UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2011/2012 Academic Session

June 2012

MSG 252 – Linear and Integer Programming
[Pengaturcaraan Linear dan Integer]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of EIGHT pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all nine** [9] questions.

*[**Arahan:*** *Jawab **semua sembilan** [9] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Use the revised simplex method to solve the following problem:

$$\begin{aligned} \text{Minimize } z = {} & x_1 + x_2 \\ \text{Subject to } & x_1 + 3x_2 \le 12 \\ & 3x_1 + x_2 \ge 13 \\ & x_1 + x_2 = 3 \\ & x_1, x_2 \ge 0 \end{aligned}$$

[10 marks]

1. Selesaikan masalah berikut dengan kaedah simpleks tertilik semula:

$$\begin{aligned} \textit{Minimumkan } z = {} & x_1 + x_2 \\ \textit{Terhadap } & x_1 + 3x_2 \le 12 \\ & 3x_1 + x_2 \ge 13 \\ & x_1 + x_2 = 3 \\ & x_1, x_2 \ge 0 \end{aligned}$$

[10 markah]

2. Consider the following LP problem.

$$\begin{aligned} \text{Maximize } z = {} & 20x_1 + 6x_2 + 8x_3 \\ \text{Subject to } & 8x_1 + 2x_2 + 3x_3 \le 200 \\ & 4x_1 + 3x_2 \le 100 \\ & 2x_1 + x_3 \le 50 \\ & x_3 \le 20 \\ & x_1, x_2, x_3, x_4 \ge 0 \end{aligned}$$

Let s_1, s_2, s_3 and s_4 denote the slack variables for the first through fourth constraints, respectively. Suppose after some number of iterations of the simplex method, a portion of the current simplex tableau is as follows:

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Solution
z				$\frac{9}{4}$	$\frac{1}{2}$	0	0	500
x_1				$\frac{3}{16}$	$-\frac{1}{8}$	0	0	25
x_2				$-\frac{1}{4}$	$\frac{1}{2}$	0	0	0
s_3				$-\frac{3}{8}$	$\frac{1}{4}$	1	0	0
s_4				0	0	0	1	20

From the tableau,

(a) indicate which of the missing numbers would be generated by the revised simplex in order to perform the next iteration.

(b) find the set of basic variables and their values in the next iteration.

[10 marks]

2. Pertimbangkan masalah PL berikut:

$$\begin{aligned} &\textit{Maksimumkan} && z = 20x_1 + 6x_2 + 8x_3 \\ &\textit{Subject to} && 8x_1 + 2x_2 + 3x_3 \le 200 \\ & && 4x_1 + 3x_2 \le 100 \\ & && 2x_1 + x_3 \le 50 \\ & && x_3 \le 20 \\ & && x_1, x_2, x_3, x_4 \ge 0 \end{aligned}$$

Biar s_1, s_2, s_3 dan s_4 masing-masing mewakili pembolehubah -pembolehubah lalai bagi kekangan pertama hingga keempat. Katakan selepas beberapa lelaran simpleks, sebahagian daripada tablo semasa adalah seperti berikut:

Asas	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Penyelesaian*
z				$\frac{9}{4}$	$\frac{1}{2}$	*0*	*0*	*500*
x_1				$\frac{3}{16}$	$-\frac{1}{8}$	*0*	*0*	*25*
x_2				$-\frac{1}{4}$	$\frac{1}{2}$	*0*	*0*	*0*
s_3				$-\frac{3}{8}$	$\frac{1}{4}$	*1*	*0*	*0*
s_4				*0*	*0*	*0*	*1*	*20*

Daripada tablo tersebut

(a) tunjukkan nombor-nombor tertinggal yang akan dijana oleh kaedah simpleks tertilik semula untuk melakukan lelaran seterusnya.

(b) tentukan set pembolehubah-pembolehubah asas dan nilai mereka dalam lelaran berikutnya.

[10 markah]

3. Find the dual of

$$\begin{aligned} &\text{Maximize} && z = 4x_1 - x_2 + 2x_3 \\ &\text{Subject to} && x_1 + x_2 \le 5 \\ & && 2x_1 + x_2 \le 7 \\ & && 2x_2 + x_3 \ge 6 \\ & && x_1 + x_3 = 4 \\ & && x_1 \ge 0,\ x_2 \text{ and } x_3, \text{ unrestricted} \end{aligned}$$

[10 marks]

3. Tuliskan dual bagi

$$\begin{aligned} &\textit{Maksimumkan} && z = 4x_1 - x_2 + 2x_3 \\ &\textit{Terhadap} && x_1 + x_2 \le 5 \\ & && 2x_1 + x_2 \le 7 \\ & && 2x_2 + x_3 \ge 6 \\ & && x_1 + x_3 = 4 \\ & && x_1 \ge 0,\ x_2 \textit{ dan } x_3, \textit{ tak tersekat} \end{aligned}$$

[10 markah]

4. Given this problem and its final simplex tableau:

$$\begin{aligned} \text{Maximize} \quad & z = 9x_1 + 6x_2 \\ \text{Subject to} \quad & 12x_1 + 5x_2 \le 600 \\ & x_2 \le 72 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Basic	x_1	x_2	s_1	s_2	solution
z	0	0	$\frac{9}{12}$	$\frac{27}{12}$	612
x_1	1	0	$\frac{1}{12}$	$-\frac{5}{12}$	20
x_2	0	1	0	1	72

The slack variables of the first and second constraints are represented by s_1 and s_2 respectively.

