UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2011/2012 Academic Session

June 2012

MSG 265 – Design and Analysis of Experiments
[Rekabentuk dan Analisis Uji Kaji]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIFTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) List two advantages of factorial design.

[20 marks]

(b) A soft drink bottler is interested in obtaining more uniform fill heights in the bottles produced by his manufacturing process. He would like to study the sources of the variability and to reduce it. There are three variables that can be controlled during the filling process: the percent carbonation (X), the operating pressure in the filler (Y) and the line speed (Z). The factor levels are as shown:

	Factor levels	
Factors	Low	High
X (%)	10	12
Y (psi)	25	30
Z (b/m)	200	250

Suppose that only four runs could be made on each shift. Hence the process engineer set up a 2^3 design in two blocks with two replicates. The data, fill height deviation, obtained is as follows:

Replicate I		Replicate II	
Block 1	Block 2	Block 1	Block 2
$(1) = -3$	$xyz = 6$	$(1) = -1$	$xyz = 5$
$xz = 2$	$x = 0$	$xz = 1$	$x = 1$
$xy = 2$	$y = -1$	$xy = 3$	$y = 0$
$yz = 1$	$z = -1$	$yz = 1$	$z = 0$

(i) Show that this is a 2^3 design in two blocks with *XYZ* confounded with blocks.

(ii) Construct an ANOVA table. Conduct an analysis of variance and comment on your findings.

(iii) Construct a 2^3 design with *XYZ* confounded in the first replicate and *YZ* confounded in the second. Construct the design and outline the analysis of variance.

(iv) Name the design mentioned in part (iii) and state the advantage of using this design.

[80 marks]

1. (a) *Nyatakan dua kelebihan reka bentuk faktorial.*

[20 markah]

(b) *Pengusaha minuman ringan berminat untuk mendapatkan pengisian ketinggian minuman yang lebih seragam dalam botol yang dihasilkan melalui proses pembuatan di kilangnya. Pengusaha tersebut berminat untuk mengkaji punca ubahan dan mengurangkannya. Terdapat tiga pemboleh ubah yang dikawal semasa proses pengisian: peratus karbonasi (X), tekanan semasa operasi (Y) dan kelajuan baris (Z). Tahap setiap faktor diberikan seperti di bawah:*

	Tahap faktor	
Faktor	Rendah	Tinggi
X (%)	10	12
Y (psi)	25	30
Z (b/m)	200	250

Diandaikan bahawa hanya empat larian dapat dilakukan pada setiap syif. Maka, jurutera proses menetapkan reka bentuk 2^3 *dalam dua blok dengan dua replika. Data bagi sisihan ketinggian minuman yang diisi adalah seperti berikut:*

Replika I		Replika II	
Blok 1	Blok 2	Blok 1	Blok 2
$(1) = -3$	$xyz = 6$	$(1) = -1$	$xyz = 5$
$xz = 2$	$x = 0$	$xz = 1$	$x = 1$
$xy = 2$	$y = -1$	$xy = 3$	$y = 0$
$yz = 1$	$z = -1$	$yz = 1$	$z = 0$

(i) *Tunjukkan bahawa ini adalah reka bentuk* 2^3 *dalam dua blok dengan XYZ terbaur dengan blok.*

(ii) *Bina jadual ANOVA. Jalankan analisis varians dan komen dapatan anda.*

(iii) *Bina reka bentuk* 2^3 *dengan pembauran XYZ dalam replika pertama dan pembauran YZ dalam replika kedua. Bangunkan reka bentuk dan tuliskan analisis varians.*

(iv) *Namakan reka bentuk yang dinyatakan dalam bahagian (iii) dan nyatakan kebaikan menggunakan reka bentuk tersebut.*

[80 markah]

2. An experiment is conducted to study the effect of coating machine, operators and material type on the density of a coated fabric. Each of the four operators (Bob, Mary, Sam and Ben) ran each of the two machines in the plant (machines 15 and 21) with both types of material (A and B). Two repeated tests were obtained for each of the process conditions. Data from the study are shown in the table below.

