai  $x$  yang mana  $\sum_{k=1}^{\infty} \frac{x+4^k}{k3^k}$  menumpu secara mutlak.

[13 markah]

9. (a) The kinetic energy of the body of mass  $m$  and velocity  $v$  is  $K = \frac{1}{2}mv^2$ .

Show that  $\frac{\partial K}{\partial m} \frac{\partial^2 K}{\partial v^2} = K$ .

(b) Show that  $f' u = \frac{1}{2} \left( \frac{1}{a} \frac{\partial z}{\partial t} + \frac{\partial z}{\partial x} \right)$  if  $z = f(x+at) + g(x-at)$ , where  $f$  and  $g$  are differentiable.

[14 marks]

9. (a) Tenaga kinetik badan yang jisimnya  $m$  dan halaju  $v$  adalah  $K = \frac{1}{2}mv^2$ .

Tunjukkan bahawa  $\frac{\partial K}{\partial m} \frac{\partial^2 K}{\partial v^2} = K$ .

(b) Tunjukkan bahawa  $f' u = \frac{1}{2} \left( \frac{1}{a} \frac{\partial z}{\partial t} + \frac{\partial z}{\partial x} \right)$  jika  $z = f(x+at) + g(x-at)$ ,  $f$  dan  $g$  terbezakan.

[14 markah]

10. Evaluate the following integrals.

(a)  $\int_0^{\pi/4} \int_x^{\pi/4} \frac{\sin y}{y} dy dx$

(b)  $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \sqrt{x^2+y^2} dz dy dx$  by changing to cylindrical coordinates.

[15 marks]

10. Nilakan kamiran berikut.

$$(b) \int_0^{\pi/4} \int_x^{\pi/4} \frac{\sin y}{y} dy dx$$

$$(b) \int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} \sqrt{x^2 + y^2} dz dy dx \text{ dengan menukar ke koordinat silinder.}$$

[15 markah]