UNIVERSITI SAINS MALAYSIA

First Semester Examination
2011/2012 Academic Session

January 2012

MSG 388 – Mathematical Algorithms for Computer Graphics
[Algoritma Matematik untuk Grafik Komputer]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all three** [3] questions.

*[**Arahan:** Jawab **semua tiga** [3] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Suppose the polyline of sequence $\{\boldsymbol{P}_0, \boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_3, \boldsymbol{P}_4\}$ given in Figure 1 is refined once by using Chaikin's subdivision scheme, find the resulting sequence of points and sketch its polyline.

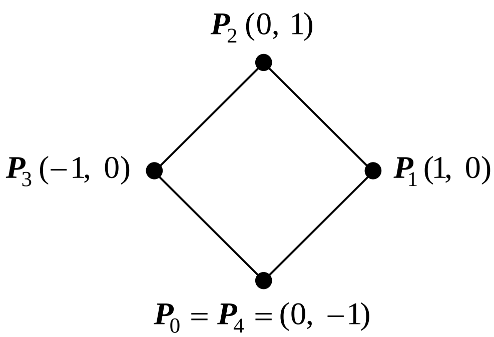


Figure 1

(b) Suppose the Chaikin's subdivision is applied to a given polyhedron. What is the limit produced when this process being iteratively applied infinitely many times onto the resulting sequence of polyhedrons?

(c) Let the Bernstein polynomial of degree n be defined as

$$B_i^n(t) = \frac{n!}{i!(n-i)!}(1-t)^{n-i}t^i, \qquad t \in [0,\ 1], \qquad i \in 0,\ 1,\ \ldots,\ n\ .$$

Given a quadratic rational Bézier curve

$$\boldsymbol{R}(t) = \frac{\boldsymbol{C}_0 B_0^2(t) + w\boldsymbol{C}_1 B_1^2(t) + \boldsymbol{C}_2 B_2^2(t)}{B_0^2(t) + wB_1^2(t) + B_2^2(t)}, \qquad t \in [0,\ 1]$$

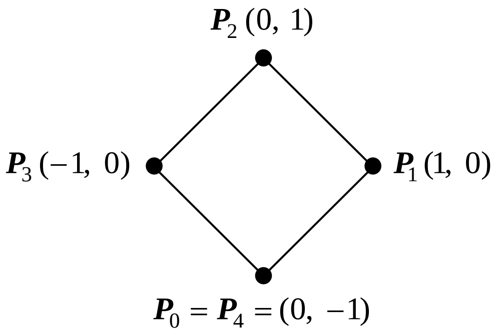
where $w \geq 0$, $\boldsymbol{C}_0 = (1,\ 1)$, $\boldsymbol{C}_1 = (2,\ -1)$ and $\boldsymbol{C}_2 = (3,\ 1)$.

(i) Suppose $w = 2$, use the de Casteljau algorithm to evaluate the point of $\boldsymbol{R}$ at $t = 0.5$.

(ii) Evaluate the parameter w such that the curve $\boldsymbol{R}(t)$ is a circular arc.

[100 marks]

1. (a) *Andaikan poligaris daripada jujukan* $\{\boldsymbol{P}_0, \boldsymbol{P}_1, \boldsymbol{P}_2, \boldsymbol{P}_3, \boldsymbol{P}_4\}$ *yang diberi dalam Rajah 1 diperhaluskan sekali dengan skema subdivisi Chaikin, cari jujukan titik yang dihasilkan dan lakarkan poligaris berkenaan.*



Rajah 1

(b) *Andaikan subdivisi Chaikin digunakan pada suatu polihedron. Apakah had yang dihasilkan apabila proses ini digunakan berulang kali tak terhingga banyaknya ke atas jujukan polihedron yang dihasilkan?*

(c) *Katakan polinomial Bernstein berdarjah* n *ditakrif sebagai*

$$B_i^n(t) = \frac{n!}{i!(n-i)!}(1-t)^{n-i}t^i, \qquad t \in [0,\ 1], \qquad i \in \ 0,\ 1,\ \ldots,\ n \ .$$

Diberi lengkung Bézier nisbah kuadratik

$$\boldsymbol{R}(t) = \frac{\boldsymbol{C}_0 B_0^2(t) + w\boldsymbol{C}_1 B_1^2(t) + \boldsymbol{C}_2 B_2^2(t)}{B_0^2(t) + wB_1^2(t) + B_2^2(t)}, \qquad t \in [0,\ 1]$$

di mana $w \geq 0$, $\boldsymbol{C}_0 = (1,\ 1)$, $\boldsymbol{C}_1 = (2,\ -1)$ *dan* $\boldsymbol{C}_2 = (3,\ 1)$.