Machine 15			Machine 21		
Operator	Material	Density	Operator	Material	Density
Bob	A	190.1	Bob	A	209.4
Bob	B	182.0	Bob	B	194.2
Mary	A	200.8	Mary	A	224.6
Mary	B	196.0	Mary	B	189.1
Sam	A	205.6	Sam	A	202.1
Sam	B	191.8	Sam	B	183.1
Ben	A	183.5	Ben	A	191.8
Ben	B	170.5	Ben	B	186.7
Bob	A	179.4	Bob	A	216.2
Bob	B	177.8	Bob	B	188.3
Mary	A	191.0	Mary	A	210.1
Mary	B	180.7	Mary	B	186.1
Sam	A	198.3	Sam	A	207.2
Sam	B	206.9	Sam	B	198.0
Ben	A	182.1	Ben	A	184.4
Ben	B	187.5	Ben	B	187.5

The sums of squares are given below:

$SS_{Total} = 4633.475$ $SS_{Machine*Operator} = 530.965$

$SS_{Machine} = 567.845$ $SS_{Machine*Material} = 283.220$

$SS_{Operator} = 1064.545$ $SS_{Operator*Material} = 287.710$

$SS_{Material} = 907.380$ $SS_{Machine*Operator*Material} = 172.690$

(i) State the linear statistical model for this experiment.

(ii) Analyze the data and state the conclusion. Use $\alpha = 0.05$.

(iii) Analyze the residuals for the model obtained in part (ii). (Refer to the Appendix)

(iv) Obtain the point estimates of the mean density for each machine and material types.

(v) Assume that the replicates are used as blocking variables. Present the ANOVA table for the full model of the three-factor factorial design. Compare the results obtained in part (v) with part (ii).

[100 marks]

2. *Satu eksperimen dilakukan bagi mengkaji kesan salutan mesin, operator dan jenis bahan terhadap kepadatan fabrik. Setiap operator (Bob, Mary, Sam dan Ben) melarikan dua mesin (mesin 15 dan 21) dengan dua jenis bahan (A dan B). Uji kaji diulang sebanyak dua kali pada setiap keadaan proses. Data daripada kajian tersebut diberikan dalam jadual di bawah:*

Mesin 15			*Mesin 21*		
Operator	*Bahan*	*Kepadatan*	*Operator*	*Bahan*	*Kepadatan*
Bob	*A*	*190.1*	*Bob*	*A*	*209.4*
Bob	*B*	*182.0*	*Bob*	*B*	*194.2*
Mary	*A*	*200.8*	*Mary*	*A*	*224.6*
Mary	*B*	*196.0*	*Mary*	*B*	*189.1*
Sam	*A*	*205.6*	*Sam*	*A*	*202.1*
Sam	*B*	*191.8*	*Sam*	*B*	*183.1*
Ben	*A*	*183.5*	*Ben*	*A*	*191.8*
Ben	*B*	*170.5*	*Ben*	*B*	*186.7*
Bob	*A*	*179.4*	*Bob*	*A*	*216.2*
Bob	*B*	*177.8*	*Bob*	*B*	*188.3*
Mary	*A*	*191.0*	*Mary*	*A*	*210.1*
Mary	*B*	*180.7*	*Mary*	*B*	*186.1*
Sam	*A*	*198.3*	*Sam*	*A*	*207.2*
Sam	*B*	*206.9*	*Sam*	*B*	*198.0*
Ben	*A*	*182.1*	*Ben*	*A*	*184.4*
Ben	*B*	*187.5*	*Ben*	*B*	*187.5*

Hasil tambah kuasa dua diberikan di bawah:

$SS_{Jumlah} = 4633.475$ $SS_{Mesin*Operator} = 530.965$

$SS_{Mesin} = 567.845$ $SS_{Mesin*Bahan} = 283.220$

$SS_{Operator} = 1064.545$ $SS_{Operator*Bahan} = 287.710$

$SS_{Bahan} = 907.380$ $SS_{Mesin*Operator*Bahan} = 172.690$

(i) Nyatakan model linear statistik bagi uji kaji ini.

(ii) Jalankan analisis data dan nyatakan kesimpulan anda. Guna $\alpha = 0.05$.

(iii) Jalankan analisis reja bagi model yang diperoleh dalam bahagian (ii). (Rujuk lampiran)

(iv) Dapatkan anggaran titik bagi min kepadatan untuk setiap mesin dan jenis bahan.

