
UNIVERSITI SAINS MALAYSIA

First Semester Examination
2011/2012 Academic Session

January 2012

MAT251 – Introduction to Operations Research
[Pengantar Penyelidikan Operasi]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of EIGHT pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LAPAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all eight** [8] questions.

Arahan: Jawab **semua lapan** [8] soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. A soup company wants to determine the optimal ingredients for its vegetable soup. The main ingredients are:
- Vegetables: Potatoes, Carrots, Onions
 - Meat
 - Water
 - Flavorings

The soup must meet these specifications:

- (i) No more than half of the soup can be vegetables.
- (ii) The ratio of water to meat should be 8:1
- (iii) The amount of meat should be between 5 and 6 percent of the soup.
- (iv) The flavorings should weigh no more than 10 grams.

The cost per gram of the ingredients is RM0.02 for the vegetables, RM0.08 for the meat, RM0.015 for the water and RM0.05 for the flavorings.

Formulate an LP model that will determine the optimal amounts of the various ingredients to produce 200-gram cans of soup at minimum cost.

[10 marks]

1. *Sebuah syarikat pengeluar sup ingin menentukan kuantiti yang optimum bagi bahan-bahan yang digunakan untuk membuat sup sayuran. Bahan-bahan utama sup ialah:*

*Sayur: Kentang, lobak, bawang
Daging
Air
Perasa*

Sup tersebut mesti memenuhi spesifikasi berikut :

- (i) Tidak lebih daripada separuh sup mengandungi sayur.*
- (ii) Nisbah air terhadap daging mestilah 8:1.*
- (iii) Amaun daging mestilah di antara 5 dan 6 peratus daripada sup.*
- (iv) Berat perasa mesti tidak lebih daripada 10 gram.*

Kos per gram setiap bahan masing-masing ialah RM0.02 bagi sayur, RM0.08 untuk daging, RM0.015 bagi air dan RM0.05 untuk perasa.

Rumuskan suatu model PL yang menentukan amaun optimum bahan-bahan sup bagi menghasilkan tin-tin sup 200-gram pada kos minimum.

[10 markah]

2. Consider the following linear programming problem.

$$\begin{aligned} \text{Minimize } z &= 0.4x_1 + 0.5x_2 \\ \text{Subject to } & 0.3x_1 + 0.1x_2 \leq 2.7 \\ & 0.5x_1 + 0.5x_2 = 6 \\ & 0.6x_1 + 0.4x_2 \geq 6 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Let s_1 = slack variable of the first constraint
 R_2 = artificial variable of the second constraint
 s_3 = surplus variable of the third constraint
 R_3 = artificial variable of the third constraint

In Phase I, the objective is to minimize $r = R_2 + R_3$ and the following final tableau is obtained:

Basic	x_1	x_2	s_1	R_2	s_3	R_3	Solution
r	0	0	0	-1	0	-1	0
x_1	1	0	0	-4	-5	5	6
s_1	0	0	1	3/5	1	-1	0.3
x_2	0	1	0	6	5	-5	6

Carry out the Phase II iterations until the optimal solution is obtained.

[10 marks]

2. *Pertimbangkan masalah pengaturcaraan linear berikut:*

Minimumkan $z = 0.4x_1 + 0.5x_2$
Terhadap

$$\begin{aligned} 0.3x_1 + 0.1x_2 &\leq 2.7 \\ 0.5x_1 + 0.5x_2 &= 6 \\ 0.6x_1 + 0.4x_2 &\geq 6 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Biar s_1 = pembolehkan lalai bagi kekangan pertama.
 R_2 = pembolehkan buatan bagi kekangan kedua.
 s_3 = pembolehkan lebihan bagi kekangan ketiga.
 R_3 = pembolehkan buatan bagi kekangan ketiga.

Dalam fasa I, objektifnya ialah meminimumkan $r = R_2 + R_3$ dan tablo terakhir berikut diperolehi:

Asas	x_1	x_2	s_1	R_2	s_3	R_3	Penyelesaian
r	0	0	0	-1	0	-1	0
x_1	1	0	0	-4	-5	5	6
s_1	0	0	1	3/5	1	-1	0.3
x_2	0	1	0	6	5	-5	6

Jalankan lelaran-lelaran fasa II sehingga penyelesaian optimum diperolehi.

[10 markah]

3. For each of the following phenomena, summarize the way in which you become aware of it when solving graphically and when solving by the simplex method.
- (a) Alternative optimal solutions.
 - (b) No feasible solution.
 - (c) Unbounded solution.

