UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2011/2012 Academic Session

June 2012

MSG 253 – Queueing Systems and Simulation
[Sistem Giliran dan Simulasi]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of THIRTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TIGA BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all three** [3] questions.

*[**Arahan:** Jawab **semua tiga** [3] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Consider an M/M/1 queueing system with an arrival rate of λ and a service rate of μ.

(i) Draw a rate-diagram to represent the queueing system.
(ii) Using the birth and death process and under the assumption that the system is stable, show that the probability that the system is in state n is:

$$P_n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n \quad \text{for} \quad n = 0, 1, 2, \ldots$$

(iii) Next, show that the average state of the system is:

$$L = \frac{\lambda}{\mu - \lambda}$$

[50 marks]

(b) There are four bays for unloading trucks at a warehouse. Trucks arrive at the warehouse can be serviced at any one of the four bays. A dock worker is assigned to each bay and is responsible for unloading the truck. It takes a dock worker an average of 1 hour to unload a truck completely. The unloading times follow an exponential distribution. The trucks arrive according to a Poisson distribution with a mean of three trucks per hour. Calculate the following operating characteristics:

(i) Average waiting time in the queue.
(ii) Average waiting time in the system.
(iii) Average length of the queue.
(iv) Average number in the system.
(v) The probability that there will be no trucks in the system.
(vi) Probability that the waiting time in the queue will exceed 3 hours.
(vii) Probability that the time in the system will exceed 4 hours.
(viii) Suppose that it is proposed that an additional unloading bay is added to the system. The increase in capital costs and employee wages resulted from this action amount to RM40 per hour. The cost of idle trucks (trucks waiting in queue and being loaded) is RM50 per hour, reflecting driver wages and equipment charges. Which of the two systems would result in a lower cost per hour?

[50 marks]

1. (a) *Pertimbangkan sistem giliran M/M/1 dengan kadar ketibaan* λ *dan kadar layanan* μ.

   (i) *Lukiskan gambar rajah kadar bagi sistem giliran itu.*

   (ii) *Dengan menggunakan proses lahir-mati dan di bawah andaian bahawa sistem berkeadaan mantap, tunjukkan bahawa kebarangkalian sistem berkeadaan n adalah:*

$$P_n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n \quad \textit{untuk} \quad n = 0, 1, 2, \ldots$$

   (iii) *Seterusnya, tunjukkan bahawa keadaan purata sistem adalah:*

$$L = \frac{\lambda}{\mu - \lambda}$$

[50 markah]

(b) *Terdapat empat pelantar untuk memunggah truk di sebuah gudang. Truk yang tiba di gudang boleh dilayan di mana-mana pelantar yang ada. Seorang pekerja ditugaskan di setiap pelantar dan beliau bertanggungjawab untuk memunggah truk di pelantar berkenaan. Pada puratanya, seorang pekerja mengambil masa 1 jam untuk memunggah sebuah truk sepenuhnya. Masa memunggah adalah mengikut agihan eksponen. Ketibaan truk adalah mengikut agihan Poisson dengan min tiga truk sejam. Tentukan ciri-ciri pengoperasian berikut:*

   (i) *Masa purata menunggu di dalam barisan menunggu.*

   (ii) *Masa purata berada di dalam sistem.*

   (iii) *Panjang purata barisan menunggu.*

   (iv) *Bilangan purata truk di dalam sistem.*

   (v) *Kebarangkalian bahawa tidak ada sebuah trukpun di dalam sistem.*

   (vi) *Kebarangkalian bahawa masa menunggu di dalam barisan menunggu akan melebihi 3 jam.*

   (vii) *Kebarangkalian bahawa masa berada di dalam sistem akan melebihi 4 jam.*

   (viii) *Katakan ada cadangan untuk menambah satu lagi pelantar memunggah ke dalam sistem. Peningkatan kos kapital dan gaji pekerja akibat daripada tindakan ini adalah RM40 sejam. Kos truk bersenang (truk menunggu in dalam barisan menunggu dan sedang dipunggah) adalah RM50 sejam, ini melibatkan gaji pemandu dan bayaran peralatan. Sistem manakah yang akan melibatkan kos sejam yang lebih rendah?*

[50 markah]

2. (a) Suppose that a one person tailor shop is in business of making men suits. Each suit requires four distinct tasks to be performed before it is completed. Assume all four tasks must be completed on each suit before another is started. The time to perform each task has an exponential distribution with a mean of 2 hours. If orders for a suit come at the average rate of 5.5 per week (assume an 8-hour per day and 6 day per week), how long can a customer expect to wait to have a suit made?

