UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2011/2012 Academic Session

June 2012

MAT 516 – Curve and Surface Methods for CAGD
[Kaedah Lengkung dan Permukaan untuk RGBK]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Let $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$. Show that

(i) $\dfrac{r}{n}\binom{n}{r} = \binom{n-1}{r-1}$

(ii) $\sum_{j=0}^{n} \dfrac{j}{n} B_j^n(t) = t$ where $B_j^n(t) = \binom{n}{j}(1-t)^{n-j} t^j : 0 \le t \le 1.$

(b) Given $B_j^n(t) = \binom{n}{j}(1-t)^{(n-j)} t^j : 0 \le t \le 1.$

(i) Show that $(1-t)B_{j+1}^n(t) + tB_j^n(t) = B_{j+1}^{n+1}(t)$.

(ii) If $\boldsymbol{p}(t) = \sum_{j=0}^{n} B_j^n(t)\boldsymbol{p}_j$ show that $\boldsymbol{p}'(0) = n(\boldsymbol{p}_1 - \boldsymbol{p}_0)$

(iii) Indicate the process of evaluating the point $\boldsymbol{p}(0.5)$ of a cubic Bezier curve with control points $\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{p}_2$ and $\boldsymbol{p}_3$ using de Casteljau algotithm.

[25 marks]

1. (a) Andai $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$. *Tunjukkan bahawa*

(i) $\dfrac{r}{n}\binom{n}{r} = \binom{n-1}{r-1}$

(ii) $\sum_{j=0}^{n} \dfrac{j}{n} B_j^n(t) = t$ *dengan* $B_j^n(t) = \binom{n}{j}(1-t)^{n-j} t^j : 0 \le t \le 1.$

(b) Diberi $B_j^n(t) = \binom{n}{j}(1-t)^{(n-j)} t^j : 0 \le t \le 1.$

(i) Tunjukkan bahawa $(1-t)B_{j+1}^n(t) + tB_j^n(t) = B_{j+1}^{n+1}(t)$.

(ii) Jika $\boldsymbol{p}(t) = \sum_{j=0}^{n} B_j^n(t)\boldsymbol{p}_j$ *tunjukkan bahawa* $\boldsymbol{p}'(0) = n(\boldsymbol{p}_1 - \boldsymbol{p}_0)$

(iii) Tunjukkan proses untuk menilai titik $\boldsymbol{p}(0.5)$ *suatu lengkung Bezier kubik dengan titik kawalan* $\boldsymbol{p}_0, \boldsymbol{p}_1, \boldsymbol{p}_2$ *dan* $\boldsymbol{p}_3$ *dengan algoritma de Casteljau.*

[25 markah]

2. (a) Given a line joining the points $0,0$ and $10,10$, and a parabola

$$\boldsymbol{r}\ \ t\ = 2t\ \ 1-t\ \ \ 0,10\ + t^2\ \ 5,0\ \ ,\ 0 \le t \le 1.$$

(i) Determine the points of intersection between the line and the parabola.

(ii) Determine the point on the parabola such that the tangent vector at this point is parallel to the vector defining the straight line.

(b) A line joining two points $\boldsymbol{P}$ and $\boldsymbol{Q}$ is given in the rational form as

$$\boldsymbol{r}\ \ t\ = \frac{1-t\ \ \alpha \boldsymbol{P} + t\beta \boldsymbol{Q}}{1-t\ \ \alpha + t\beta}, 0 \le t \le 1.$$

(i) Give an example of the values of such that the point $\boldsymbol{r}(0.5)$ divides the segment $\boldsymbol{PQ}$ in the ratio of 4:3.

(ii) Determine the location of $\boldsymbol{r}(0.5)$ when both α and β approach to infinity simultaneusly.

(c) The quadratic function $f\ \ u\ = 2u^2 - 3u - 1,\ 2 \le \mathrm{u} \le 5$ can be rewritten as

$$\mathrm{f}\ \ \mathrm{t}\ = (1-\mathrm{t})^2 \mathrm{f}_0 + 2\ \ 1-\mathrm{t}\ \ \mathrm{t}\mathrm{f}_1 + \mathrm{t}^2 \mathrm{f}_2,\ \text{when } \mathrm{t} = \frac{\mathrm{u}-2}{3}.$$

Find $\mathrm{f}_0, \mathrm{f}_1,$ and f_2.

[25 marks]

2. (a) Diberi suatu garis menghubongkan $0,0$ *dan* $10,10$, *dan suatu parabola*

$$\boldsymbol{r}\ \ t\ = 2t\ \ 1-t\ \ \ 0,10\ + t^2\ \ 5,0\ \ ,\ 0 \le t \le 1.$$

(i) Tentukan titik persilangan diantara garis dengan parabola.

(ii) Tentukan titik pada parabola supaya vektor tangen pada titik ini adalah selari denagn vektor yang menakrifkan garis lurus.