(v) Andaikan replika digunakan sebagai pemboleh ubah blok. Berikan jadual ANOVA bagi model penuh reka bentuk faktorial tiga faktor. Bandingkan keputusan yang diperoleh dalam bahagian (v) dengan bahagian (ii).

[100 markah]

3. (a) In an experiment on the preparation of chocolate cakes, three recipes for preparing the batter were compared. In addition, six different baking temperatures were tested: 175°, 185°, 195°, 205°, 215° and 225°. Each time that the mix was made by any recipe, enough batter was prepared for six cakes, each of which was baked at a different temperature. Thus, the recipes are the whole plot treatments, while the baking temperatures are the subplot treatments. There were 15 replications, treated as blocks. The data consists of measurements made on the texture of the cakes produced. The sum of squares of the effects and the total are given in the table below.

Effects	Sum of squares
Blocks	10,204
Recipes	135
Temperatures	2,100
Blocks x Recipes	1,199
Recipes x Temperatures	206
Total	18,143

(i) State the experimental design used and write down the appropriate model for this design. Construct the ANOVA table.
(ii) Which effects are significant? State your conclusions.

[50 marks]

(b) The effects of strength (A) and time (B) on the density of photographic plate film are being studied. Three strengths and three times are used. A single replicate experiment is run.

(i) What is the name of the design used?
(ii) Draw the geometric view with all treatment combinations for the design chosen in (i).
(iii) If all the treatment combinations involved in this experiment cannot be run at one time, confounding in blocks is often necessary. How many blocks do you need to run the experiment? List the assignment of the treatment combinations in each block. Use factor AB as the confounding effect.

[50 marks]

3. (a) *Dalam satu uji kaji terhadap penyediaan kek coklat, tiga resipi penyediaan adunan telah dibanding. Sebagai tambahan, enam suhu membakar kek telah diuji:* 175°, 185°, 195°, 205°, 215° *and* 225°. *Setiap kali campuran dibuat mengikut sebarang resipi, adunan dibuat secukupnya untuk enam biji kek, setiap satu dibakar pada suhu yang berlainan. Oleh itu, resipi adalah rawatan plot keseluruhan, sementara suhu adalah rawatan subplot. Terdapat 15 replika yang dijalankan dan dianggap sebagai blok. Data terdiri daripada ukuran yang dibuat terhadap tekstur kek yang dimasak. Hasil tambah kuasa dua kesan dan hasil tambah kuasa dua jumlah diberi dalam jadual di bawah:*

Kesan	*Hasil tambah kuasa dua*
Blok	*10,204*
Resipi	*135*
Suhu	*2,100*
Blok x Resipi	*1,199*
Resipi x Suhu	*206*
Jumlah	*18,143*

(i) *Namakan reka bentuk uji kaji yang digunakan dan tuliskan model yang bersesuaian dengan reka bentuk tersebut. Bina jadual ANOVA yang berkaitan.*

(ii) *Kesan manakah yang bererti? Nyatakan kesimpulan anda.*

[50 markah]

(b) *Kesan kekuatan (A) dan masa (B) terhadap kepadatan piring filem dikaji. Tiga kekuatan dan tiga masa digunakan. Eksperimen replika tunggal dilarikan.*

(i) *Apakah nama reka bentuk uji kaji yang digunakan?*

(ii) *Lukis gambaran geometri dengan semua kombinasi rawatan bagi reka bentuk yang dipilih dalam (i).*

(iii) *Sekiranya semua kombinasi rawatan yang terlibat dalam eksperimen ini tidak dapat dilakukan dalam satu masa, pembauran dalam blok adalah perlu. Berapakah bilangan blok yang diperlukan bagi menjalankan eksperimen ini? Nyatakan pengumpukkan kombinasi rawatan bagi setiap blok. Gunakan faktor AB sebagai kesan pembauran.*

[50 markah]

4. An experiment was conducted to determine which factors were affecting the color of a product produced by a chemical process. A team of employees identified five factors that were considered to be possible candidates in reducing variation in the product color. These factors and the levels chosen for consideration are given in the table below:

Factors		Levels	
	Name	Low (-1)	High (+1)
A	Solvent	Low	High
B	Catalyst	0.025	0.035
C	Temperature	150	160
D	Reactant Purity	92	96
E	Reactant pH	8.0	8.7

