UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2011/2012 Academic Session

June 2012

MGM 562 – Probability Theory
[Teori Kebarangkalian]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all eight** [8] questions.

*[**Arahan:** Jawab **semua lapan** [8] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. There are 97 men and 3 women in an organization. A committee of 5 people is chosen at random, and one of these 5 is randomly designated as a chairperson.
   (a) What is the probability that the committee includes all 3 women and has one of the women as chairperson?
   (b) What is the expectation and variance of the number of women chosen in this committee?

[15 marks]

1. Terdapat 97 lelaki dan 3 wanita di dalam sesuatu organisasi. Satu jawatankuasa seramai 5 orang dipilih secara rawak, dan seorang daripada lima ahli itu dilantik sebagai pengerusi.
   (a) Apakah kebarangkalian jawatankuasa itu termasuk 3 orang wanita dan salah seorang daripadanya adalah pengerusi?
   (b) Apakah jangkaan dan varians bagi bilangan wanita yang dipilih di dalam jawatankuasa tersebut?

[15 markah]

2. Let A, B, C and D be the events such that $B = A'$, $C \cap D = \varnothing$, and $\Pr(A) = 1/4$, $\Pr(B) = 3/4$, $\Pr(C \mid A) = 1/2$, $\Pr(C \mid B) = 3/4$, $\Pr(D \mid A) = 1/4$ and $\Pr(D \mid B) = 1/8$. Calculate
   (a) $\Pr(C \cup D)$.
   (b) $\Pr(B \mid D)$.

[10 marks]

2. Biarkan A, B, C dan D suatu peristiwa yang mana $B = A'$, $C \cap D = \varnothing$, dan $Kb(A) = 1/4$, $Kb(B) = 3/4$, $Kb(C \mid A) = 1/2$, $Kb(C \mid B) = 3/4$, $Kb(D \mid A) = 1/4$ dan $Kb(D \mid B) = 1/8$. Kira
   (a) $Kb(C \cup D)$.
   (b) $Kb(B \mid D)$.

[10 markah]

3. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \frac{1}{9}x(4-x) & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the mean and variance of *X*.
(b) Use Markov's Theorem to show that the probability that *X* is greater than 2 is less than 0.875.
(c) Find the probability that *X* is greater than 2.

[15 marks]

3. Biarkan X menjadi pemboleh ubah rawak dengan fungsi ketumpatan kebarangkalian

$$f(x) = \begin{cases} \frac{1}{9}x(4-x) & 0 < x < 3 \\ 0 & \textit{selainnya} \end{cases}$$

(a) Cari min dan varians bagi X.
(b) Gunakan Teorem Markov untuk menunjukkan bahawa kebarangkalian X lebih besar daripada 2 adalah kurang daripada 0.875.
(c) Cari kebarangkalian X lebih besar daripada 2.

[15 markah]

4. Let X have the density function

$$f(x) = \begin{cases} \frac{2x}{k^2} & 0 < x < k \\ 0 & \text{otherwise} \end{cases}$$

(a) For what value of k is the variance of X equal to 2?
(b) Let X_1 and X_2 be two independent, identically distributed random variables each with density function as given above. Let $Y = \max(X_1, X_2)$. Find $\Pr[Y > E(X)]$.

[10 marks]

4. Biarkan X mempunyai fungsi taburan

$$f(x) = \begin{cases} \frac{2x}{k^2} & 0 < x < k \\ 0 & \textit{selainnya} \end{cases}$$

(a) Apakah nilai k supaya varians X bersamaan dengan 2?
(b) Biarkan X_1 dan X_2 sebagai dua pemboleh ubah rawak yang merdeka dan sepunya, dengan fungsi taburan di atas. Biarkan $Y = maks(X_1, X_2)$. Cari $Kb[Y > E(X)]$.

[10 markah]

5. (a) If X has a Poisson distribution with mean 5, what is the value of $E(X^2)$? Find the probability that X is less than 6 given that it is greater than or equal to 2.

(b) Let X have a Poisson distribution with mean 10. If $Y = 2X + 3$, find $\ln[M_Y(t)]$ where $M_Y(t)$ is the moment generating function of Y.

[15 marks]

5. (a) Jika X mempunyai taburan Poisson dengan min 5, apakah nilai $E(X^2)$*? Cari kebarangkalian X lebih kecil daripada 6 diberi bahawa ia adalah lebih besar atau sama dengan 2.*

(b) Biarkan X mempunyai taburan Poisson dengan min 10. Jika Y = 2X + 3, cari $\ln[M_Y(t)]$ *yang mana* $M_Y(t)$ *adalah fungsi penjana momen bagi Y.*

[15 markah]

6. Job 1 takes X minutes to complete, where X is modelled as a $N(28, 2^2)$ random variable. Job 2, independent of Job 1, takes Y minutes to complete, and begins 5 minutes after Job 1 begins. Y is modelled as a $N(25, 1^2)$ random variable.

(a) If Job 1 and Job 2 both complete at the same time, find the relationship between X and Y.

(b) Calculate the probability that Job 1 be the last to be completed.

[10 marks]

6. Tugasan 1 mengambil masa X minit untuk disiapkan, yang mana X dimodelkan sebagai taburan rawak $N(28, 2^2)$*. Tugasan 2, yang tidak bersandaran dengan Tugasan 1, mengambil masa Y minit untuk diselesaikan, dan bermula 5 minit selepas Tugasan 1 bermula. Y dimodelkan sebagai taburan rawak* $N(25, 1^2)$*.*

(a) Jika Tugasan 1 dan Tugasan 2 kedua-duanya selesai pada masa yang sama, cari kaitan antara X dan Y.

