UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2011/2012 Academic Session

June 2012

MGM 511 Linear Algebra
*[*Aljabar Linear*]*

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer all six [6] questions.

[Arahan: Jawab semua enam [6] soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

(1) Solve the following system of linear equations

$$\begin{aligned} x_1 + x_2 \quad +x_3 &= 2, \\ 2x_1 + 4x_2 \; +x_3 &= 3, \\ -3x_1 + x_2 \; -2x_3 &= -8, \end{aligned}$$

by LU decomposition method.

[16 marks]

(1) Selesaikan sistem persamaan linear berikut

$$\begin{aligned} x_1 + x_2 \quad +x_3 &= 2, \\ 2x_1 + 4x_2 \; +x_3 &= 3, \\ -3x_1 + x_2 \; -2x_3 &= -8, \end{aligned}$$

dengan kaedah penghuraian LU.

[16 markah]

(2) (a) Let V be a vector space and $\mathbf{x}, \mathbf{y}, \mathbf{z} \in V$. Assume that $0\mathbf{0} = \mathbf{0}$. Show that

$$\mathbf{x} + \mathbf{y} = \mathbf{x} + \mathbf{z} \Longrightarrow \mathbf{y} = \mathbf{z}.$$

By using this cancellation law, prove also that

$$(-1)\mathbf{x} = -\mathbf{x}.$$

[5 marks]

(b) Show that the null space of a matrix A of order $m \times n$ is a subspace of $\mathbb{R}^n$.

[4 marks]

(c) Is the set of vectors

$$\{(1,0,0), (1,4,0), (2,5,7)\}$$

a basis for the vector space $\mathbb{R}^3$? Justify your answer.

[8 marks]

(2) (a) Biar V suatu ruang vektor dan $\mathbf{x}, \mathbf{y}, \mathbf{z} \in V$. Andaikan $0\mathbf{0} = \mathbf{0}$. Tunjukkan bahawa

$$\mathbf{x} + \mathbf{y} = \mathbf{x} + \mathbf{z} \Longrightarrow \mathbf{y} = \mathbf{z}.$$

Dengan menggunakan hukum pembatalan, buktikan bahawa

$$(-1)\mathbf{x} = -\mathbf{x}.$$

[5 markah]

(b) Tunjukkan bahawa ruang nol suatu matriks A berperingkat $m \times n$ ialah subruang untuk $\mathbb{R}^n$.

[4 markah]

(c) Adakah set vektor

$$\{(1,0,0),(1,4,0),(2,5,7)\}$$

suatu asas untuk ruang vektor $\mathbb{R}^3$? Jelaskan jawapan anda.

[8 markah]

(3) (a) Show that a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ can be written as

$$T(\mathbf{x}) = A\mathbf{x}$$

for some matrix A of order $m \times n$.

[4 marks]

(b) Let V and W be vector spaces. Show that the kernel of a linear transformation $T : V \to W$ is a subspace of V.

[4 marks]

(c) Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ denote the transformation of reflection of the vector $\mathbf{x} = (x_1, x_2)$ along x_1-axis. Determine the matrix A such that

$$L(\mathbf{x}) = A\mathbf{x}.$$

What is the kernel of L?

[4 marks]

(d) Let

$$\mathbf{y}_1 = (1,1,1)^T, \quad \mathbf{y}_2 = (1,1,0)^T, \quad \mathbf{y}_3 = (1,0,0)^T$$

and $L : \mathbb{R}^3 \to \mathbb{R}^3$ be given by

$$L(c_1\mathbf{y}_1 + c_2\mathbf{y}_2 + c_3\mathbf{y}_3) = (c_1 + c_2 + c_3)\mathbf{y}_1 + (2c_1 + c_3)\mathbf{y}_2 - (2c_2 + c_3)\mathbf{y}_3.$$

Find the matrix representation of L with respect to the basis $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$. Find $L(\mathbf{x})$ when $\mathbf{x} = (7,5,2)^T$ and $\mathbf{x} = (3,2,1)^T$.

