UNIVERSITI SAINS MALAYSIA

First Semester Examination
2011/2012 Academic Session

January 2012

MAA 111 - Linear Algebra for Science Students
[Aljabar untuk Pelajar Sains]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all eight** [8] questions.

*[Arahan: Jawab **semua lapan** [8] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

(1) Let A be a 2×2 matrix with $A^2 = I$.

(a) Show that either $A = I$ or $A = -I$ or $\mathrm{tr}(A) = 0$.

(b) Give an example of a matrix with $A \neq \pm I$ for which $A^2 = I$.

[10 marks]

(1) Biar A suatu matriks 2×2 dengan $A^2 = I$.

(a) Tunjukkan bahawa $A = I$ atau $A = -I$ atau $\mathrm{tr}(A) = 0$.

(b) Beri satu contoh suatu matriks $A \neq \pm I$ yang mana $A^2 = I$.

[10 markah]

(2) Prove that the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

is invertible and represent it as a product of elementary matrices.

[10 marks]

(2) Buktikan bahawa matriks

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

adalah tersongsangkan dan wakilkan matriks tersebut sebagai hasil darab matriks permulaan.

[10 markah]

(3) Let the matrix A be given by

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}.$$

(a) List all minors and all cofactors of A, and find A^{-1}.

(b) Solve the system $A\boldsymbol{x}=\boldsymbol{b}$ where

$$\boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}.$$

[10 marks]

(3) Biar matriks

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}.$$

(a) Senaraikan semua minor dan kofaktor bagi A, dan dapatkan A^{-1}.

(b) Selesaikan sistem $A\boldsymbol{x}=\boldsymbol{b}$ yang mana

$$\boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}.$$

[10 markah]

(4) Determine the values of k for which the matrix A fails to be invertible:

(a)

$$A = \begin{pmatrix} 2-k & -1 \\ -1 & 2-k \end{pmatrix},$$

(b)

$$A = \begin{pmatrix} 2 & k-1 & 1 \\ 1+k & 2 & 3 \\ 3 & 4k & -1 \end{pmatrix}.$$

[10 marks]

(4) Apakah nilai k supaya A gagal tersongsangkan:

(a)

$$A = \begin{pmatrix} 2-k & -1 \\ -1 & 2-k \end{pmatrix},$$

(b)

$$A = \begin{pmatrix} 2 & k-1 & 1 \\ 1+k & 2 & 3 \\ 3 & 4k & -1 \end{pmatrix}.$$

[10 markah]

(5) (a) If $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors in $\mathbb{R}^n$, show that

(i)
$$\|\boldsymbol{u} \pm \boldsymbol{v}\|^2 = \|\boldsymbol{u}\|^2 + \|\boldsymbol{v}\|^2 \pm 2\boldsymbol{u} \cdot \boldsymbol{v}.$$

(ii) the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are orthogonal if and only if $\|\boldsymbol{u}+\boldsymbol{v}\| = \|\boldsymbol{u}-\boldsymbol{v}\|$.

(b) (i) State the Cauchy-Schwarz inequality.

(ii) Use it to show that $\|\boldsymbol{u}+\boldsymbol{v}\| \leq \|\boldsymbol{u}\| + \|\boldsymbol{v}\|$.

[10 marks]

(5) (a) Jika $\boldsymbol{u}$ dan $\boldsymbol{v}$ adalah vector-vektor dalam $\mathbb{R}^n$, tunjukkan bahawa

(i)
$$\|\boldsymbol{u} \pm \boldsymbol{v}\|^2 = \|\boldsymbol{u}\|^2 + \|\boldsymbol{v}\|^2 \pm 2\boldsymbol{u} \cdot \boldsymbol{v}.$$

(ii) vektor $\boldsymbol{u}$ dan $\boldsymbol{v}$ adalah ortogon jika dan hanya jika $\|\boldsymbol{u}+\boldsymbol{v}\| = \|\boldsymbol{u}-\boldsymbol{v}\|$.

(b) (i) Nyatakan ketaksamaan Cauchy-Schwarz.

(ii) Gunakan ketaksamaan tersebut untuk tunjukkan bahawa $\|\boldsymbol{u}+\boldsymbol{v}\| \leq \|\boldsymbol{u}\| + \|\boldsymbol{v}\|$.

[10 markah]

(6) (a) (i) Show that $W = \{(x, y, 0) | x, y \in \mathbb{R}\}$ is a vector subspace of $\mathbb{R}^3$.

(ii) Is the set $B = \{(1, 0, 0), (1, 1, 0)\}$ a basis for W?

(iii) What is the dimension of W?

(b) Let $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ be vectors in $\mathbb{R}^n$. Prove that

$$\boldsymbol{u} + \boldsymbol{w} = \boldsymbol{v} + \boldsymbol{w} \Rightarrow \boldsymbol{u} = \boldsymbol{v}.$$

[10 marks]

(6) (a) (i) Tunjukkan bahawa $W = \{(x, y, 0) | x, y \in \mathbb{R}\}$ *adalah suatu subruang vektor bagi* $\mathbb{R}^3$.

(ii) Adakah set $B = \{(1, 0, 0), (1, 1, 0)\}$ *suatu basis bagi* W*?*

(iii) Apakah dimensi bagi W*?*

(b) Biar $\boldsymbol{u}$, $\boldsymbol{v}$ *dan* $\boldsymbol{w}$ *vektor-vektor dalam* $\mathbb{R}^n$. *Buktikan bahawa*

$$\boldsymbol{u} + \boldsymbol{w} = \boldsymbol{v} + \boldsymbol{w} \Rightarrow \boldsymbol{u} = \boldsymbol{v}.$$

[10 markah]

(7) Determine λ and μ so that the system of equations

$$\begin{aligned} x + y + z &= 6, \\ x + 2y + 3x &= 10, \\ x + 2y + \lambda z &= \mu \end{aligned}$$

has

(a) no solution
(b) unique solution, or
(c) infinite number of solutions.

[10 marks]

(7) Tentukan λ *dan* μ *supaya sistem persamaan*

$$\begin{aligned} x + y + z &= 6, \\ x + 2y + 3x &= 10, \\ x + 2y + \lambda z &= \mu \end{aligned}$$

mempunyai

(a) *tiada penyelesaian*
(b) *penyelesaian unik, atau*
(c) *penyelesaian tak terhingga banyak.*

[10 markah]

(8) Orthogonally diagonalize the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}.$$

[10 marks]

(8) Pepenjurukan secara ortogon matriks

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}.$$

[10 markah]

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