UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
2012/2013 Sidang Akademik

Ogos 2013

MAT 101 Calculus
[Kalkulus]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>FIVE</u> pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>LIMA</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

<u>Instructions</u>: Answer **all eight** [8] questions.

*[<u>Arahan:</u> Jawab **semua lapan** [8] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. (a) Verify that $x = \frac{1}{2}$ is a solution of $|6x - 7| = |3 + 2x|$. Also find the other solution of $|6x - 7| = |3 + 2x|$.

   (b) Consider the function $f : \mathbb{R}^+ \to \mathbb{R}^+$ defined by $f(x) = x^2$, where $\mathbb{R}^+$ is the set of all nonnegative real numbers.
   (i) Show that f is one-to-one.
   (ii) Is f onto? Justify your answer.

[14 marks]

1. (a) *Tentusahkan bahawa $x = \frac{1}{2}$ ialah penyelesaian untuk $|6x - 7| = |3 + 2x|$. Juga carikan satu lagi penyelesaian untuk $|6x - 7| = |3 + 2x|$.*

   (b) *Pertimbangkan fungsi $f : \mathbb{R}^+ \to \mathbb{R}^+$ ditakrifkan sebagai $f(x) = x^2$, dengan $\mathbb{R}^+$ set semua nombor nyata tak negatif.*
   (i) Tunjukkan bahawa f satu ke satu.
   (ii) Adakah f keseluruh? Jelaskan jawapan anda.

[14 markah]

2. (a) Find the following limits without using the L'Hospital rule.
   (i) $\lim_{x \to 1} \frac{x^2 + 3x + 2}{6x^2 + x - 1}$ (ii) $\lim_{x \to \infty} \frac{2x - 1}{e^x(x+3)}$

   (b) (i) If $f(0) = 0$ and the limit $\lim_{x \to 0} \frac{f(x)}{x}$ exists, show that $\lim_{x \to 0} f(x) = 0$.
   (ii) Give an example that illustrates the result in (i) is still true even if the condition "$f(0) = 0$" is omitted.
   (iii) Is f in part (i) continuous? Give your reason.

[15 marks]

2. (a) *Cari had-had berikut tanpa menggunakan petua L'Hospital.*
   (i) $\lim_{x \to 1} \frac{x^2 + 3x + 2}{6x^2 + x - 1}$ (ii) $\lim_{x \to \infty} \frac{2x - 1}{e^x(x+3)}$

   (b) *(i) Jika $f(0) = 0$ dan had $\lim_{x \to 0} \frac{f(x)}{x}$ wujud, tunjukkan bahawa $\lim_{x \to 0} f(x) = 0$.*
   (ii) Beri satu contoh yang menunjukkan keputusan (i) masih benar walaupun syarat "$f(0) = 0$" diabaikan.
   (iii) Adakah f di bahagian (i) selanjar? Beri alasan anda.

[15 markah]

3.   Let f be a real function defined by

$$f(x) = \begin{cases} 1 + x, & x \le -1; \\ 1 - x^2, & -1 < x \le 2; \\ 1 - x, & x > 2. \end{cases}$$

(a) Show that f is continuous at -1 but not at 2.
(b) Is f differentiable at -1 and at 2? Give your reasons.

[15 marks]

3.   *Andaikan f fungsi nyata ditakrif sebagai*

$$f(x) = \begin{cases} 1 + x, & x \le -1; \\ 1 - x^2, & -1 < x \le 2; \\ 1 - x, & x > 2. \end{cases}$$

(a) Tunjukkan bahawa f selanjar pada -1 tapi tidak pada 2.
(b) Adakah f terbezakan pada -1 dan pada 2? Beri alasan anda.

[15 markah]

4.   Let $f(x) = \frac{x+1}{x-1}$.
(a) What are the values of $f(0)$ and $f(2)$?
(b) Show that there is no value of c such that $f'(c) = \frac{f(2)-f(0)}{2}$.
Why does this not contradict the Mean Value Theorem?
(c) Can we conclude by the Intermediate Value Theorem that there exists α in the interval $(0,2)$ such that $f(\alpha) = 0$? Give your reason.