(a) Identify the dual prices and interpret their values.

(b) Determine the range of feasibility for right-hand side value of the first constraint and interpret your answer.

(c) Determine the range of optimality for the coefficient of x_2 in the objective function.

(d) If the right-hand side of the first constraint is changed to 450, what impact would that have on the values of the basic variables and z?

(e) A new variable with coefficients $\begin{bmatrix} c_3 \\ a_{13} \\ a_{23} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ is added to the original problem. What impact would that have on the current solution?

(f) A new constraint $x_1 + x_2 \le 80$ is added to the problem. What impact would that have on the current solution?

[15 marks]

4. Diberikan masalah berikut dan tablo simpleks terakhirnya:

$$\begin{aligned} \textit{Maksimumkan} \quad & z = 9x_1 + 6x_2 \\ \textit{Subject to} \quad & 12x_1 + 5x_2 \le 600 \\ & x_2 \le 72 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Asas	x_1	x_2	s_1	s_2	*Penyelesaian*
z	*0*	*0*	$\frac{9}{12}$	$\frac{27}{12}$	*612*
x_1	*1*	*0*	$\frac{1}{12}$	$-\frac{5}{12}$	*20*
x_2	*0*	*1*	*0*	*1*	*72*

Pembolehubah lalai bagi kekangan pertama dan kedua masing-masing diwakili oleh s_1 dan s_2.

(a) Camkan harga-harga dual dan terjemahkan nilai-nilainya.

(b) Tentukan julat ketersauran nilai sebelah kanan bagi kekangan pertama dan terjemahkan jawapan anda.

(c) Tentukan julat keoptimuman bagi pekali x_2 di dalam fungsi matlamat.

(d) Jika nilai sebelah kanan kekangan pertama diubah kepada 450, apakah kesannya terhadap nilai-nilai pembolehubah asas dan z?

(e) Suatu pembolehubah baru dengan pekali-pekali $\begin{bmatrix} c_3 \\ a_{13} \\ a_{23} \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ *ditambahkan kepada masalah asal. Apakah kesannya ke atas penyelesaian semasa?*

(f) Suatu kekangan baru $x_1 + x_2 \leq 80$ *ditambahkan kepada masalah tersebut. Apakah kesannya ke atas penyelesaian semasa?*

[15 markah]

5. Consider the following integer programming and the optimal simplex tableau of its LP relaxation problem.

Maximize $z = 2x_1 - 4x_2$

Subject to $2x_1 + x_2 \leq 5$

$-4x_1 + 4x_2 \leq 5$

$x_1, x_2 \geq 0$ and integer

z	x_1	x_2	s_1	s_2	Solution
1	0	0	$-\frac{2}{3}$	$-\frac{5}{6}$	$-\frac{15}{2}$
0	1	0	$\frac{1}{3}$	$-\frac{1}{12}$	$\frac{5}{4}$
0	0	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{5}{2}$

The surplus variables of the first and second constraints are represented by s_1 and s_2 respectively.

Use the cutting plane algorithm to find the integer solution.

[10 marks]

5. *Pertimbangkan masalah pengaturcaraan integer berikut dan tablo simpleks optimum bagi masalah PL tak ketat .*

Maksimumkan $z = 2x_1 - 4x_2$

Terhadap $2x_1 + x_2 \leq 5$

$-4x_1 + 4x_2 \leq 5$

$x_1, x_2 \geq 0$ *dan integer*

z	x_1	x_2	s_1	s_2	*Penyelesaian*
1	0	0	$-\frac{2}{3}$	$-\frac{5}{6}$	$-\frac{15}{2}$
0	1	0	$\frac{1}{3}$	$-\frac{1}{12}$	$\frac{5}{4}$
0	0	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{5}{2}$

Pembolehubah- pembolehubah lebihan bagi kekangan pertama dan kedua masing-masing diwakili oleh s_1 *dan* s_2.

Gunakan algorithma satah potongan untuk mendapatkan penyelesaian integer.