[5 marks]

(3) *(a)* *Tunjukkan bahawa penjelmaan linear* $T : \mathbb{R}^n \to \mathbb{R}^m$ *boleh ditulis sebagai*

$$T(\mathbf{x}) = A\mathbf{x}$$

untuk suatu matriks A *berperingkat* $m \times n$.

[4 markah]

(b) *Biar* V *dan* W *dua ruang vektor. Tunjukkan bahawa inti penjelmaan linear* $T : V \to W$ *ialah subruang untuk* V.

[4 markah]

(c) *Biar* $L : \mathbb{R}^2 \to \mathbb{R}^2$ *menandakan penjelmaan untuk pantulan vektor* $\mathbf{x} = (x_1, x_2)$ *sepanjang paksi* x_1. *Tentukan matriks* A *sedemikian*

$$L(\mathbf{x}) = A\mathbf{x}.$$

Apakah inti untuk L*?*

[4 markah]

(d) *Biar*

$$\mathbf{y}_1 = (1, 1, 1)^T, \quad \mathbf{y}_2 = (1, 1, 0)^T, \quad \mathbf{y}_3 = (1, 0, 0)^T$$

dan $L : \mathbb{R}^3 \to \mathbb{R}^3$ *diberi oleh*

$$L(c_1\mathbf{y}_1 + c_2\mathbf{y}_2 + c_3\mathbf{y}_3) = (c_1 + c_2 + c_3)\mathbf{y}_1 + (2c_1 + c_3)\mathbf{y}_2 - (2c_2 + c_3)\mathbf{y}_3.$$

Cari matriks perwakilan untuk L *dengan merujuk kepada asas* $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$. *Cari* $L(\mathbf{x})$ *apabila* $\mathbf{x} = (7, 5, 2)^T$ *dan* $\mathbf{x} = (3, 2, 1)^T$.

[5 markah]

(4) (a) State and prove the Cauchy-Schwarz inequality for two vectors in an inner product space V.

[8 marks]

(b) Let $\{\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_n\}$ be an orthonormal basis for an inner product space V, $\mathbf{u} = \sum_{i=1}^{n} a_i\mathbf{u}_i$ and $\mathbf{v} = \sum_{i=1}^{n} b_i\mathbf{u}_i$. Show that $a_i = \langle \mathbf{u}, \mathbf{u}_i \rangle$ and hence $\langle \mathbf{u}, \mathbf{v} \rangle = \sum_{i=1}^{n} a_i b_i$.

[9 marks]

(4) *(a)* *Nyata dan buktikan ketaksamaan Cauchy-Schwarz untuk dua vektor di dalam ruang hasil darab terkedalam* V.

[8 markah]

(b) *Biar* $\{\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_n\}$ *suatu asas ortonormal untuk ruang hasil darab terkedalam* V, $\mathbf{u} = \sum_{i=1}^{n} a_i\mathbf{u}_i$ *dan* $\mathbf{v} = \sum_{i=1}^{n} b_i\mathbf{u}_i$. *Tunjukkan bahawa* $a_i = \langle \mathbf{u}, \mathbf{u}_i \rangle$ *dan dengan itu* $\langle \mathbf{u}, \mathbf{v} \rangle = \sum_{i=1}^{n} a_i b_i$.

[9 markah]

(5) Find an orthonormal basis for the column space of the matrix

$$A = \begin{pmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix}.$$

Also determine its QR factorization.

[17 marks]

(5) Cari asas ortonormal bagi ruang lajur untuk matriks

$$A = \begin{pmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix}.$$

Juga tentukan pemfaktoran QR.

[17 markah]

(6) Define binary code and linear code. Show that a binary code C is a linear code if and only if it is a subspace of the vector space B^n.

[16 marks]

(6) Takrifkan kod perduaan dan kod linear. Tunjukkan bahawa suatu kod perduaan C ialah kod linear jika dan hanya jika ia adalah suatu subruang untuk ruang vektore B^n.

[16 markah]

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