UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2011/2012 Academic Session

June 2012

MAA 111 – Algebra for Sciences Students
[Aljabar untuk Pelajar Sains]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Find the matrix A from the followings:

(i) $(I+2A)^{-1}=\begin{pmatrix}-1 & 2\\ 4 & 5\end{pmatrix}$, (ii) $(5A^T)^{-1}=\begin{pmatrix}-3 & -1\\ 5 & 2\end{pmatrix}$.

(b) Let $A=\begin{pmatrix}1 & 0 & -2\\ 0 & 4 & 3\\ 0 & 0 & 1\end{pmatrix}$.

(i) Find elementary matrices E_1, E_2, E_3 such that $E_3E_2E_1A=I_3$.

(ii) Write A as a product of elementary matrices.

(c) Find h,k such that the system

$$\begin{aligned} x_1 + hx_2 &= 2 \\ 4x_1 + 8x_2 &= k \end{aligned}$$

has

(i) no solution;

(ii) unique solution;

(iii) many solutions.

(d) For each of the following statements, determine whether it is **True** or **False**. Justify your answers.

(i) Whenever a system has free variables, the solution set contains many solutions.

(ii) If one row in an echelon form of an augmented matrix is $\begin{bmatrix}0 & 0 & 0 & 5 & 0\end{bmatrix}$, then the associated linear system is inconsistent.

(iii) A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix

(iv) In order for a matrix B to be the inverse of A, both equations $AB=I$ and $BA=I$ must be true.

[100 marks]

1. (a) Cari matriks A daripada maklumat-maklumat berikut:

(i) $(I+2A)^{-1}=\begin{pmatrix}-1 & 2\\ 4 & 5\end{pmatrix}$, *(ii)* $(5A^T)^{-1}=\begin{pmatrix}-3 & -1\\ 5 & 2\end{pmatrix}$.

(b) Biar $A=\begin{pmatrix}1 & 0 & -2\\ 0 & 4 & 3\\ 0 & 0 & 1\end{pmatrix}$.

(i) Cari matriks-matriks asas E_1, E_2, E_3 supaya $E_3E_2E_1A=I_3$.

(ii) Tulis A sebagai hasildarab matriks-matriks asas.

(c) Cari h,k supaya sistem berikut

$$\begin{aligned} x_1 &+ hx_2 = 2 \\ 4x_1 &+ 8x_2 = k \end{aligned}$$

mempunyai

(i) tiada penyelesaian;

(ii) penyelesaian unik;

(iii) banyak penyelesaian.

*(d) Untuk setiap pernyataan berikut, tentukan samada ia **Benar** atau **Palsu**. Berikan alasan kepada jawapan anda.*

(i) Apabila sesuatu sistem mempunyai pembolehubah bebas, set penyelesaiannya mengandungi banyak penyelesaian.

(ii) Jika satu baris dalam bentuk eselon suatu matriks imbuhan ialah

$\begin{bmatrix} 0 & 0 & 0 & 5 & 0 \end{bmatrix}$ *, maka sistem linear yang berkaitan tidak konsisten.*

(iii) Pembolehubah asas dalam suatu sistem linear ialah pembolehubah yang merujuk kepada lajur pivot dalam matriks koefisien.

(iv) Untuk matriks B menjadi matriks songsang kepada A, kedua-dua persamaan $AB = I$ dan $BA = I$ mestilah benar.

[100 markah]

2. (a) Show that $A(3,0,2), B(4,3,0), C(8,1,-1)$ are vertices of a right triangle. At which vertex is the right angle?

(b) Find two unit vectors that are orthogonal to $(-3,4)$.

(c) Find a vector $\vec{v}$ that is orthogonal to the vector $\vec{u} = (2,-5,5)$.

(d) Show that

$$\begin{vmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & 1 \\ 1 & 1 & 1 & a \end{vmatrix} = (a+3)(a-1)^3 .$$

(e) Let $T(1,0) = (0,1)$ and $T(0,1) = (1,0)$

(i) Determine $T(x, y)$ for any (x, y).

(ii) Give a geometric description of transfformation T.

[100 marks]

2. (a) Tunjukkan $A(3,0,2), B(4,3,0), C(8,1,-1)$ adalah titik pepenjuru segitiga tepat. Pada pepenjuru manakah sudut tepat itu berada?

(b) Cari dua vektor unit yang berortogon dengan $(-3,4)$.

(c) Cari suatu vektor $\vec{v}$ yang berortogon dengan vektor $\vec{u} = (2,-5,5)$.

(d) Tunjukkan

$$\begin{vmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & 1 \\ 1 & 1 & 1 & a \end{vmatrix} = (a+3)(a-1)^3 .$$

(e) *Biar* $T(1,0) = (0,1)$ *dan* $T(0,1) = (1,0)$

(i) *Tentukan* $T(x, y)$ *untuk setiap* (x, y).