[10 markah]

6. Solve the following 0-1 problem.

$$\begin{aligned} \text{Maximize } z &= 30x_1 + 90x_2 + 40x_3 + 15x_4 \\ \text{Subject to } & 35x_1 + 10x_2 + 25x_3 + 90x_4 \leq 120 \\ & 4x_1 + 2x_2 + 7x_3 + 3x_4 \leq 12 \\ & x_1 + x_2 \leq 1 \\ & x_1, x_2, x_3, x_4 = 0, 1 \end{aligned}$$

[10 marks]

6. Selesaikan masalah 0-1 berikut.

$$\begin{aligned} \textit{Maksimumkan } z &= 30x_1 + 90x_2 + 40x_3 + 15x_4 \\ \textit{Terhadap } & 35x_1 + 10x_2 + 25x_3 + 90x_4 \leq 120 \\ & 4x_1 + 2x_2 + 7x_3 + 3x_4 \leq 12 \\ & x_1 + x_2 \leq 1 \\ & x_1, x_2, x_3, x_4 = 0, 1 \end{aligned}$$

[10 markah]

7. Consider the following pure integer programming problem.

$$\begin{aligned} \text{Maximize } z &= 120x_1 + 50x_2 \\ \text{Subject to } & 14x_1 + 5x_2 \leq 70 \\ & 2x_1 + 3x_2 \leq 18 \\ & x_1, x_2 \geq 0 \text{ and integer} \end{aligned}$$

(a) Graph the problem and identify the LP relaxation solution on the graph.
(b) Solve using the branch and bound method and show the subproblems on your graph.

[10 marks]

7. Pertimbangkan masalah pengaturcaraan integer tulen berikut:

$$\begin{aligned} \textit{Maksimumkan } z &= 120x_1 + 50x_2 \\ \textit{Terhadap } & 14x_1 + 5x_2 \leq 70 \\ & 2x_1 + 3x_2 \leq 18 \\ & x_1, x_2 \geq 0 \textit{ dan integer} \end{aligned}$$

(a) Grafkan masalah ini dan tentukan penyelesaian bagi masalah PL tak ketat di atas graf.
(b) Selesaikan dengan kaedah cabang dan batas dan tunjukkan submasalah-submasalah di atas graf.

[10 markah]

8. Given the following goal model:

$$\begin{aligned} \text{Minimize } z &= P_1 v_1 + P_2 u_2 + P_3 u_3 \\ \text{Subject to } & 2x_1 + 4x_2 + u_1 - v_1 = 80 \\ & 2x_1 + 4x_2 + u_2 - v_2 = 120 \\ & x_1 + u_3 - v_3 = 30 \\ & x_1, x_2, u_i, v_i \geq 0 \quad i = 1, 2, 3 \end{aligned}$$

(a) Determine the values of the deviation variables in the objective function that will minimize the objective.
(b) Find the values of the decision variables that result from your solution.

[10 marks]

8. Diberikan model gol berikut:

$$\begin{aligned} \textit{Minimumkan } z &= P_1 v_1 + P_2 u_2 + P_3 u_3 \\ \textit{Terhadap } & 2x_1 + 4x_2 + u_1 - v_1 = 80 \\ & 2x_1 + 4x_2 + u_2 - v_2 = 120 \\ & x_1 + u_3 - v_3 = 30 \\ & x_1, x_2, u_i, v_i \geq 0 \quad i = 1, 2, 3 \end{aligned}$$

(a) Dapatkan nilai-nilai pembolehubah sisihan di dalam fungsi objektif yang meminimumkan objektif.
(b) Tentukan nilai-nilai pembolehubah keputusan yang terhasil daripada penyelesaian anda.

[10 markah]

9. There are four lecturers of Operations Research in the School. Each semester, 200 students take each of the following courses: A, B, C, and D. The "effectiveness" of each lecturer in teaching each class is given in the table below. Each lecturer can teach a total of 200 students during the semester. The dean has set a goal of obtaining an average teaching effectiveness level of about 6 in each course. Deviations from this goal in any course are considered equally important. Formulate a goal programming model that can be used to determine the semester's teaching assignments.

Lecturer	Course A	Course B	Course C	Course D
1	7	5	8	2
2	7	8	9	4
3	3	5	7	9
4	5	5	6	7

[15 marks]

9. *Terdapat empat pensyarah Penyelidikan Operasi di pusat pengajian. Setiap semester,* 200 *pelajar mengambil setiap kursus yang berikut: A, B, C, dan D. "Keberkesanan" setiap pensyarah dalam pengajaran setiap kelas diberikan dalam jadual di bawah. Setiap pensyarah boleh mengajar seramai* 200 *orang pelajar pada semester tertentu. Dekan telah menetapkan matlamat untuk mendapatkan keberkesanan pengajaran tahap purata kira-kira* 6 *dalam setiap kursus. Penyimpangan daripada matlamat ini dalam sebarang kursus dianggap sama penting. Rumuskan suatu model pengaturcaraan gol yang boleh digunakan untuk menentukan tugas pengajaran semester tersebut.*

Pensyarah	*Kursus A*	*Kursus B*	*Kursus C*	*Kursus D*
1	7	5	8	2
2	7	8	9	4
3	3	5	7	9
4	5	5	6	7

[15 markah]

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