[10 marks]

3. Bagi setiap fenomena berikut, ringkaskan cara bagaimana anda menyedarinya semasa menyelesaikan secara bergraf dan semasa menyelesaikannya dengan kaedah simpleks.
- (a) Penyelesaian optimum alternatif.
 - (b) Tiada penyelesaian tersaur.
 - (c) Penyelesaian tak terbatas.

[10 markah]

4. Consider the following LP problem and its optimal tableau.

$$\begin{array}{lll}
 \text{Maximize} & z = 20x_1 + 12x_2 & \text{(Profit)} \\
 \text{Subject to} & 3x_1 + 2x_2 \leq 180 & \text{(Raw material 1)} \\
 & x_1 \leq 50 & \text{(Raw material 2)} \\
 & x_2 \leq 50 & \text{(Raw material 3)} \\
 & x_1, x_2 \geq 0 &
 \end{array}$$

Basic	x_1	x_2	x_3	x_4	x_5	Solution
z	0	0	6	2	0	1180
x_2	0	1	$\frac{1}{2}$	$-\frac{3}{2}$	0	15
x_1	1	0	0	1	0	50
x_5	0	0	$-\frac{1}{2}$	$\frac{3}{2}$	1	35

The slack variables are represented by x_3 , x_4 and x_5 . Based on the given tableau, answer the following questions.

- (a) Determine the status of each resource.
- (b) In terms of the optimal profit, determine the dual prices for raw material 1, 2 and 3.
- (c) Determine the range of feasibility for raw material 1.
- (d) Determine the range of optimality for the coefficient of x_2 in objective function.
- (e) Which of these two possible changes would have a greater impact on the value of the objective function in the final solution?
 - (i) Increase the raw material 2 by 10 units.
 - (ii) Increase the raw material 3 by 40 units.

[20 marks]

4. Pertimbangkan masalah PL berikut dan tablo optimumnya.

$$\begin{array}{lll}
 \text{Maksimumkan} & z = 20x_1 + 12x_2 & (\text{Keuntungan}) \\
 \text{Terhadap} & 3x_1 + 2x_2 \leq 180 & (\text{Bahan mentah 1}) \\
 & x_1 \leq 50 & (\text{Bahan mentah 2}) \\
 & x_2 \leq 50 & (\text{Bahan mentah 3}) \\
 & x_1, x_2 \geq 0 &
 \end{array}$$

Asas	x_1	x_2	x_3	x_4	x_5	Penyelesaian
z	0	0	6	2	0	1180
x_2	0	1	$\frac{1}{2}$	$-\frac{3}{2}$	0	15
x_1	1	0	0	1	0	50
x_5	0	0	$-\frac{1}{2}$	$\frac{3}{2}$	1	35

Pembolehubah-pembolehubah lalai diwakili oleh x_3 , x_4 dan x_5 . Berdasarkan tablo yang diberi, jawab soalan-soal berikut.

- Tentukan status setiap sumber.
- Dalam sebutan keuntungan optimum, tentukan harga dual bagi bahan mentah 1, 2 dan 3.
- Tentukan julat ketersauran bagi bahan mentah 1.
- Tentukan julat keoptimuman bagi pekali x_2 dalam fungsi matlamat.
- Yang mana satu daripada dua perubahan-perubahan berikut akan memberi impak yang lebih ke atas nilai fungsi matlamat dalam penyelesaian akhir?
 - Tambahkan bahan mentah 2 sebanyak 10 unit.
 - Tambahkan bahan mentah 3 sebanyak 40 unit.

[20 markah]

5. A mineral water company has two bottling plants. Plant 1 can produce 10,000 bottles per day, and Plant 2 can produce 6,000 bottles per day. The bottling plants processes three types of sizes: 0.5 liter bottle, 1 liter bottle and 1.5 liter bottle. The processing cost per bottle depends on the plant (see Table below). Each day, 5,000 bottles of each type must be processed.

Bottle	Plant	
	1	2
0.5 liter	5	3
1 liter	4	4
1.5 liter	2	5

- Formulate a balanced transportation problem to minimize the daily costs.
- Identify the basic feasible solutions through the Vogel's method.
- Conduct a transportation simplex and solve for the optimal solution.

[15 marks]

5. Sebuah syarikat air mineral mempunyai dua kilang pembotolan. Kilang 1 boleh menghasilkan 10,000 botol setiap hari, dan kilang 2 boleh menghasilkan 6,000 botol setiap hari. Kilang pembotolan memproses tiga jenis saiz: botol 0.5 liter, botol 1 liter dan botol 1.5 liter. Kos pemprosesan setiap botol bergantung kepada kilang tersebut (lihat jadual di bawah). Setiap hari, 5000 botol untuk setiap jenis perlu diproses.

