UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2011/2012 Academic Session

June 2012

MSG 284 – Introduction to Geometric Modelling
[Pengenalan kepada Pemodelan Geometri]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all three** [3] questions.

*[**Arahan:** Jawab **semua tiga** [3] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. (a) Let $\boldsymbol{P}(s)$ be a regular curve in terms of the arc length parameter $s \geq 0$. Show that the unit tangent $\boldsymbol{T}$, the principal normal $\boldsymbol{N}$ and the binormal $\boldsymbol{B}$ of curve $\boldsymbol{P}$ are mutually orthogonal.

(b) Given a circle $x^2 + y^2 - 2x - 3 = 0$, rewrite the circle in standard form and then evaluate its curvature.

(c) Find a polynomial function that interpolates the points $(1,3)$, $(2,2)$ and $(4,1)$.

(d) Let $\boldsymbol{P}(u)$ be a piecewise curve composed of quadratic polynomial $\boldsymbol{F}(u)$ and cubic polynomial $\boldsymbol{G}(u)$ as

$$\boldsymbol{P}(u) = x(u), y(u) = \begin{cases} \boldsymbol{F}(u), & 0 \leq u \leq 1 \\ \boldsymbol{G}(u), & 1 < u \leq 3 \end{cases}$$

on the xy-plane. With $x(u) = u + 1$, determine the $\boldsymbol{F}$ and $\boldsymbol{G}$ in terms of u such that the curve $\boldsymbol{P}$ is G^2 geometrically continuous at $u = 1$ and satisfies

$$\boldsymbol{P}(0) = (1,3),$$
$$\boldsymbol{P}(1) = (2,2),$$
$$\boldsymbol{P}(2) = (4,1)$$

and

$$\frac{\mathrm{d}}{\mathrm{d}u}\boldsymbol{P}(1) = (1,0).$$

[100 marks]

1. (a) *Katakan* $\boldsymbol{P}(s)$ *ialah satu lengkung nalar berparamater panjang lengkok* $s \geq 0$. *Tunjukkan bahawa tangen unit* $\boldsymbol{T}$, *normal prinsipal* $\boldsymbol{N}$ *dan binormal* $\boldsymbol{B}$ *bagi lengkung* $\boldsymbol{P}$ *adalah saling berkeserenjangan.*

   (b) *Diberi bulatan* $x^2 + y^2 - 2x - 3 = 0$, *tulis semula bulatan ini dalam bentuk piawai kemudian nilaikan kelengkungannya.*

   (c) *Cari fungsi polinomial yang menginterpolasi titik-titik* $(1,3)$, $(2,2)$ *dan* $(4,1)$.

   (d) *Katakan* $\boldsymbol{P}(u)$ *ialah satu lengkung bercebisan yang digubah dengan polinomial kuadratik* $\boldsymbol{F}(u)$ *dan polinomial kubik* $\boldsymbol{G}(u)$ *sebagai*

$$\boldsymbol{P}(u) = x(u), y(u) = \begin{cases} \boldsymbol{F}(u), & 0 \leq u \leq 1 \\ \boldsymbol{G}(u), & 1 < u \leq 3 \end{cases}$$

pada satah- xy. *Dengan* $x(u) = u + 1$, *tentukan* $\boldsymbol{F}$ *dan* $\boldsymbol{G}$ *dalam sebutan* u *supaya lengkung* $\boldsymbol{P}$ *adalah selanjar secara bergeometric* G^2 *pada* $u = 1$ *dan memenuhi*

$$\boldsymbol{P}(0) = (1,3),$$

$$\boldsymbol{P}(1) = (2,2),$$

$$\boldsymbol{P}(2) = (4,1)$$

dan

$$\frac{\mathrm{d}}{\mathrm{d}u}\boldsymbol{P}(1) = (1,0).$$

[100 markah]

2. (a) Let the Bernstein polynomial of degree n be denoted as

$$B_i^n(t) = \frac{n!}{i!(n-i)!} t^i \left(1-t\right)^{n-i}, \qquad 0 \le t \le 1, \qquad \text{for } i = 0, 1, \ldots, n.$$

Given $\sum_{i=0}^{n} B_i^n(t) = 1$, show that $\sum_{i=0}^{n} i B_i^n(t) = nt$.

(b) Find a cubic Bézier function that matches with $y(x) = 1 + 2x^2$, $0 \le x \le 1$.