A half-fraction of a 2^5 factorial experiment was selected, but no repeated test runs could be included. The results obtained were as follow:

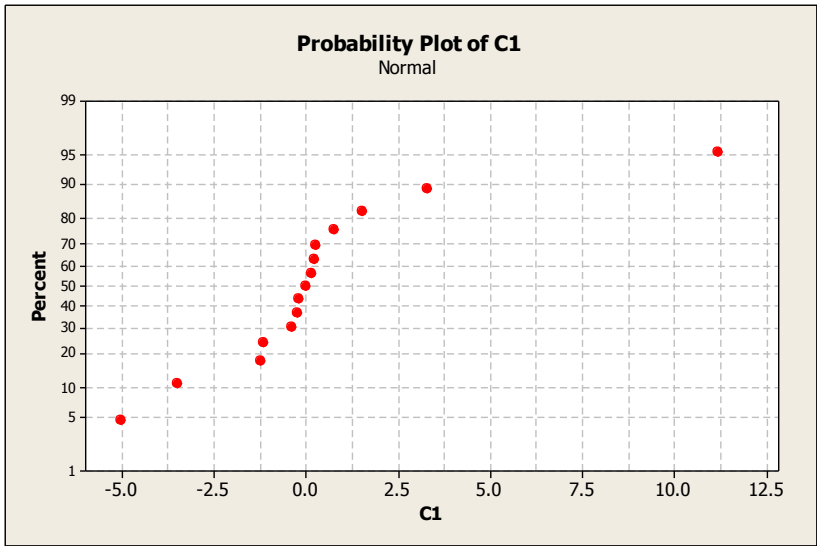
Factors					Color Measurement
Solvent	Catalyst	Temperature	Purity	pH	
1	-1	1	1	-1	24.76
1	1	-1	1	-1	17.36
1	-1	-1	1	1	18.94
-1	1	-1	-1	-1	0.64
1	1	-1	-1	1	2.68
-1	-1	1	1	1	16.44
1	1	1	-1	-1	9.86
-1	1	1	1	-1	14.60
-1	-1	-1	1	-1	19.58
-1	-1	-1	-1	1	4.74
1	-1	-1	-1	-1	11.02
-1	-1	1	-1	-1	10.12
-1	1	-1	1	1	12.90
-1	1	1	-1	1	1.82
1	1	1	1	1	14.10
1	-1	1	-1	1	8.44

(i) What type of design has been used? What is the resolution of this design? Identify the defining relation and the alias relationships.

(ii) List all the treatment combinations that were run in this experiment.

(iii) The table below shows the estimates of the factor effects and its normal probability plot.

$A = 3.29$	$AB = 0.22$	$BD = -0.18$
$B = -5.01$	$AC = 0.255$	$BE = 0.745$
$C = 1.535$	$AD = -0.38$	$CD = -1.255$
$D = 11.17$	$AE = -1.225$	$CE = -1.15$
$E = -3.485$	$BC = 0.165$	$DE = 0.005$



Based on the above normal probability plot, which factors can be identified as important. Perform an appropriate statistical analysis to test the hypothesis that the factors identified have a significant effect in reducing variation in the product color.

(iv) Fit a model that could be used to predict the variation in the product color in terms of the factors that you have identified as important in part (iii).

(v) Find the predicted variation in the product color at treatment combination number 2 (*abd*) and find its residual. Use the coded levels.

(vi) Show how the runs of the alternate fraction can be constructed.

[100 marks]

4. *Satu eksperimen dijalankan bagi mengenalpasti faktor-faktor yang memberi kesan terhadap warna produk yang dihasilkan melalui proses kimia. Sekumpulan pekerja telah mengenalpasti lima faktor yang berkemungkinan dapat mengurangkan variasi dalam warna produk. Faktor dan aras yang dipilih disenaraikan dalam jadual di bawah:*

	Faktor	Aras	
	Nama	Rendah (-1)	Tinggi (+1)
A	Pelarut	Rendah	Tinggi
B	Katalis	0.025	0.035
C	Suhu	150	160
D	Ketulenan Reaktan	92	96
E	pH Reaktan	8.0	8.7