(i) *Andaikan* $w = 2$, *gunakan algoritma de Casteljau untuk menilai titik* $\boldsymbol{R}$ *pada* $t = 0.5$.

(ii) *Nilaikan parameter* w *supaya lengkung* $\boldsymbol{R}(t)$ *ialah satu lengkok bulatan.*

[100 markah]

2. (a) State one difference between natural spline and periodic spline.

(b) Let the normalised basis spline of order k be defined recursively as

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad k > 1,$$

and

$$N_i^1(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

over a knot vector $\boldsymbol{u} = (u_0, u_1, \ldots, u_{n+k})$, $n \ge k-1$. Given a cubic B-spline curve defined with four control points $\boldsymbol{D}_i \in \mathbb{R}^2$, $i = 0, 1, 2, 3$, as

$$\boldsymbol{P}(u) = \sum_{i=0}^{3} \boldsymbol{D}_i N_i^4(u), \qquad u_3 \le u \le u_4.$$

(i) Suppose $\boldsymbol{u} = (0, 1, 2, 2, 3, 3, 4, 5)$, $\boldsymbol{D}_0 = (1, 1)$, $\boldsymbol{D}_1 = (1, 2)$, $\boldsymbol{D}_2 = (2, 2)$ and $\boldsymbol{D}_3 = (2, 1)$, use the de Boor algorithm to evaluate the point of $\boldsymbol{P}$ at $u = 2.5$.

(ii) Suppose $\boldsymbol{u} = (-3, -2, -1, 0, 1, 2, 3, 4)$, find the control points of $\boldsymbol{P}$ such that

$$\boldsymbol{P}(u) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} u^2 + \begin{pmatrix} 4 \\ 2 \end{pmatrix} 1-u \;\; u + \begin{pmatrix} 3 \\ 2 \end{pmatrix} 1-u \;\; ^2, \qquad u \in [0, 1].$$

(c) Why a circular arc can not be accurately represented by B-spline?

[100 marks]

2. (a) *Nyatakan satu perbezaan antara splin asli dan splin berkala.*

(b) *Katakan splin asas ternormal berperingkat* k *ditakrif secara rekursi sebagai*

$$N_i^k(u) = \frac{u - u_i}{u_{i+k-1} - u_i} N_i^{k-1}(u) + \frac{u_{i+k} - u}{u_{i+k} - u_{i+1}} N_{i+1}^{k-1}(u), \qquad k > 1,$$

dan

$$N_i^1(u) = \begin{cases} 1, & u_i \le u < u_{i+1} \\ 0, & \textit{di tempat lain} \end{cases}$$

pada suatu vektor simpulan $\boldsymbol{u} = (u_0, u_1, \ldots, u_{n+k})$, $n \ge k-1$. *Diberi lengkung splin-B kubik yang ditakrif dengan empat titik kawalan* $\boldsymbol{D}_i \in \mathbb{R}^2$, $i = 0, 1, 2, 3$, *sebagai*

$$\boldsymbol{P}(u) = \sum_{i=0}^{3} \boldsymbol{D}_i N_i^4(u), \qquad u_3 \le u \le u_4.$$

(i) *Andaikan* $\boldsymbol{u} = (0, 1, 2, 2, 3, 3, 4, 5)$, $\boldsymbol{D}_0 = (1, 1)$, $\boldsymbol{D}_1 = (1, 2)$, $\boldsymbol{D}_2 = (2, 2)$ *dan* $\boldsymbol{D}_3 = (2, 1)$, *gunakan algoritma de Boor untuk menilai titik* $\boldsymbol{P}$ *pada* $u = 2.5$.

(ii) *Andaikan* $\boldsymbol{u} = (-3, -2, -1, 0, 1, 2, 3, 4)$, *cari titik-titik kawalan bagi* $\boldsymbol{P}$ *supaya*

$$\boldsymbol{P}(u) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} u^2 + \begin{pmatrix} 4 \\ 2 \end{pmatrix} 1-u \;\; u + \begin{pmatrix} 3 \\ 2 \end{pmatrix} 1-u \;\; ^2, \qquad u \in [0, 1].$$

(c) *Mengapa lengkok bulatan tidak boleh tepat diwakili oleh splin-B?*

[100 markah]

3. (a) Given a biquadratic Bézier surface

$$\boldsymbol{S}(u,v)=\sum_{i=0}^{2}\sum_{j=0}^{2}\boldsymbol{C}_{i,j}\,\frac{4}{i!\,j!(2-i)!(2-j)!}u^i v^j(1-u)^{2-i}(1-v)^{2-j}\,,$$

where $0\le u\le 1$, $0\le v\le 1$ and $\boldsymbol{C}_{i,j}$ are the Bézier points as

$\boldsymbol{C}_{0,0}=(0,\ 0,\ 1)$, $\boldsymbol{C}_{1,0}=(1,\ 0,\ 2)$, $\boldsymbol{C}_{2,0}=(2,\ 0,\ 1)$,
$\boldsymbol{C}_{0,1}=(0,\ 1,\ 2)$, $\boldsymbol{C}_{1,1}=(1,\ 1,\ 3)$, $\boldsymbol{C}_{2,1}=(2,\ 1,\ 2)$,
$\boldsymbol{C}_{0,2}=(0,\ 2,\ 1)$, $\boldsymbol{C}_{1,2}=(1,\ 2,\ 2)$, $\boldsymbol{C}_{2,2}=(2,\ 2,\ 1)$.