[10 marks]

4.   *Andaikan $f(x) = \frac{x+1}{x-1}$.*
(a) Apakah nilai-nilai untuk $f(0)$ dan $f(2)$?
(b) Tunjukkan bahawa tiada nilai c sedemikian $f'(c) = \frac{f(2)-f(0)}{2}$.
Mengapa ini tak bercanggah dengan Teorem Nilai Min?
(c) Bolehkah kita menyimpulkan dengan Teorem Nilai Pertengahan bahawa wujud α dalam selang $(0,2)$ sedemikian $f(\alpha) = 0$? Beri alasan anda.

[10 markah]

5. (a) Evaluate the following integrals.
   (i) $\int \frac{2x^2+x+3}{x^2}\,dx$
   (ii) $\int_1^1 x^9\sqrt{1+x^{10}}\,dx$

   (b) Given that $\int_4^9 \sqrt{f(x)}\,dx = \frac{38}{3}$, find $1 + \int_9^4 \sqrt{f(t)}\,dt$.

   (c) (i) Express the definite integral $\int_0^\pi \sin(5x)\,dx$ as a limit of sum.
   (ii) Express the limit of sum $\lim_{n\to\infty}\sum_{k=1}^n \frac{k^2}{n^3}$ as a definite integral.

   [15 marks]

5. (a) *Nilaikan kamiran berikut.*
   (i) $\int \frac{2x^2+x+3}{x^2}\,dx$
   (ii) $\int_1^1 x^9\sqrt{1+x^{10}}\,dx$

   (b) *Diberi* $\int_4^9 \sqrt{f(x)}\,dx = \frac{38}{3}$, *cari* $1 + \int_9^4 \sqrt{f(t)}\,dt$.

   (c) (i) *Ungkapkan kamiran tentu* $\int_0^\pi \sin(5x)\,dx$ *sebagai satu had hasil tambah.*
   (ii) *Ungkapkan had* $\lim_{n\to\infty}\sum_{k=1}^n \frac{k^2}{n^3}$ *sebagai satu kamiran tentu.*

   [15 markah]

6. (a) Setup, without evaluating, the integral for the arc length of the curve $y = x^3$ from $x = 1$ to $x = 2$.

   (b) Setup, without evaluating, the integral for the volume of the solid generated by rotating about $y = 1$ the region bounded by $y = x^3$, $y = 1$ and $x = 0$.

   [10 marks]

6. (a) *Bentukkan, tanpa menilai, kamiran untuk panjang lengkok bagi lengkung* $y = x^3$ *dari* $x = 1$ *ke* $x = 2$.

   (b) *Bentukkan, tanpa menilai, kamiran untuk isipadu bongkah yang terjana dengan memutar sekitar* $y = 1$ *rantau yang dibatasi oleh* $y = x^3$, $y = 1$ *dan* $x = 0$.

   [10 markah]

7. (a) Find $F'(t)$ if
   (i) $F(t) = \int_0^1 [1 + \sqrt{\sin(x^2)}]\, dx$;   (ii) $F(t) = \int_{t^2}^1 \sqrt{1 + x^3}\, dx$, $t \in [1, 2]$.

   (b) By considering the function F in part (ii), determine whether F is increasing or decreasing on the interval $[1, 2]$. Give you reason.

                                                                              [8 marks]

7. *(a) Cari $F'(t)$ jika*
   (i) $F(t) = \int_0^1 [1 + \sqrt{\sin(x^2)}]\, dx$;   (ii) $F(t) = \int_{t^2}^1 \sqrt{1 + x^3}\, dx$, $t \in [1, 2]$.

   (b) Dengan mempertimbangkan fungsi F di bahagian (ii), tentukan sama ada F menokok atau menyusut pada selang $[1, 2]$. Beri alasan anda.

                                                                              [8 markah]

8. (a) If $f(x) = 3 + x + e^x$, find $f'(4)$ and $(f^{-1})'(4)$.

   (b) (i) A function f is *odd* if $f(-x) = -f(x)$. Show that the function $f(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$ is odd.
   (ii) Find the inverse function of f in part (i).

                                                                              [14 marks]

8. *(a) Jika $f(x) = 3 + x + e^x$, cari $f'(4)$ dan $(f^{-1})'(4)$.*

   (b) (i) Satu fungsi f adalah ganjil *jika $f(-x) = -f(x)$. Tunjukkan bahawa fungsi $f(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$ adalah ganjil.*
   (ii) Cari fungsi songsang untuk f di bahagian (i).

                                                                              [14 markah]

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