Pecahan separa bagi eksperimen 2^5 *faktoran dipilih dan tiada uji kaji ulangan dilakukan. Keputusan yang diperoleh adalah seperti berikut:*

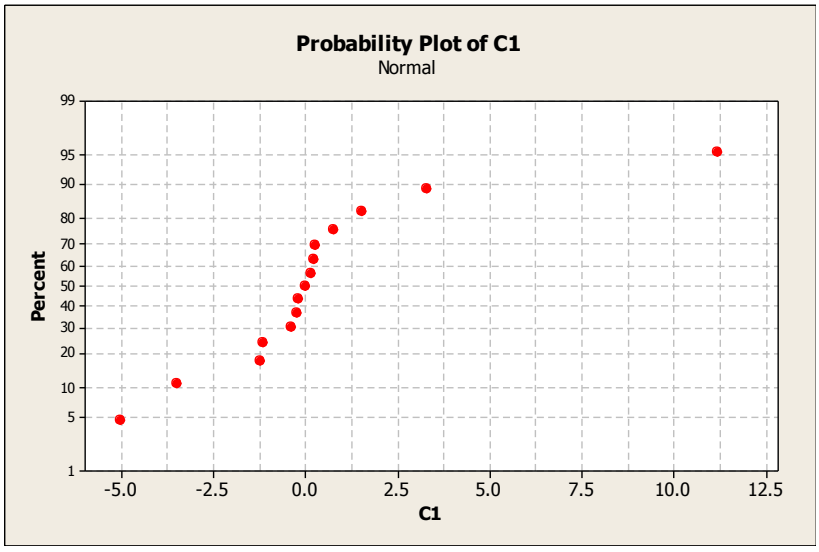
Faktor					Pengukuran
Pelarut	Katalis	Suhu	Ketulenan	pH	Warna
1	-1	1	1	-1	24.76
1	1	-1	1	-1	17.36
1	-1	-1	1	1	18.94
-1	1	-1	-1	-1	0.64
1	1	-1	-1	1	2.68
-1	-1	1	1	1	16.44
1	1	1	-1	-1	9.86
-1	1	1	1	-1	14.60
-1	-1	-1	1	-1	19.58
-1	-1	-1	-1	1	4.74
1	-1	-1	-1	-1	11.02
-1	-1	1	-1	-1	10.12
-1	1	-1	1	1	12.90
-1	1	1	-1	1	1.82
1	1	1	1	1	14.10
1	-1	1	-1	1	8.44

(i) Apakah jenis reka bentuk yang diguna? Apakah resolusi reka bentuk ini? Camkan hubungan pentakrif dan hubungan aliasnya.

(ii) Senaraikan semua gabungan rawatan yang dilarikan dalam eksperimen ini.

(iii) Jadual berikut menunjukkan anggaran kesan faktor dan plot kebarangkalian normal bagi anggaran-anggaran tersebut.

$A = 3.29$	$AB = 0.22$	$BD = -0.18$
$B = -5.01$	$AC = 0.255$	$BE = 0.745$
$C = 1.535$	$AD = -0.38$	$CD = -1.255$
$D = 11.17$	$AE = -1.225$	$CE = -1.15$
$E = -3.485$	$BC = 0.165$	$DE = 0.005$



Berdasarkan plot kebarangkalian normal di atas, faktor manakah yang boleh dikenalpasti sebagai penting. Jalankan analisis statistik yang sesuai untuk menguji bahawa faktor-faktor yang dikenalpasti mempunyai kesan bererti terhadap variasi dalam warna produk yang dihasilkan.