[20 marks]

(b) At a queueing system with 4 waiting positions customers arrive according to a Poisson process with rate $\lambda = 80$ customers per minute. The system has two servers. The first server has a service rate of 40 customers per minute and works all the time. The second server has the same service rate of 40 customers per minute but works only if there are more than 3 customers in the system. The service time is exponentially distributed. Find the average number of customers in the system.

[25 marks]

(c) Visitors parking at *Aman Clinic* is limited to only five spaces. Cars making use of this space arrive according to a Poisson distribution at the rate of six cars per hour. Parking time is exponentially distributed with a mean of 30 minutes. Visitors who cannot find an empty space immediately on arrival may temporarily wait inside the lot until a parked car leaves. That temporary space can hold only three cars. All other cars that cannot park or find a temporary waiting space must go elsewhere. Determine the following:

(i) The probability P_n of n cars being in the system.
(ii) The effective rate at which cars arrive at the lot.
(iii) The average number of cars in the lot.
(iv) The average time a car waits for a parking space inside the lot.
(v) The average number of occupied parking space.

[30 marks]

(d) A laundry shop has five washing machines. A typical machine breaks down once every 5 days. A repairer can repair a machine in an average of 2.5 days. Currently, three repairers are on duty. The owner of the laundry shop has the option of replacing them with a super worker, who can repair a machine in an average of $5/6$ of a day. The salary of the super worker equals the total pay of the three regular employees. Breakdown and service times are exponential. Should the laundry shop replace the repairers with the super worker?

[25 marks]

2. (a) *Katakan sebuah kedai jahitan perseorangan berurusan membuat pakaian sut lelaki. Setiap sut perlu diproses menggunakan empat langkah berlainan. Andaikan bahawa keempat-empat langkah itu mesti disiapkan terlebih dahulu terhadap sesuatu sut sebelum proses membuat sut berikutnya boleh dimulakan. Masa untuk melaksanakan setiap langkah itu adalah mengikut agihan eksponen dengan min 2 jam. Jika pesanan untuk membuat sut diterima dengan kadar purata 5.5 seminggu (andaikan 8 jam sehari dan 6 hari seminggu) mengikut agihan Poisson, berapa lama seorang pelanggan dijangka menunggu supaya pesanannya dapat disiapkan?*

[20 markah]

(b) *Di dalam suatu sistem giliran yang mempunyai 4 tempat menunggu, pelanggan tiba mengikut proses Poisson dengan kadar $\lambda = 80$ pelanggan seminit. Sistem itu mempunyai dua pelayan. Pelayan pertama berupaya melayan 40 pelanggan seminit dan bertugas setiap masa. Pelayan kedua juga berupaya melayan 40 pelanggan seminit tetapi beliau akan bertugas hanya apabila terdapat lebih daripada 3 pelanggan di dalam sistem. Masa layanan adalah mengikut agihan eksponen. Tentukan bilangan purata pelanggan di dalam sistem.*

[25 markah]

(c) *Tempat meletak kereta pelawat di Klinik Aman mempunyai ruang untuk lima kenderaan sahaja. Ketibaan kereta ke tempat itu adalah mengikut agihan Poisson dengan kadar enam sejam. Masa meletak kereta adalah mengikut agihan eksponen dengan min 30 minit. Pelawat yang tidak dapat mencari ruang yang kosong semasa tiba boleh menunggu sementara di dalam lot sehingga terdapat kekosongan. Ruang menunggu sementara hanya mampu menempatkan tiga kereta sahaja. Semua kereta yang tiba semasa tempat meletak kereta penuh dan juga tiada ruang menunggu sementara akan pergi ke tempat yang lain. Tentukan perkara-perkara berikut:*

(i) *Kebarangkalian P_n bahawa n buah kereta berada di dalam sistem.*
(ii) *Kadar berkesan ketibaan kereta di lot.*
(iii) *Bilangan purata kereta di dalam lot.*
(iv) *Masa purata sesebuah kereta menunggu untuk mendapatkan tempat letak kereta di dalam lot.*
(v) *Bilangan purata tempat letak kereta yang digunakan.*

[30 markah]

(d) *Sebuah kedai dobi mempunyai lima mesin pembasuh. Sesebuah mesin biasanya mengalami kerosakan sekali setiap 5 hari. Seorang pembaiki mesin pada puratanya mengambil masa 2.5 hari untuk membaiki mesin. Buat masa sekarang, tiga pembaiki mesin ditempatkan di kedai itu. Pemilik kedai dobi itu mempunyai pilihan untuk menggantikan mereka dengan seorang pekerja mahir yang mampu membaiki mesin dalam masa purata $5/6$ hari. Gaji pekerja mahir itu adalah bersamaan dengan jumlah gaji tiga pembaiki mesin biasa. Masa kerosakan dan masa layanan adalah eksponen. Patutkah kedai dobi itu menggantikan ketiga-tiga pembaiki sedia ada dengan pekerja mahir?*