(b) Suatu garis menghubongkan dua titik $\boldsymbol{P}$ *dan* $\boldsymbol{Q}$ *adalah diberi dalam bentuk nisbah*

$$\boldsymbol{r}\ \ t\ = \frac{1-t\ \ \alpha \boldsymbol{P} + t\beta \boldsymbol{Q}}{1-t\ \ \alpha + t\beta}, 0 \le t \le 1.$$

(i) Beri suatu contoh nilai supaya titik $\boldsymbol{r}(0.5)$ *membahagikan segmen* $\boldsymbol{PQ}$ *dalam nisbah 4:3.*

(ii) Tentukan kedudukan titik $\boldsymbol{r}(0.5)$ *bila kedua dua* α dan β *secara serentak menumpu ke infiniti.*

(c) Fungsi kuadratik $f\ \ u\ = 2u^2 - 3u - 1,\ 2 \le \mathrm{u} \le 5$ *boleh ditulis semula sebagai*

$$f\ \ t\ = (1-t)^2 f_0 + 2\ \ 1-t\ \ tf_1 + t^2 f_2\ ,\ bila\ t = \frac{u-2}{3}.$$

Cari $f_0, f_1,$ *dan* f_2 .

[25 markah]

3. (a) A rational quadratic curve is defined as

$$\boldsymbol{r}(t) = \frac{(1-t)^2\boldsymbol{P}_0 + 2(1-t)tw\boldsymbol{H} + t^2\boldsymbol{P}_1}{(1-t)^2 + 2(1-t)tw + t^2}, 0 \le t \le 1, w \text{ is positive weight.}$$

Let $\boldsymbol{P}_0 = (-1,0)$, $\boldsymbol{H} = (0,\tan\alpha)$, $\boldsymbol{P}_1 = (1,0)$ and the angle $< P_0P_1H = \alpha$.

(i) If the curve represents a circular arc, then show that $w = cos\,\alpha.$

(ii) Determine the absolute value of the curvature $\boldsymbol{r}(t)$ in terms of α

(iii) If tends to $\frac{\pi}{2}$, determine the limiting value of w and the location of the point $\boldsymbol{H}$.

(b) Let the curve

$$\boldsymbol{r}(t) = \frac{(1-t)^2(1-2t)\boldsymbol{P}_0 + 4(1-t)^2tv\boldsymbol{Q}_1 + 4(1-t)t^2v\boldsymbol{Q}_2 + t^2(2t-1)\boldsymbol{P}_1}{(1-t)^2(1-2t) + 4(1-t)^2tv + 4(1-t)t^2v + t^2(2t-1)}$$

be a semicircle.

(i) If $\boldsymbol{P}_0 = (-1,0)$ and $\boldsymbol{P}_1 = (1,0)$, find the points $\boldsymbol{Q}_1$ and $\boldsymbol{Q}_2$.

(ii) Show that $v = \frac{1}{2}$.

(c) A quadratic function on the interval $2 \le x \le 5$ can be written as

$$f(t) = \frac{(1-t)^2y_0 + 2(1-t)tvh + t^2y_1}{(1-t)^2 + 2(1-t)tv + t^2}, \text{ where } t = \frac{x-2}{3}.$$

If the value of slope at $x = 2$ is d_0, write h in terms of y_0, d_0 and v.

[25 marks]

3. (a) Suatu lengkung nisbah kuadratik ditakrifkan sebagai

$$\boldsymbol{r}(t) = \frac{(1-t)^2\boldsymbol{P}_0 + 2(1-t)tw\boldsymbol{H} + t^2\boldsymbol{P}_1}{(1-t)^2 + 2(1-t)tw + t^2}, 0 \le t \le 1, w \text{ ialah pemberat positif}.$$

Biar $\boldsymbol{P}_0 = (-1,0)$, $\boldsymbol{H} = (0,\tan\alpha)$, $\boldsymbol{P}_1 = (1,0)$ *dan sudut* $< P_0P_1H = \alpha$.

(i) Jika lengkung mewakili suatu lengkuk bulatan, maka tunjukkan bahawa $w = cos\,\alpha.$

(ii) Tentukan nilai mutlak kelengkungan $\boldsymbol{r}(t)$ *dalam sebutan* α.

(iii) Jika nilai α *menuju* $\frac{\pi}{2}$, *tentukan nilai tumpuan* w *dan kedudukan titik* $\boldsymbol{H}$.

(b) *Andai lengkung*

$$\boldsymbol{r}(t) = \frac{(1-t)^2(1-2t)\boldsymbol{P}_0 + 4(1-t)^2 tv\boldsymbol{Q}_1 + 4(1-t)t^2 v\boldsymbol{Q}_2 + t^2(2t-1)\boldsymbol{P}_1}{(1-t)^2(1-2t) + 4(1-t)^2 tv + 4(1-t)t^2 v + t^2(2t-1)}$$

sebagai suatu semi bulatan.

(i) *Jika* $\boldsymbol{P}_0 = (-1,0)$ *dan* $\boldsymbol{P}_1 = (1,0)$, *dapatkan titik* $\boldsymbol{Q}_1$ *dan* $\boldsymbol{Q}_2$.