(b) Kira kebarangkalian bahawa Tugasan 1 adalah yang terakhir diselesaikan.

[10 markah]

7. A wheel spun has equal probabilities of $\frac{1}{3}$ for the numbers 1, 2 and 3 appeared each. If number 1 appears, the player gets a score of 1.0; if number 2 appears, the player gets a score of 2.0; if number 3 appears, the player gets a score of X, where X is a normal random variable with mean 3 and standard deviation 1.

(a) If W represents the player's score on 1 spin of the wheel and A represents the random variable of wheel spun, then define the respective conditional probabilities, $\Pr(W = w \mid A = a)$ for $A = 1, 2, 3$.

(b) What is $\Pr(W \leq 1.5)$?

(c) Find the expectation and variance of the score.

[15 marks]

7. Suatu roda yang diputarkan mempunyai kebarangkalian yang sama iaitu $\frac{1}{3}$ bagi dengan nombor-nombor 1, 2 dan 3 setiap satu. Jika nombor 1 muncul, pemain itu memperolehi skor 1.0; jika nombor 2 muncul, dia memperolehi skor 2.0; jika nombor 3 muncul, dia memperolehi skor X, yang mana X adalah taburan rawak normal dengan min 3 dan sisihan piawai 1.

(a) Jika W mewakili skor pemain untuk satu putaran roda tersebut, dan A mewakili pemboleh ubah rawak bagi putaran roda, nyatakan kebarangkalian bersyarat masing-masing, iaitu $Kb(W = w \mid A = a)$ untuk $A = 1, 2, 3$.

(b) Apakah $Kb(W \leq 1.5)$?

(c) Cari jangkaan dan varians bagi skor tersebut.

[15 markah]

8. Let $X_1, X_2, ..., X_N$ be independent random variables, each having a gamma distribution with mean 4 and variance 4. Let $S = X_1 + X_2 + ... + X_N$. If N is an integer random variable having binomial distributed with mean 10, and variance 8

(a) Find the mean and variance of S

(b) Calculate an approximate value for the probability S exceeds 35.

[Use normal approximation to calculate the probabilities]

[10 marks]

8. Biarkan $X_1, X_2, ..., X_N$ sebagai pemboleh ubah rawak tidak bersandar, setiap satu mempunyai taburan Gamma dengan min 4 dan varians 4. Biarkan $S = X_1 + X_2 + ... + X_N$. Jika N ialah pemboleh ubah rawak integer taburan Binomial dengan min 10, dan varians 8

(a) Cari min dan varians bagi S

(b) Kira nilai hampir bagi kebarangkalian S lebih daripada 35.

[Gunakan penghampiran normal untuk mengira kebarangkalian tersebut]

[10 markah]

APPENDIX / ***LAMPIRAN***

$E\ Y\ = E\left[E\ Y \mid X\ \right]$ $Var\ Y\ = E\left[Var\ Y \mid X\ \right] + Var\left[E\ Y \mid X\ \right]$ For $S \mid N = X_1 + X_2 + \ldots + X_N$ $E\ S\ = E\left[NE\ X\ \right]$ $Var\ S\ = E\left[NVar\ X\ \right] + Var\left[NE\ X\ \right]$	$Y \sim Hypergeometric\ \ A, B, n, N = A + B$ $\Pr\ Y = y\ = \begin{cases} \dfrac{\binom{A}{n}\binom{B}{n-y}}{\binom{N}{n}} & y = 0,1,\ldots,\ \min\ n, A \\ 0 & otherwise \end{cases}$
$Y \sim binomial\ \ n, p$ $\Pr\ Y = y\ = \begin{cases} \binom{n}{y} p^y \times 1 - p\ ^{n-y} & y = 0,1,\ldots,n \\ 0 & otherwise \end{cases}$ $E\ Y\ = np$ and $Var\ Y\ = np\ 1 - p$ $M_Y\ t\ = \ pe^t + 1 - p\ ^n$	$Y \sim Poisson\ \ m$ $\Pr\ Y = y\ = \begin{cases} \dfrac{e^{-m} m^y}{y!} & y = 0,1,2,\ldots \\ 0 & otherwise \end{cases}$ $E\ Y\ = Var\ Y\ = m$ $M_Y\ t\ = e^{m\ e^t - 1}$
$Y \sim Normal\ \ m, s^2$ $f_Y\ Y\ = \begin{cases} \dfrac{1}{\sqrt{2\pi s^2}} \times \exp\left[-\dfrac{1}{2}\left(\dfrac{y-m}{s}\right)^2\right] & -\infty < y < \infty \\ 0 & o/w \end{cases}$ $E\ Y\ = m$ and $Var\ Y\ = s^2$ $M_Y\ t\ = \exp\left(mt + \dfrac{1}{2}s^2 t^2\right)$	$Y \sim gamma\ \ \alpha, \theta$ $f_Y\ Y\ = \begin{cases} \dfrac{y^{\alpha-1} e^{-y/\theta}}{\Gamma\ \alpha\ \theta^\alpha} & y > 0 \\ 0 & otherwise \end{cases}$ $E\ Y\ = \alpha\theta$ and $Var\ Y\ = \alpha\theta^2$ $M_Y\ t\ = \ 1 - \theta t\ ^{-\alpha}$

-ooo000ooo-