(c) Given a quadratic Bézier curve

$$\boldsymbol{P}(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} B_0^2(t) + \begin{pmatrix} 2 \\ 3 \end{pmatrix} B_1^2(t) + \begin{pmatrix} 3 \\ 2 \end{pmatrix} B_2^2(t), \qquad 0 \le t \le 1,$$

where $B_i^2(t)$, $i = 0, 1, 2$, are the Bernstein polynomials of degree 2. Suppose the curve is truncated to a small curve segment in the parametric interval $0.25 \le t \le 0.75$, find the three new Bézier points defining that curve segment.

(d) Given a functional biquadratic Bézier surface

$$z(x, y) = \sum_{i=0}^{2} \sum_{j=0}^{2} C_{i,j} B_j^2(y) B_i^2(x), \qquad 0 \le x, y \le 1,$$

where $B_s^2(t)$, $0 \le t \le 1$, $s = 0, 1, 2$, are the Bernstein polynomials of degree 2 and $C_{i,j}$ are the Bézier ordinates given as

$$\begin{array}{lll} C_{0,0} = 2, & C_{0,1} = 0, & C_{0,2} = 2, \\ C_{1,0} = 1, & C_{1,1} = -1, & C_{1,2} = 0, \\ C_{2,0} = 2, & C_{2,1} = 1, & C_{2,2} = 2. \end{array}$$

Determine the unit normal vector of tangent plane to the surface z at $(x, y) = (0.5, 0.5)$.

[100 marks]

2. (a) *Katakan polinomial Bernstein berdarjah* n *ditanda sebagai*

$$B_i^n(t) = \frac{n!}{i!\,(n-i)!}\, t^i \left(1-t\right)^{n-i}, \qquad 0 \le t \le 1, \qquad \text{untuk } i = 0, 1, \ldots, n.$$

Diberi $\sum_{i=0}^{n} B_i^n(t) = 1$, *tunjukkan bahawa* $\sum_{i=0}^{n} i B_i^n(t) = nt$.

(b) *Cari satu fungsi Bézier kubik yang memadankan* $y(x) = 1 + 2x^2, \quad 0 \le x \le 1$.

(c) *Diberi satu lengkung Bézier kuadratik*

$$\boldsymbol{P}(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} B_0^2(t) + \begin{pmatrix} 2 \\ 3 \end{pmatrix} B_1^2(t) + \begin{pmatrix} 3 \\ 2 \end{pmatrix} B_2^2(t), \qquad 0 \le t \le 1,$$

di mana $B_i^2(t)$, $i = 0, 1, 2$, *adalah polinomial Bernstein berdarjah* 2. *Andaikan lengkung ini disingkatkan kepada segmen lengkung kecil dalam selang* $0.25 \le t \le 0.75$, *cari tiga titik Bézier baru yang menakrif segmen lengkung berkenaan.*

(d) *Diberi satu permukaan Bézier dwikuadratik fungsian*

$$z(x, y) = \sum_{i=0}^{2} \sum_{j=0}^{2} C_{i,j}\, B_j^2(y)\, B_i^2(x), \qquad 0 \le x, y \le 1,$$

di mana $B_s^2(t)$, $0 \le t \le 1$, $s = 0, 1, 2$, *adalah polinomial Bernstein berdarjah* 2 *dan* $C_{i,j}$ *ialah ordinat Bézier diberikan sebagai*

$$C_{0,0} = 2, \qquad C_{0,1} = 0, \qquad C_{0,2} = 2,$$
$$C_{1,0} = 1, \qquad C_{1,1} = -1, \qquad C_{1,2} = 0,$$
$$C_{2,0} = 2, \qquad C_{2,1} = 1, \qquad C_{2,2} = 2.$$

Tentukan vektor unit normal bagi satah tangen kepada permukaan z *pada* $(x, y) = (0.5, 0.5)$.

