
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2011/2012 Academic Session

June 2012

MAT 102 – Advanced Calculus
[Kalkulus Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

[Arahan: Jawab **semua sepuluh** [10] soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Show that $\lim_{x \rightarrow \infty} x^{1/x^n} = 1$ for every positive integer n .

[5 marks]

1. Tunjukkan bahawa $\lim_{x \rightarrow \infty} x^{1/x^n} = 1$ bagi setiap integer positif n .

[5 markah]

2. If a_n is convergent, show that $\lim_{n \rightarrow \infty} a_{n+1} = \lim_{n \rightarrow \infty} a_n$.

[5 marks]

2. Jika a_n menumpu, tunjukkan bahawa $\lim_{n \rightarrow \infty} a_{n+1} = \lim_{n \rightarrow \infty} a_n$.

[5 markah]

3. Test the convergence of the series.

(a) $\sum_{k=1}^{\infty} \left(\frac{3}{k^2} - \frac{3}{(k+1)^2} \right)$

(b) $\sum_{k=1}^{\infty} \frac{-2^{k+1}}{k+5^k}$

[10 marks]

3. Uji penumpuan siri.

(a) $\sum_{k=1}^{\infty} \left(\frac{3}{k^2} - \frac{3}{(k+1)^2} \right)$

(b) $\sum_{k=1}^{\infty} \frac{-2^{k+1}}{k+5^k}$

[10 markah]

4. Find all values of x for which $\sum_{n=1}^{\infty} \frac{nx^n}{(n+1)(2x+1)^n}$ converges absolutely.

[12 marks]

4. Dapatkan semua nilai x yang mana $\sum_{n=1}^{\infty} \frac{nx^n}{(n+1)(2x+1)^n}$ menumpu secara mutlak.

[12 markah]

5. Find the power series representation for $\frac{1}{1-x^2}$. Hence, find the first five terms of the power series representation of $f(x) = \frac{1+x^2}{1-x^2}$ which holds for $|x| < 1$. [7 marks]

5. Dapatkan perwakilan siri kuasa bagi $\frac{1}{1-x^2}$. Seterusnya, dapatkan lima sebutan pertama perwakilan siri kuasa $f(x) = \frac{1+x^2}{1-x^2}$ yang sah bagi $|x| < 1$. [7 markah]

6. Determine whether the following integrals is convergent or divergent.

(a) $\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}}$

(b) $\int_1^{\infty} \frac{2+e^{-x}}{x} dx$

[10 marks]

6. Tentukan sama ada kamiran berikut menumpu atau mencapah.

(a) $\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}}$

(b) $\int_1^{\infty} \frac{2+e^{-x}}{x} dx$

[10 markah]

7. (a) Suppose f is a differentiable function of x and y , and $g(r,s) = f(2r-s, s^2-4r)$. Use the table of values, to calculate $g_r(1,2)$ and $g_s(1,2)$.

	f	g	f_x	f_y
0,0	3	6	4	8
1,2	6	3	2	5

- (b) If $f(x,y) = \frac{x^4}{x^2+y^2}$, does $\lim_{x,y \rightarrow 0,0} f(x,y)$ exist?

[15 marks]

7. (a) Katakan f fungsi bagi x dan y yang terbezakan, dan $g(r, s) = f(2r - s, s^2 - 4r)$. Menggunakan jadual nilai, hitung $g_r(1, 2)$ dan $g_s(1, 2)$.

	f	g	f_x	f_y
0,0	3	6	4	8
1,2	6	3	2	5

- (b) Jika $f(x, y) = \frac{x^4}{x^2 + y^2}$, adakah $\lim_{x, y \rightarrow 0,0} f(x, y)$ wujud?

[15 markah]

8. Show that there is no direction $\underline{u}$ in which the rate of change of $f(x, y) = x^2 - 3xy + 4y^2$ at $P(1, 2)$ equals 14.

[6 marks]

8. Tunjukkan bahawa tiada arah $\underline{u}$ yang mana kadar perubahan $f(x, y) = x^2 - 3xy + 4y^2$ pada $P(1, 2)$ bersamaan 14.

[6 markah]

9. Show that the maximum value of $x^2 y^2 z^2$ on a sphere of radius r centered at the origin of a Cartesian xyz -coordinate system is $\left(\frac{r^2}{3}\right)^3$.

[15 marks]

9. Tunjukkan bahawa nilai maksima $x^2 y^2 z^2$ di sfera berjejari r berpusat di asalan system koordinat Cartesian- xyz adalah $\left(\frac{r^2}{3}\right)^3$.

[15 markah]

10. (a) Evaluate $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-x^2-y^2}^{x^2+y^2} 21xy^2 dz dy dx$ by changing to cylindrical coordinates.

- (b) Find the volume of the region bounded above by the paraboloid $z = x^2 + y^2$ and below by the triangle enclosed by the lines $y = x$, $x = 0$, and $x + y = 2$ in the xy -plane.

[15 marks]

10. (a) Nilai $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-x^2-y^2}^{x^2+y^2} 21xy^2 dz dy dx$ dengan menukar ke koordinat silinder.
- (b) Dapatkan isipadu rantau yang dibatasi bahagian atasnya oleh paraboloid $z = x^2 + y^2$ dan bahagian bawahnya oleh segitiga yang dibatasi oleh garis $y = x, x = 0$, dan $x + y = 2$ di satah xy .

[15 markah]