Botol	Kilang	
	1	2
0.5 liter	5	3
1 liter	4	4
1.5 liter	2	5

- (a) Rumuskan masalah pengangkutan yang seimbang untuk meminimumkan kos harian.
(b) Kenalpastikan penyelesaian asas tersaur melalui kaedah Vogel.
(c) Jalankan kaedah simpleks pengangkutan dan cari penyelesaian optimum.
[15 markah]

6. There are five individuals in your group. In order to complete a group assignment, four tasks have to be completed: Task 1, Task 2, Task 3, and Task 4. The time it takes each individual member in your group to do each task is shown in the following table.

- (a) Who is best suited for each task?
(b) What is the best total time to complete your assignment?

Group Member	Time (Hours)			
	Task 1	Task 2	Task 3	Task 4
1	6	5	2	1
2	9	8	7	3
3	8	5	9	4
4	7	7	8	3
5	5	5	6	4

[10 marks]

6. Terdapat lima individu dalam kumpulan anda. Dalam usaha untuk melengkapkan kerja kursus secara kumpulan, empat tugas perlu disiapkan: Tugas 1, Tugas 2, Tugas 3, dan Tugas 4. Masa yang diperlukan oleh setiap individu dalam kumpulan anda untuk melakukan tugas ditunjukkan dalam jadual berikut.

- (a) Siapakah yang paling sesuai untuk setiap tugas?
(b) Apakah jumlah masa terbaik untuk menyiapkan tugas anda?

[10 markah]

7. A transportation company must move 40 units of a product from Penang to Singapore. The table below indicates transportation costs (in ringgit per unit) between the company's various depots across cities. Blank entries in the table show that transportation cannot occur between the corresponding cities. Find the cheapest shipping schedule.

	Georgetown	Ipoh	Kuala Lumpur	Shah Alam	Seremban	Johor	Singapore
Georgetown	---	6	7	---	38	---	90
Ipoh	6	---	21	16	---	36	80
Kuala Lumpur	7	21	---	12	25	27	---
Shah Alam	---	16	12	---	29	17	---
Seremban	38	---	25	29	---	14	20
Johor	---	36	27	17	14	---	13
Singapore	90	80	---	---	20	13	---

[10 marks]

7. Sebuah syarikat pengangkutan mesti menghantar sejumlah 40 unit suatu produk dari Pulau Pinang ke Singapura. Jadual di bawah menunjukkan kos pengangkutan (dalam ringgit seunit) antara gudang-gudang syarikat yang terletak di beberapa bandar. Catatan-catatan kosong dalam jadual menunjukkan bahawa pengangkutan tidak boleh berlaku di antara bandar-bandar tersebut. Tentukan jadual penghantaran termurah.

	Georgetown	Ipoh	Kuala Lumpur	Shah Alam	Seremban	Johor	Singapura
Georgetown	---	6	7	---	38	---	90
Ipoh	6	---	21	16	---	36	80
Kuala Lumpur	7	21	---	12	25	27	---
Shah Alam	---	16	12	---	29	17	---
Seremban	38	---	25	29	---	14	20
Johor	---	36	27	17	14	---	13
Singapura	90	80	---	---	20	13	---

[10 markah]

8. Consider the simplified list of activities in constructing a house. The description of activities, predecessors and duration of the project are shown in the table below.

Activity	Description	Immediate Predecessors	Duration (Days)
A	Build foundation	-	10
B	Build walls and ceilings	A	16
C	Build roof	B	20
D	Do electrical wiring	B	10
E	Put in windows	B	8
F	Put on siding	E	12
G	Paint house	C,F	6

As the project manager, you are required to:

- Draw a project network,
- Determine the critical path,
- Find the total float of each activity, and
- Find the free float of each activity.

[15 marks]

8. *Pertimbangkan senarai aktiviti dalam pembinaan sebuah rumah. Huraian aktiviti, kegiatan pendahulu, serta tempoh setiap kegiatan dalam menjayakan projek tersebut dinyatakan dalam jadual di bawah.*

Aktiviti	Huraian	Pendahulu	Tempoh (hari)
A	Bina asas susuk rangka	-	10
B	Bina tembok-tembok dan siling-siling	A	16
C	Bina Bumbung	B	20
D	Pendawaian elektrik	B	10
E	Masukkan tingkap-tingkap	B	8
F	Masukkan papan dinding	E	12
G	Cat rumah	C,F	6

Sebagai seorang pengurus projek, anda dikehendaki:

- melukis rangkaian projek pembinaan rumah tersebut ,*
- menentukan laluan genting,*
- mencari jumlah apungan setiap aktiviti, dan*
- mencari apungan bebas setiap aktiviti*

[15 markah]