(ii) *Beri huraian geometri untuk tranformasi* T .

[100 markah]

3\. (a) Explain why the following sets ARE NOT vector spaces? (the usual rule for addition and scalar multiplication are assumed unless specified otherwise).

(i) The set of vectors on the line $y = 3x - 1$;

(ii) The set $S = \ a,b \ : a,b \in \Re$ where addition and scalar multiplication on S is defined by

$$x_1, x_2 \ + \ y_1, y_2 \ = \ x_1 + y_1, 0 \ ,$$

$$k \ x_1, x_2 \ = \ kx_1, kx_2 \ .$$

(iii) The set

$$L = \left\{ \begin{pmatrix} 1 & a \\ 0 & b \end{pmatrix} : a, b \in \Re \right\}.$$

(b) Find the solution space of the system $\mathbf{Ax} = \mathbf{0}$, where the matrix $\mathbf{A}$ is given below. Consequently, determine whether the solution space is a line through the origin, a plane through the origin, or the origin only.

(i) $\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 \\ -3 & 6 & -9 \\ -2 & 4 & -6 \end{pmatrix}$, (ii) $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$, (iii) $\mathbf{A} = \begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix}$.

(c) Let

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}, \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 12 \\ 6 \\ 18 \end{pmatrix}$$

(i) Find the least squares solution of the linear system $\mathbf{Ax} = \mathbf{b}$.

(ii) Hence, or otherwise, find the orthogonal projection of $\mathbf{b}$ on the column space of $\mathbf{A}$.

[100 marks]

3. (a) *Terangkan mengapa set-set berikut BUKAN ruang vektor? (peraturan biasa bagi penambahan dan pendaraban skalar diandaikan, melainkan dinyatakan sebaliknya).*

   (i) Set vektor di atas garis $y = 3x - 1$*;*

   (ii) Set $S = \{(a,b) : a,b \in \Re\}$ *di mana penambahan dan pendaraban skalar pada S ditakrifkan sebagai*

$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, 0),$$
$$k(x_1, x_2) = (kx_1, kx_2).$$

   (iii) Set

$$L = \left\{ \begin{pmatrix} 1 & a \\ 0 & b \end{pmatrix} : a, b \in \Re \right\}.$$

(b) *Cari ruang penyelesaian sistem* $\mathbf{Ax} = \mathbf{0}$*, di mana matriks* $\mathbf{A}$ *diberikan di bawah. Sehubungan itu, tentukan samada ruang penyelesaian itu suatu garis melalui asalan, satah melalui asalan, atau asalan sahaja*

(i) $\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 \\ -3 & 6 & -9 \\ -2 & 4 & -6 \end{pmatrix}$, *(ii)* $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$, *(iii)* $\mathbf{A} = \begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix}$.

(c) *Biar*

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix}, \quad \text{dan} \quad \mathbf{b} = \begin{pmatrix} 12 \\ 6 \\ 18 \end{pmatrix}$$

   (i) Cari penyelesaian kuasa dua terkecil bagi sistem linear $\mathbf{Ax} = \mathbf{b}$.

   (ii) Oleh yang demikian, atau sebaliknya, cari unjuran ortogon $\mathbf{b}$ *pada ruang jalur* $\mathbf{A}$.

[100 markah]

4. Consider the matrices below,

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 1 & 0 & 2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 0 & 0 \\ -1 & 4 & 0 \\ -3 & 6 & 2 \end{pmatrix}.$$

(a) Determine the eigenspaces of $\mathbf{A}$ and $\mathbf{B}$.

(b) Which matrix is diagonalizable? Why? Factor the diagonalizable matrix into the product $\mathbf{SDS}^{-1}$ where $\mathbf{D}$ is diagonal.

(c) Show that any matrix of the form

$$\begin{pmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & b \end{pmatrix}$$

is not diagonalizable.

[100 marks]

4. *Pertimbangkan matriks-matriks berikut,*

$$\mathbf{A}=\begin{pmatrix}2&0&0\\0&4&0\\1&0&2\end{pmatrix},\qquad \mathbf{B}=\begin{pmatrix}2&0&0\\-1&4&0\\-3&6&2\end{pmatrix}.$$

(a) *Tentukan ruang eigen matriks* $\mathbf{A}$ *dan* $\mathbf{B}$.

(b) *Matriks manakah terpepenjuru? Mengapa? Faktorkan matriks yang terpepenjurukan itu kepada hasildarab* $\mathbf{SDS}^{-1}$ *di mana* $\mathbf{D}$ *ialah matriks pepenjuru.*

(c) *Tunjukkan bahawa mana-mana matriks dalam bentuk*

$$\begin{pmatrix}a&1&0\\0&a&1\\0&0&b\end{pmatrix}$$

tidak terpepenjurukan.

[100 markah]

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