UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2011/2012 Academic Session

June 2012

MSS 212 – Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all five** [5] questions.

*[**Arahan:** Jawab **semua lima** [5] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Let A be a 4×4 matrix. Show that if $\det A \neq 0$, then A has minimally four non-zero entries.

[30 marks]

(b) Find $\det A$ where $A=\begin{pmatrix} 1 & b & c & d \\ -1 & x & c & d \\ -1 & -b & y & d \\ -1 & -b & -c & z \end{pmatrix}$.

[30 marks]

(c) Solve the following system of linear equations by using Cramer's rule

$$x+z+w=1$$
$$x+y+w=0$$
$$x+y+z=2$$
$$y+z+w=2$$

[60 marks]

1. (a) Biar A suatu matriks 4×4. *Tunjukkan bahawa jika* $\det A \neq 0$, *maka A mempunyai sekurang-kurangmya empat permasukan yang bukan kosong*

[30 markah]

(b) Cari $\det A$ *dengan* $A=\begin{pmatrix} 1 & b & c & d \\ -1 & x & c & d \\ -1 & -b & y & d \\ -1 & -b & -c & z \end{pmatrix}$.

[30 markah]

(c) Selesaikan sistem persamaan linear berikut dengan menggunakan petua Cramer

$$x+z+w=1$$
$$x+y+w=0$$
$$x+y+z=2$$
$$y+z+w=2$$

[60 markah]

2. Let $W = \{(a,b,0,c) \mid a,b,c \in \mathbb{R}\}$

(a) Show that W is a subspace of $\mathbb{R}^4$ over $\mathbb{R}$?

[40 marks]

(b) Find a basis of W over $\mathbb{R}$ and thus gives the dimension of W over $\mathbb{R}$.

[40 marks]

(c) Find a subspace U of P_3 □ over □ such that U is isomorphic to W over □ . Justify your answer.

[40 marks]

2. *Biar* $W = \; a,b,0,c \;\big|\, a,b,c \in$ □

(a) Tunjukkan bahawa W ialah subruang bagi □ ^{4} *atas* □ *?*

[40 markah]

(b) Cari satu asas bagi W atas □ *dan seterusnya berikan dimensi W atas* □ *.*

[40 markah]

(c) Cari satu subruang U bagi P_3 □ *atas* □ *yang berisomorfisma dengan W atas* □ *. Jelaskan jawapan anda.*

[40 markah]

3. (a) Define $T : M_{2x2}$ □ $\longrightarrow P_3$ □ such that $\begin{pmatrix} a & b \\ c & d \end{pmatrix} T = a + bx + \; c - d \;\; x^3.$

(i) Show that T is a linear transformation over □ .

[20 marks]

(ii) Find $\ker \; T$ and $\text{Im} \; T$

[20 marks]

(iii) Is T onto? Why?

[20 marks]

(iv) Is T one to one? Why?

[20 marks]

(b) Construct a linear transformation over □ from M_{2x2} □ to P_3 □ which is onto but NOT one to one

[30 marks]

(c) Construct a linear transformation over □ from M_{2x2} □ to P_3 □ which is one to one but NOT onto

[30 marks]

3. *(a) Takrifkan* $T : M_{2x2}$ □ $\longrightarrow P_3$ □ *sedemikian hingga*

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} T = a + bx + \; c - d \;\; x^3.$$

(i) Tunjukkan T ialah satu transformasi linear atas □ *.*

[20 markah]

(ii) *Cari* $\ker\ T$ *dan* $\operatorname{Im}\ T$

[20 markah]

(iii) *Adakah T keseluruh? Mengapa?*

[20 markah]

(iv) *Adakah T satu ke satu? Mengapa?*

[20 markah]

(b) *Bina satu transformasi linear atas* $\mathbb{R}$ *dari* $M_{2x2}(\mathbb{R})$ *ke* $P_3(\mathbb{R})$ *yang keseluruh tetapi BUKAN satu ke satu*

[30 markah]

(c) *Binak satu transformasi linear atas* $\mathbb{R}$ *dari* $M_{2x2}(\mathbb{R})$ *ke* $P_3(\mathbb{R})$ *yang satu ke satu tetapi BUKAN keseluruh*

[30 markah]

4. Let $A = \begin{pmatrix} 2 & 2 & -1 \\ -1 & -1 & 1 \\ -1 & -2 & 2 \end{pmatrix}$.

(a) Can A be diagonalised?

[70 marks]

(b) Find a basis β of $\mathbb{R}^3$ over $\mathbb{R}$ that will give the Jordan Cannonical form of A.

[30 marks]

4. *Biar* $A = \begin{pmatrix} 2 & 2 & -1 \\ -1 & -1 & 1 \\ -1 & -2 & 2 \end{pmatrix}$.

(a) *Adakah A boleh diperpenjurukan?*

[70 markah]

(b) *Cari suatu asas* β *bagi* $\mathbb{R}^3$ *atas* $\mathbb{R}$ *yang akan memberikan bentuk Jordan berkanun A.*

[30 markah]

5. (a) Let $A = [a_{ij}],\ B = [b_{ij}] \in M_{2\times 2}(\mathbb{R})$. Show that $\langle\ \rangle$ from $M_{2\times 2}(\mathbb{R}) \times M_{2\times 2}(\mathbb{R})$ to $\mathbb{R}$ such that

$$< A, B >= a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$$

is an inner product of $M_{2\times 2}$ □

[30 marks]

(b) Give an orthonormal basis of $M_{2\times 2}$ □ . Justify your answer.

[40 marks]

(c) Let $T : M_{2\times 2}$ □ $\rightarrow M_{2\times 2}$ □ be a linear transformation such that $A \;\; T = A^T$. By using part (b) or otherwise, show that T is a self-adjoint linear transformation.

[40 marks]

(d) Can T be diagonalised? Give your reason.

[10 marks]

5. (a) *Biar* $A = \;\; a_{ij} \;\; , B = \;\; b_{ij} \;\; \in M_{2\times 2}$ □ . *Tunjukkan* $\langle \;\; \rangle$ *dari* $M_{2\times 2}$ □ $\times M_{2\times 2}$ □ *ke* □ *sedemikian hingga*

$$< A, B >= a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$$

ialah suatu hasil darab terkedalaman bagi $M_{2\times 2}$ □

[30 markah]

(b) *Beri satu asas ortonormal bagi* $M_{2\times 2}$ □ . *Jelaskan jawapan anda.*

[40 markah]

(c) *Biar* $T : M_{2\times 2}$ □ $\rightarrow M_{2\times 2}$ □ *ialah satu transformasi linear sedemikian* $A \;\; T = A^T$. *Gunakan bahagian (b) atau cara lain untuk menunjukkan* T *ialah suatu transformasi linear yang swaadjoin.*

[40 markah]

(d) *Bolehkah T diperpenjurukan? Beri alas anda.*

[10 markah]

- ooo O ooo -