[25 markah]

3 (a) Ships arrive at a harbor at the rate of 1 ± ½ hours. There are six berths to accommodate them. They also need the service of a crane for unloading and there are five cranes. After unloading, 10% of the ships stay to refuel before leaving; the others leave immediately. Ships do not need the cranes for refueling. It takes 7½ ± 3 hours to unload and 1 ± ½ hours to refuel a ship. Write a GPSSPC program to simulate the clearance of 100 ships from the harbor. The queueing patterns for berths and cranes are to be observed.

[50 marks]

(b) Referring to problem 3(a), perform a hand simulation for the clearance of 10 ships from the harbor. Assume that there are only three berths and two cranes. The inter-arrival time of ships is either 1, 2 or 3 hours, each with equal chances of happening. The unloading time is either 2, 3, 4 or 5 hours with probabilities of 0.1, 0.3, 0.4 and 0.2, respectively. The refueling time is either 1, 1½ , or 2½ hours, each with equal chances of happening. The answers to the following questions are to be provided:

(i) What is the average time that a ship spent at the harbor?
(ii) What proportion of time are the cranes idle?
(iii) What is the average time that a ship spent waiting for a berth to be available?

(Use the enclosed 2-digit random number table with the first column for *inter-arrival time*, second column for *unloading time*, the third column for *determining whether to refuel or not* and the fourth column for *refueling time.*)

[50 marks]

3 (a) *Kapal tiba di pelabuhan dengan kadar 1 $\pm$ ½ jam. Enam pelantar disediakan untuk kegunaan kapal. Kapal juga memerlukan kren untuk pemunggahan dan terdapat lima kren di perlabuhan itu. Selepas siap pemunggahan, 10% daripada kapal akan terus berlabuh untuk mengisi minyak; yang lainnya akan terus beredar. Semasa pengisian minyak, kren tidak diperlukan. Masa memunggah ialah 7½ $\pm$ 3 jam dan masa mengisi minyak ialah 1 $\pm$ ½ jam bagi setiap kapal. Tulis satu aturcara GPSSPC untuk melakukan simulasi pemprosesan 100 buah kapal di pelabuhan. Corak barisan menunggu untuk kegunaan pelantar dan kren haruslah diperhatikan.*

[50 markah]

(b) *Dengan merujuk kepada masalah 3(a), lakukan simulasi dengan tangan pemprosesan 10 buah kapal di pelabuhan. Andaikan bahawa terdapat hanya tiga pelantar dan dua kren. Masa antara ketibaan kapal ialah sama ada 1, 2, atau 3 jam, dengan setiap satunya sama mungkin akan berlaku. Masa memunggah ialah sama ada 2, 3, 4 atau 5 jam dengan kebarangkalian masing-masingnya ialah 0.1, 0.3, 0.4 dan 0.2. Masa untuk mengisi minyak pula ialah 1, 1½ atau 2½ jam, dengan setiap satunya sama mungkin akan berlaku. Jawapan kepada soalan-soalan berikut mestilah diberikan:*

(i) Berapakah masa purata sesebuah kapal berada di pelabuhan?
(ii) Berapakah peratusan masa bersenang kren?
(iii) Berapakah masa purata sesebuah kapal menuggu untuk menggunakan pelantar?

(Gunakan sifir nombor rawak 2-digit yang disertakan dengan lajur pertama untuk masa antara ketibaan, lajur kedua untuk masa memunggah, lajur ketiga untuk menentukan sama ada perlu mengisi minyak ataupun tidak dan lajur keempat untuk masa mengisi minyak)

[50 markah]

APPENDIX / *LAMPIRAN*

Formulas for Queueing Theory:

1. *M/M/1*:

$$\rho = \lambda / \mu$$

$$P_n = (1-\rho)\rho^n \quad \text{for } n = 0,1,2,\ldots$$

$$L = \frac{\lambda}{\mu - \lambda}$$

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$W = \frac{1}{\mu-\lambda} \quad , \quad W_q = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$P[w > t] = e^{-t/w}$$

$$P[w_q > t] = \rho e^{-t/w}$$

2. *M/M/s*:

$$\rho = \frac{\lambda}{s\mu}$$

$$P_0 = \left[\frac{(\lambda/\mu)^s}{s!} \frac{1}{(1-\rho)} \quad + \quad \sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} \right]^{-1}$$

$$P_n = \begin{cases} \dfrac{(\lambda/\mu)^n}{n!} P_0 & , \quad \text{if } 0 \le n \le s \\[2ex] \dfrac{(\lambda/\mu)^n}{s!\,s^{n-s}} P_0 & , \quad \text{if } n \le s \end{cases}$$

$$L_q = \frac{(\lambda/\mu)^s \rho}{s!(1-\rho)^2} P_0$$

$$W_q = \frac{L_q}{\lambda} \quad , \quad W = W_q + 1/\mu$$

$$L = L_q + \lambda/\mu$$

$$P[w_q > t] = e^{-\mu t} \left[1 + \frac{P_0 \left(\dfrac{\lambda}{\mu} \right)^s}{s!(1-\rho)} \left(\frac{1 - e^{\mu t (s-1-\lambda/\mu)}}{s-1-\lambda/\mu} \right) \right]$$

$$P[w_q > t] = \left[1 - P\{w_q = 0\} \right] e^{-s\mu(1-\rho)t}$$

$$\text{where} \quad P\{w_q = 0\} = \sum_{n=0}^{s-1} P_n$$

3. *M/M/s*: finite population of size *M*.

$$P_0 = \left[\sum_{n=0}^{s-1} \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{M} \binom{M}{n} \frac{n!}{s^{n-s} s!} \left(\frac{\lambda}{\mu}\right)^n \right]^{-1}$$

$$P_n = \begin{cases} P_0 \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n & , \quad \text{if } 0 \le n \le s \\ P_0 \binom{M}{n} \left(\frac{n!}{s^{n-s} s!}\right) \left(\frac{\lambda}{\mu}\right)^n & , \quad \text{if } s < n \le M \\ 0 & , \quad \text{if } n > M \end{cases}$$

$$L = P_0 \left[\sum_{n=0}^{s-1} n \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s}^{M} n \binom{M}{n} \frac{n!}{s^{n-s} s!} \left(\frac{\lambda}{\mu}\right)^n \right]$$

$$L_q = L - s + P_0 \sum_{n=0}^{s-1} (s-n) \binom{M}{n} \left(\frac{\lambda}{\mu}\right)^n$$

$$W = \frac{L}{\lambda(M-L)} \quad , \quad W_q = \frac{L_q}{\lambda(M-L)}$$

4. *M/G/1*:

$$P_0 = 1 - \rho$$

$$L_q = \frac{\lambda^2 \sigma^2 + \rho^2}{2(1-\rho)}$$

$$L = \rho + L_q$$

$$W_q = \frac{L_q}{\lambda} \quad , \quad W = w_q + \frac{1}{\mu}$$

5. $M / E_k / 1$:

$$L_q = \frac{1+k}{2k} \frac{\lambda^2}{\mu(\mu - \lambda)}$$

$$W_q = \frac{1+k}{2k} \frac{\lambda}{\mu(\mu - \lambda)}$$

$$W = W_q + 1/\mu$$

$$L = \lambda W$$

6. *M/M/1/k*:

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{k+1}} & (\rho \neq 1) \\ \dfrac{1}{k+1} & (\rho = 1) \end{cases}$$

For $\rho \neq 1$

$$L = \frac{\rho\left[1-(k+1)\rho^k + k\rho^{k+1}\right]}{\left(1-\rho^{k+1}\right)(1-\rho)}$$

$$L_q = L - (1-P_0) = L - \frac{\rho\left(1-\rho^k\right)}{1-\rho^{k+1}}$$

$$W = L/\lambda' \quad , \quad \lambda' = \mu\left(L - L_q\right)$$

$$W_q = W - 1/\mu = L_q/\lambda'$$

For $\rho = 1$

$$L = \frac{k}{2}$$

7. *M/M/s/k*:

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & (0 \le n < s) \\ \dfrac{1}{s^{n-s}s!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & (s \le n \le k) \end{cases}$$

$$P_0 = \begin{cases} \left[\displaystyle\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{(\lambda/\mu)^s}{s!}\,\frac{1-\left(\dfrac{\lambda}{s\mu}\right)^{k-s+1}}{1-\dfrac{\lambda}{s\mu}}\right]^{-1} & \text{for} \quad \left(\dfrac{\lambda}{s\mu} \neq 1\right) \\ \left[\displaystyle\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{(\lambda/\mu)^s}{s!}(k-s+1)\right]^{-1} & \text{for} \quad \left(\dfrac{\lambda}{s\mu} = 1\right) \end{cases}$$

