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UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2011/2012 Academic Session

June 2012

MAT 122 – Differential Equations I
[Persamaan Pembezaan I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** (4) questions.

*[**Arahan**: Jawab **semua empat** (4) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Solve the initial value problem

$$y' + 4xy = e^{-2x^2}\cos(x)$$
$$y(0) = 6$$

(b) Given the differential equation $y' = \frac{y-1^{\frac{2}{3}}}{x}$. Which of the following initial values will guarantee that the solution for the differential equation is unique? (Show your working and steps)

(i) $y(0)=0$, (ii) $y(0)=1$, (iii) $y(1)=0$,
(iv) $y(1)=1$, (v) $y(2)=1$, (vi) $y(0)=2$.

(c) Consider the initial value problem

$$(x^2-3x)y'' + xy' - (x+3)y = 0,$$
$$y(1) = 2, \quad y'(1) = 1.$$

The existence and uniqueness theorem for linear differential equations guarantees that there will be a unique solution. Determine the longest interval in which this solution is certain to exist.

[100 marks]

1. (a) Selesaikan masalah nilai awal

$$y' + 4xy = e^{-2x^2}\cos(x)$$
$$y(0) = 6$$

(b) Diberikan persamaan pembezaan $y' = \frac{y-1^{\frac{2}{3}}}{x}$. *Yang manakah di antara nilai awal berikut yang menjamin bahawa penyelesaian bagi persamaan pembezaan tersebut unik? (Tunjukkan jalan kerja dan langkah-langkah anda)*

(i) $y(0)=0$, *(ii)* $y(0)=1$, *(iii)* $y(1)=0$,
(iv) $y(1)=1$, *(v)* $y(2)=1$, *(vi)* $y(0)=2$.

(c) Pertimbangkan masalah nilai awal

$$(x^2-3x)y'' + xy' - (x+3)y = 0,$$
$$y(1) = 2, \quad y'(1) = 1.$$

Teorem kewujudan dan keunikan bagi persamaan pembezaan linear menjamin bahawa wujud penyelesaian unik. Tentukan selang terbesar yang penyelesaian ini pasti wujud.

[100 markah]

2. (a) Find the general solution of $y'' - 4y' + 4y = e^{2x}$.

(b) Prove that $d\left(\ln\frac{y}{x}\right) = \frac{x\,dy - y\,dx}{xy}$.

Hence, solve $x\,dy - y\,dx = 3x^3 y\,dx$.

(c) The general solution of $y'' - \frac{2y}{x^2} = 0,\ x > 0$ is $y(x) = Ax^2 + \frac{B}{x}$. Find a particular solution of the equation $y'' - \frac{2y}{x^2} = 0,\ x > 0$.

[100 marks]

2. (a) Dapatkan penyelesaian am bagi $y'' - 4y' + 4y = e^{2x}$.

(b) Buktikan bahawa $d\left(\ln\frac{y}{x}\right) = \frac{x\,dy - y\,dx}{xy}$.

Seterusnya, selesaikan $x\,dy - y\,dx = 3x^3 y\,dx$.

(c) Penyelesaian am bagi $y'' - \frac{2y}{x^2} = 0,\ x > 0$ *diberikan oleh* $y(x) = Ax^2 + \frac{B}{x}$. *Dapatkan penyelesaian khusus bagi persamaan* $y'' - \frac{2y}{x^2} = 6,\ x > 0$.

[100 markah]

3. (a) Consider the initial value problem

$$\frac{dy}{dx} = 2y - 3x, \quad y(0) = 1.$$

Use Euler's method to find the approximate value of $y(0.1)$ with step size 0.05.

(b) (i) What is the value of b so that the differential equation $(xy^2 + bx^2y)dx + (x+y)x^2dy = 0$ is exact?

(ii) Given the differential equation $y' - y = 0, \quad y(0) = 1$.

Solve for n if $y(2012) = y(1)^{n}$.

(c) Solve $(1-x^2)y'' - 6xy' - 4y = 0$ in powers of x. Determine the interval where the solution is valid.

[100 marks]

3. (a) Pertimbangkan masalah nilai awal

$$\frac{dy}{dx} = 2y - 3x, \quad y(0) = 1.$$

Gunakan kaedah Euler untuk mendapatkan nilai anggaran bagi $y(0.1)$ *dengan menggunakan saiz langkah 0.05.*

(b) (i) Apakah nilai b supaya persamaan pembezaan $(xy^2 + bx^2y)dx + (x+y)x^2dy = 0$ *tepat?*

(ii) Diberikan persamaan pembezaan $y' - y = 0, \quad y(0) = 1$.

Dapatkan nilai bagi n jika $y(2012) = y(1)^{n}$.

(c) Selesaikan $(1-x^2)y'' - 6xy' - 4y = 0$ *dalam kuasa x. Tentukan selang yang penyelesaian tersebut sah.*

[100 markah]

4. (a) Consider the system of differential equations given by the matrix equation

$$\mathbf{X}' = \begin{pmatrix} 3 & 4 \\ -1 & 3 \end{pmatrix} \mathbf{X}.$$

(i) Find the general solution of the system.
(ii) Find a fundamental matrix for the system.

(b) A model for the competition between two species with population densities x and y leads to the differential equations

$$\frac{dx}{dt} = ax - by, \quad \frac{dy}{dt} = -cx + dy,$$

where a, b, c and d are positive constants.

(i) Show that x satisfies

$$\frac{d^2x}{dt^2} - (a+d)\frac{dx}{dt} + (ad - bc)x = 0.$$

(ii) Show that x has a solution of the form

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t},$$

with at least one α_i positive.

(iii) Find the solution for y.

(iv) Using the values $a = d = 4, b = 1, c = 4$ and $x(0) = 700$, $y(0) = 3400$, determine the time t when one of the species becomes extinct.

[100 marks]

4. (a) Pertimbangkan sistem persamaan pembezaan yang diberikan oleh persamaan matriks

$$\mathbf{X}' = \begin{pmatrix} 3 & 4 \\ -1 & 3 \end{pmatrix} \mathbf{X}.$$

(i) Dapatkan penyelesaian am bagi sistem tersebut.
(ii) Dapatkan suatu matriks asas bagi sistem tersebut.

(b) Perkembangan bagi dua spesies yang bersaing untuk suatu sumber makanan diwakili oleh dua persamaan pembezaan

$$\frac{dx}{dt} = ax - by, \quad \frac{dy}{dt} = -cx + dy,$$

dengan x, y sebagai populasi kedua – dua species dan a, b, c, d ialah pemalar – pemalar positif.

(i) Tunjukkan bahawa x memenuhi

$$\frac{d^2x}{dt^2} - (a+d)\frac{dx}{dt} + (ad - bc)x = 0.$$

(ii) Tunjukkan bahawa x mempunyai penyelesaian dalam bentuk

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t},$$

dengan sekurang – kurangnya satu α_i positif.

(iii) Cari penyelesaian bagi y.

(iv) Dengan menggunakan nilai a=d=4, b=1, c=4 dan x(0)=700, y(0)=3400, tentukan bilakah salah satu spesies menjadi pupus.

[100 markah]

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