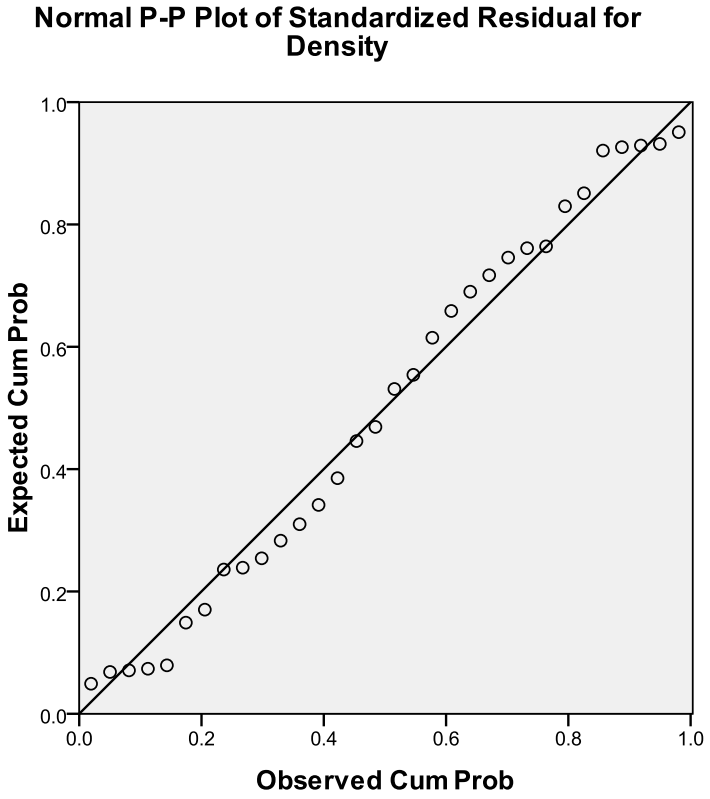
(iv) Suaikan satu model regresi yang boleh diguna untuk meramal variasi warna produk yang dihasilkan dalam sebutan faktor-faktor yang telah anda kenalpasti sebagai penting dalam bahagian (iii).

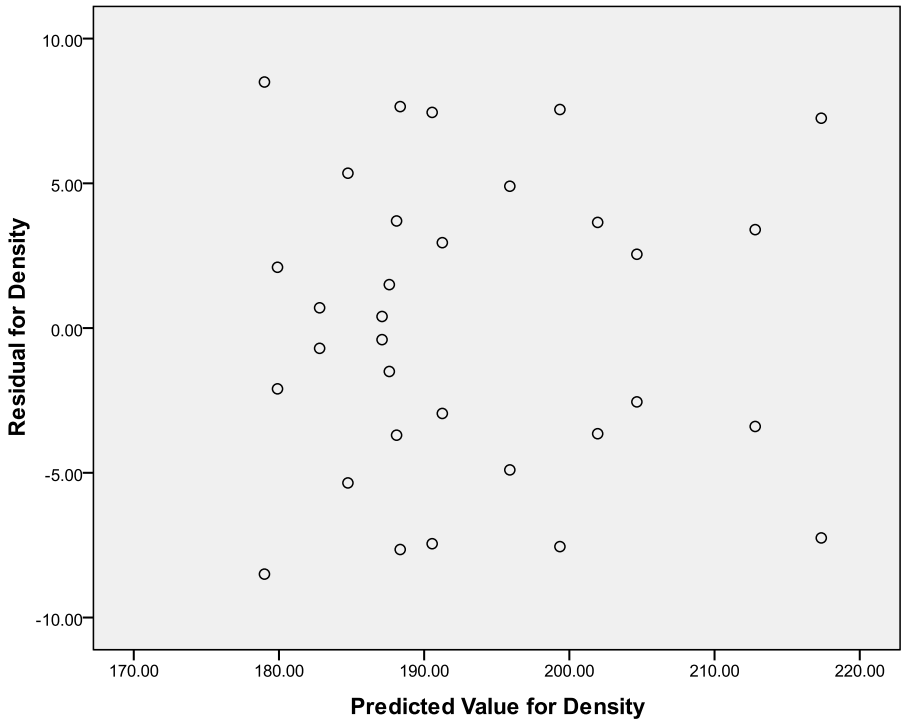
(v) Dapatkan ramalan variasi dalam warna produk pada larian nombor 2 (abd) dan dapatkan rejanya. Guna aras yang telah dikod.

(vi) Tunjukkan bagaimana larian bagi pecahan kedua reka bentuk ini boleh dibina.