$$L_q = \frac{P_0 (s\rho)^S \rho}{s!(1-\rho)^2}\left[1-\rho^{k-s+1} - (1-\rho)(k-s+1)\rho^{k-s}\right]$$

$$L = L_q + s - P_0 \sum_{n=0}^{s-1} \frac{(s-n)(\rho s)^n}{n!}$$

$$W = \frac{L}{\lambda'} \quad , \quad \lambda' = \lambda(1 - P_k)$$

$$W_q = W - \frac{1}{\mu}$$

$$W_q = \frac{L_q}{\lambda'}$$

8. *M/M/s/s*:

$$P_n = \frac{(\lambda/\mu)^n / n!}{\sum_{i=0}^{s} \left(\frac{\lambda}{\mu}\right)^i / i!} \quad \text{for} \quad (0 \le n \le s)$$

$$P_s = \frac{(s\rho)^s / s!}{\sum_{i=0}^{s} (s\rho)^i / i!} \quad \text{where} \quad \left(\rho = \frac{\lambda}{s\mu}\right).$$

$$L = \frac{\lambda}{\mu}(1 - P_s) \quad , \quad W = \frac{L}{\lambda'} \text{ where } \lambda' = \lambda(1 - P_s)$$

9. $M/M/\infty$:

$$P_n = \frac{(\lambda/\mu)^n e^{-\lambda/\mu}}{n!} \quad \text{for } n = 0, 1, 2, \ldots$$

$$L = \lambda/\mu$$

$$W = \frac{1}{\mu}$$

10. *M/M/1*: state-dependent service

$$\mu_n = \begin{cases} \mu_1 & (1 \le n \le k) \\ \mu & (n \ge k) \end{cases}$$

$$P_0 = \left[\frac{1-\rho_1^k}{1-\rho_1} + \frac{\rho\rho_1^{k-1}}{1-\rho} \right]^{-1} \quad (\rho_1 = \lambda / \mu_1, \rho = \lambda / \mu < 1)$$

$$L = P_0 \left[\frac{\rho_1 \left[1 + (k-1)\rho_1^k - k\rho_1^{k-1} \right]}{(1-\rho_1)^2} + \frac{\rho\rho_1^{k-1}\left[k - (k-1)\rho \right]}{(1-\rho)^2} \right]$$

$$L_q = L - (1 - P_0)$$

$$W = \frac{L}{\lambda} \qquad W_q = \frac{L_q}{\lambda}$$

$$W = W_q + \frac{1-P_0}{\lambda}$$

$$P_n = \begin{cases} \left(\dfrac{\lambda}{\mu_1} \right)^n P_0 & (0 \le n < k) \\ \dfrac{\lambda^n}{\mu_1^{k-1}\mu^{n-k+1}} P_0 & (n \ge k) \end{cases}$$

11. *M/M/1*: finite population of size *M*.

$$P_0 = \left[\sum_{n=0}^{M} \frac{M!}{(M-n)!} \left(\frac{\lambda}{\mu} \right)^n \right]^{-1}$$

$$P_n = \frac{M!}{(M-n)!} \left(\frac{\lambda}{\mu} \right)^n P_0 \quad \text{for } n = 1, 2, \ldots, M$$

$$L = M - \frac{\mu}{\lambda}\left[1 - P_0 \right]$$

$$L_q = M - \frac{\lambda + \mu}{\lambda}(1 - P_0)$$

$$W = \frac{L}{\lambda'} \quad , \quad W_q = \frac{L_q}{\lambda'} \quad \text{where } \lambda' = \lambda (M - L)$$

TWO-DIGIT RANDOM NUMBER TABLE

03	26	48	92	38	96	41	04	35	84
71	44	81	46	44	47	07	20	58	04
33	75	06	41	87	72	63	88	59	54
53	71	27	13	37	45	89	61	30	26
41	15	43	91	46	81	57	39	34	86
16	18	75	11	26	80	93	97	29	33
88	50	00	56	70	19	90	00	93	95
13	10	08	15	29	33	75	70	43	05
15	72	73	69	27	75	72	95	99	56
64	10	99	02	18	26	78	69	19	12
98	66	53	86	34	71	09	88	56	08
43	05	06	19	91	78	03	65	08	16
69	82	02	61	98	50	74	84	60	41
06	40	10	24	68	42	39	97	25	55
34	86	83	41	33	83	85	92	32	29
46	05	92	36	82	04	67	05	18	69
28	73	59	56	43	88	61	17	07	48
35	53	49	39	98	14	16	76	69	10
90	90	18	27	75	08	75	17	55	68
62	32	97	16	33	66	02	34	62	26

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