(ii) *Tunjukkan bahawa* $v = \frac{1}{2}$.

(c) *Suatu fungsi kuadratik pada selang* $2 \le x \le 5$ *boleh titulis sebagai*

$$f(t) = \frac{(1-t)^2 y_0 + 2(1-t)tvh + t^2 y_1}{(1-t)^2 + 2(1-t)tv + t^2}, \text{ dengan } t = \frac{x-2}{3}.$$

Jika nilai kecerunan pada $x = 2$ *ialah* d_0, *ungkapkan* h *dalam sebutan* y_0, d_0 *dan v.*

[25 markah]

4. (a) Let two curves be given as

$$\boldsymbol{r}_1(t) = (1-t)\boldsymbol{P}_0 + t\boldsymbol{P}_1, \text{ and}$$
$$\boldsymbol{r}_2(t) = (1-t)^3\boldsymbol{P}_1 + 3(1-t)^2 t\boldsymbol{P}_2 + 3(1-t)t^2\boldsymbol{P}_3 + t^3\boldsymbol{P}_4,\ 0 \le t \le 1.$$

Show that if the points $\boldsymbol{P}_1, \boldsymbol{P}_2$ and $\boldsymbol{P}_3$ are collinear then the curves are joined with curvature continuity.

(b) Let a surface be defined parametrically as

$$S(u,v) = (u+v^2, uv, u^2+v-1), \text{ where } 0 \le u,v \le 2$$

(i) Evaluate the point $S(1,1)$.

(ii) Determine the equation of tangent plane at $u = v = 1$.

(c) A B-spline quadratic curve with knot vector $1,1,1,2,4,4,4$ and control points $\{\boldsymbol{P}_0,\boldsymbol{P}_1,\boldsymbol{P}_2,\boldsymbol{P}_3\}$ is given by $\boldsymbol{p}\ \ t\ = \sum_{i=0}^{3} N_{i,2}(t)\boldsymbol{P}_i$

where $N_{i,d}\ \ t$ is defined by

$$N_{i,0} = \begin{cases} 1 & \text{if} \quad t \in [t_{i,} \ t_{i+1}) \\ 0 & \text{elsewhere} \end{cases}$$

$$N_{i,d}\ \ t\ = \frac{t-t_i}{t_{i+d}-t_i} N_{i,d-1}\ \ t\ + \frac{t_{i+d+1}-t}{t_{i+d+1}-t_{i+1}} N_{i+1,d-1}\ \ t\ .$$

(i) Evaluate $N_{0,2}\ \ 1$ and $N_{1,2}\ \ 1$

(ii) Express $\boldsymbol{p}\ \ 1$ in terms of the control points.

[25 marks]

4. *(a)* *Andaikan dua lengkung diberi sebagai*

$$\boldsymbol{r}_1\ \ t\ =\ 1-t\ \ \boldsymbol{P}_0 + t\boldsymbol{P}_1, \ dan$$
$$\boldsymbol{r}_2\ \ t\ =\ 1-t\ ^3 \boldsymbol{P}_1 + 3\ 1-t\ ^2 t\boldsymbol{P}_2 + 3\ 1-t\ t^2\boldsymbol{P}_3 + t^3\boldsymbol{P}_4,\ 0 \le t \le 1.$$

Tunjukkan jika $\boldsymbol{P}_1,\boldsymbol{P}_2$ *dan* $\boldsymbol{P}_3$ *adalah segaris tunjukkan dua garis bersambung secara keselanjaran kelengkungan.*

(b) *Biarkan suatu permukaan ditakrif secara berparameter*

$$S\ \ u,v\ = (u+v^2, uv, u^2+v-1),\ dengan\ 0 \le u,v \le 2$$

(i) *Nilaikan titik* $S(1,1)$.

(ii) *Tentukan persamaan satah tangen pada* $u = v = 1$.

(c) *Suatu lengkung splin B kuadratik dengan vector knot* $1,1,1,2,4,4,4$ *dan titik kawalan* $\{\boldsymbol{P}_0,\boldsymbol{P}_1,\boldsymbol{P}_2,\boldsymbol{P}_3\}$ *diberi sebagai* $\boldsymbol{p}\ \ t\ = \sum_{i=0}^{3} N_{i,2}(t)\boldsymbol{P}_i$

dengan $N_{i,d}\ \ t$ *ditakrifkan sebagai*

$$N_{i,0} = \begin{cases} 1 & jika \quad t \in [t_{i,} \ t_{i+1}) \\ 0 & sebaliknya \end{cases}$$

$$N_{i,d}\ \ t\ = \frac{t-t_i}{t_{i+d}-t_i} N_{i,d-1}\ \ t\ + \frac{t_{i+d+1}-t}{t_{i+d+1}-t_{i+1}} N_{i+1,d-1}\ \ t\ .$$

(i) *Nilaikan* $N_{0,2}\ \ 1$ dan $N_{1,2}\ \ 1$

(ii) *Ungkapkan* $\boldsymbol{p}\ \ 1$ *dalam sebuatan titik kawalan.*

[25 markah]

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