[100 markah]

Appendix


Normal P-P Plot of Standardized Residual for Density
Expected Cum Prob
Observed Cum Prob
0.0
0.2
0.4
0.6
0.8
1.0


Residual for Density
Predicted Value for Density
10.00
5.00
0.00
-5.00
-10.00
170.00
180.00
190.00
200.00
210.00
220.00

...13/-

Formula

1. Two-factor Factorial Design

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \sum_{i=1}^{a} \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$SS_B = \sum_{j=1}^{b} \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{abn}$$

$$SS_{Subtotal} = \sum_{i=1}^{a}\sum_{j=1}^{b} \frac{y_{ij.}^2}{n} - \frac{y_{...}^2}{abn}$$

$$SS_{AB} = SS_{Subtotal} - SS_A - SS_B$$

$$SS_E = SS_T - SS_{Subtotal} \quad \text{atau} \quad SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

Expected Mean Square
Random Effects Model:

$$E[MS_A] = \sigma^2 + n\sigma_{\tau\beta}^2 + bn\sigma_{\tau}^2$$

$$E[MS_B] = \sigma^2 + n\sigma_{\tau\beta}^2 + an\sigma_{\beta}^2$$

$$E[MS_{AB}] = \sigma^2 + n\sigma_{\tau\beta}^2$$

$$E[MS_E] = \sigma^2$$

Expected Mean Square
Mixed Model (*A*: fixed, *B*: random):

$$E[MS_A] = \sigma^2 + n\sigma_{\tau\beta}^2 + \frac{bn\sum_{i=1}^{a}\tau_i^2}{a-1}$$

$$E[MS_B] = \sigma^2 + an\sigma_{\beta}^2$$

$$E[MS_{AB}] = \sigma^2 + n\sigma_{\tau\beta}^2$$

$$E[MS_E] = \sigma^2$$

2. Three-factor Factorial Design

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\sum_{l=1}^{n} y_{ijkl}^2 - \frac{y_{....}^2}{abcn}$$

$$SS_A = \sum_{i=1}^{a}\frac{y_{i...}^2}{bcn} - \frac{y_{....}^2}{abcn}$$

$$SS_B = \sum_{j=1}^{b}\frac{y_{.j..}^2}{acn} - \frac{y_{....}^2}{abcn}$$

$$SS_C = \sum_{k=1}^{c}\frac{y_{..k.}^2}{abn} - \frac{y_{....}^2}{abcn}$$

$$SS_{Subtotal(AB)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\frac{y_{ij..}^2}{cn} - \frac{y_{....}^2}{abcn}$$

$$SS_{AB} = SS_{Subtotal(AB)} - SS_A - SS_B$$

$$SS_{Subtotal(AC)} = \sum_{i=1}^{a}\sum_{k=1}^{c}\frac{y_{i.k.}^2}{bn} - \frac{y_{....}^2}{abcn}$$

$$SS_{AC} = SS_{Subtotal(AC)} - SS_A - SS_C$$

$$SS_{Subtotal(BC)} = \sum_{j=1}^{b}\sum_{k=1}^{c}\frac{y_{.jk.}^2}{an} - \frac{y_{....}^2}{abcn}$$

$$SS_{BC} = SS_{Subtotal(BC)} - SS_B - SS_C$$

$$SS_{Subtotal(ABC)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}\frac{y_{i.k.}^2}{bn} - \frac{y_{....}^2}{abcn}$$

$SS_E = SS_T - SS_{Subtotal}$ atau $SS_E = SS_T - SS_A - SS_B - SS_{AB}$

3. Two-stage Nested Design

$$SS_T = \sum_i\sum_j\sum_k y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \sum_{i=1}^{a}\frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$SS_{B(A)} = \sum_{i=1}^{a}\sum_{j=1}^{b}\frac{y_{ij.}^2}{n} - \sum_{i=1}^{a}\frac{y_{i..}^2}{bn}$$

$$SS_E = SS_T - SS_A - SS_{B(A)}$$

4. Expected Mean Square
Split-Plot Design

$$\tau_i : \sigma^2 + ab\sigma_\tau^2$$

$$\beta_j : \sigma^2 + b\sigma_{\tau\beta}^2 + \frac{rb\sum \beta_j^2}{a-1}$$

$$\gamma_k : \sigma^2 + \frac{ra\sum \gamma_k^2}{ab-1}$$

$$\tau\beta_{ij} : \sigma^2 + b\sigma_{\tau\beta}^2$$

$$\beta\gamma_{jk} : \sigma^2 + \frac{r\sum\sum \beta\gamma_{jk}^2}{a-1 \quad b-1}$$

$$\varepsilon_{ijk} : \sigma^2$$

- ooo O ooo -