[100 markah]

3. (a) Let $\boldsymbol{u} = (u_0, u_1, \ldots, u_{n+k})$ be a non-decreasing knot vector where n and k are positive integers. The normalized B-spline basis functions of order k are defined recursively by

$$N_i^1(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

and

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad \text{for } k > 1,$$

where $i = 0, 1, \ldots, n$. Given a B-spline curve of order 3

$$\boldsymbol{P}(u) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} N_0^3(u) + \begin{pmatrix} 1 \\ 2 \end{pmatrix} N_1^3(u) + \begin{pmatrix} 2 \\ 2 \end{pmatrix} N_2^3(u) + \begin{pmatrix} 2 \\ 1 \end{pmatrix} N_3^3(u), \qquad u_2 \le u \le u_4 .$$

(i) Suppose $\boldsymbol{u} = (0, 1, 2, \ldots, 6)$, find the local support of function $N_0^3(u)$. Next, show that $\boldsymbol{P}(u)$ is a C^1 continuous curve.

(ii) Suppose $\boldsymbol{u} = (0, 1, 1, 2, 3, 3, 4)$, find the curve point $\boldsymbol{P}$ at $u = 2$.

(b) Consider a bilinearly blended Coons patch $\boldsymbol{F}(u, v)$, $0 \le u, v \le 1$, which

$$\boldsymbol{F}(0, 0) = (1, 1, 1), \qquad \boldsymbol{F}(0, 1) = (1, 5, 1),$$
$$\boldsymbol{F}(1, 0) = (5, 2, 1), \qquad \boldsymbol{F}(1, 1) = (5, 5, 1).$$

Suppose the boundary of patch $\boldsymbol{F}$ at $u = 0$ is a Bézier quadratic

$$\boldsymbol{F}(0, v) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} (1 - v)^2 + \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} 2v(1 - v) + \begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix} v^2,$$

while the other three boundaries are linear polynomials. Evaluate the point on patch $\boldsymbol{F}$ at $(u, v) = (0.5, 0.5)$.

[100 marks]

3\. (a) *Katakan* $\boldsymbol{u} = (u_0,\ u_1,\ \ldots,\ u_{n+k})$ *ialah suatu vektor knot tak menyusut di mana* n *dan* k *adalah nombor integer positif. Fungsi asas splin-B ternormal berperingkat* k *ditakrif secara rekursi sebagai*

$$N_i^1(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0, & \textit{di tempat lain} \end{cases}$$

dan

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad \textit{untuk } k > 1,$$

di mana $i = 0,\ 1,\ \ldots,\ n$. *Diberi satu lengkung splin-B berperingkat* 3

$$\boldsymbol{P}(u) = \begin{pmatrix}1\\1\end{pmatrix} N_0^3(u) + \begin{pmatrix}1\\2\end{pmatrix} N_1^3(u) + \begin{pmatrix}2\\2\end{pmatrix} N_2^3(u) + \begin{pmatrix}2\\1\end{pmatrix} N_3^3(u), \qquad u_2 \le u \le u_4.$$

(i) *Andaikan* $\boldsymbol{u} = (0,\ 1,\ 2,\ \ldots,\ 6)$, *cari sokongan setempat bagi fungsi* $N_0^3(u)$. *Seterusnya, tunjukkan bahawa* $\boldsymbol{P}(u)$ *ialah satu lengkung berkeselanjaran* C^1.

(ii) *Andaikan* $\boldsymbol{u} = (0,\ 1,\ 1,\ 2,\ 3,\ 3,\ 4)$, *cari titik lengkung* $\boldsymbol{P}$ *pada* $u = 2$.

(b) *Pertimbangkan satu tampalan Coons teraduan dwilinear* $\boldsymbol{F}(u,\ v)$, $0 \le u, v \le 1$, *di mana*

$$\boldsymbol{F}(0,\ 0) = (1,\ 1,\ 1), \qquad \boldsymbol{F}(0,\ 1) = (1,\ 5,\ 1),$$
$$\boldsymbol{F}(1,\ 0) = (5,\ 2,\ 1), \qquad \boldsymbol{F}(1,\ 1) = (5,\ 5,\ 1).$$

Andaikan sempadan tampalan $\boldsymbol{F}$ *pada* $u = 0$ *ialah satu kuadratik Bézier*

$$\boldsymbol{F}(0,\ v) = \begin{pmatrix}1\\1\\1\end{pmatrix}(1-v)^2 + \begin{pmatrix}1\\3\\0\end{pmatrix} 2v(1-v) + \begin{pmatrix}1\\5\\1\end{pmatrix} v^2,$$

manakala tiga sempadan lain adalah polinomial linear. Nilaikan titik tampalan $\boldsymbol{F}$ *pada* $(u,\ v) = (0.5,\ 0.5)$.

[100 markah]

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