(i) Find the Bézier points of the isoparametric curve $\boldsymbol{S}(u, 0.5)$.

(ii) Use the de Casteljau algorithm to evaluate the cross boundary derivative of $\boldsymbol{S}$ at $(u,v)=(1,\ 0.25)$.

(b) Let the generalised Bernstein polynomials of degree n be defined as

$$B^n_{i,j,k}(u,v,w)=\frac{n!}{i!\,j!k!}u^i v^j w^k\,,\qquad 0\le u,v,w\le 1,\qquad u+v+w=1,$$

for i, j, $k\in\{0, 1, \ldots, n\}$ and $i+j+k=n$. Given a Bézier function

$$S(u,v,w)=\sum_{\substack{i,j,k\ge 0\\ i+j+k=2}} C_{i,j,k}\,B^2_{i,j,k}(u,v,w)$$

on a triangular domain with the Bézier ordinates $C_{2,0,0}=1$, $C_{1,1,0}=3$, $C_{1,0,1}=3$, $C_{0,2,0}=1$, $C_{0,1,1}=2$ and $C_{0,0,2}=1$.

(i) Use the de Casteljau algorithm to evaluate the S at $(u,v,w) = (0.2, 0.5, 0.3)$.

(ii) Find the maximum value of S and its associated parameter (u,v,w).

[100 marks]

3\. (a) *Diberi permukaan Bézier bikuadratik*

$$\boldsymbol{S}(u,v) = \sum_{i=0}^{2}\sum_{j=0}^{2} \boldsymbol{C}_{i,j} \frac{4}{i!\,j!(2-i)!(2-j)!} u^i v^j (1-u)^{2-i}(1-v)^{2-j} ,$$

di mana $0 \le u \le 1$, $0 \le v \le 1$ *dan* $\boldsymbol{C}_{i,j}$ *ialah titik-titik Bézier sebagai*

$\boldsymbol{C}_{0,0} = (0,\ 0,\ 1)$, $\boldsymbol{C}_{1,0} = (1,\ 0,\ 2)$, $\boldsymbol{C}_{2,0} = (2,\ 0,\ 1)$,

$\boldsymbol{C}_{0,1} = (0,\ 1,\ 2)$, $\boldsymbol{C}_{1,1} = (1,\ 1,\ 3)$, $\boldsymbol{C}_{2,1} = (2,\ 1,\ 2)$,

$\boldsymbol{C}_{0,2} = (0,\ 2,\ 1)$, $\boldsymbol{C}_{1,2} = (1,\ 2,\ 2)$, $\boldsymbol{C}_{2,2} = (2,\ 2,\ 1)$.

(i) *Cari titik-titik Bézier bagi lengkung isoparameter* $\boldsymbol{S}(u, 0.5)$.

(ii) *Gunakan algoritma de Casteljau untuk menilai terbitan silang sempadan bagi* $\boldsymbol{S}$ *pada* $(u,v) = (1,\ 0.25)$.

(b) *Katakan polinomial Bernstein teritlak berdarjah* n *ditakrif sebagai*

$$B^n_{i,j,k}(u,v,w) = \frac{n!}{i!\,j!\,k!} u^i v^j w^k , \qquad 0 \le u, v, w \le 1, \qquad u+v+w=1,$$

bagi i, j, $k \in \{0,\ 1,\ \ldots,\ n\}$ *dan* $i+j+k=n$. *Diberi fungsi Bézier*

$$S(u,v,w) = \sum_{\substack{i,j,k\ge 0\\ i+j+k=2}} C_{i,j,k}\, B^2_{i,j,k}(u,v,w)$$

pada satu domain berbentuk tiga segi dengan ordinat Bézier $C_{2,0,0} = 1$, $C_{1,1,0} = 3$, $C_{1,0,1} = 3$, $C_{0,2,0} = 1$, $C_{0,1,1} = 2$ *dan* $C_{0,0,2} = 1$.

(i) *Gunakan algoritma de Casteljau untuk menilai* S *pada* $(u,v,w) = (0.2, 0.5, 0.3)$.

(ii) *Cari nilai maksimum* S *dan nilai parameter yang berkaitan* (u,v,w).

[100 markah]

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