UNIVERSITI SAINS MALAYSIA

First Semester Examination
2011/2012 Academic Session

January 2012

MAT 282 – Engineering Computation I
[Pengiraan Kejuruteraan I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Solve the following system using a LU decomposition

$$
\begin{aligned}
x_1 + 2x_2 + 3x_3 &= 5 \\
2x_1 - 4x_2 + 6x_3 &= 18 \\
3x_1 - 9x_2 - 3x_3 &= 6
\end{aligned}
$$

[50 marks]

(b) What is meant by a set of linear equations $Ax = b$ is ill-conditioned?

[10 marks]

(c) Solve the following system by using partial pivoting

$$
\begin{aligned}
8x_2 + 2x_3 &= -7 \\
3x_1 + 5x_2 + 2x_3 &= 8 \\
6x_1 + 2x_2 + 8x_3 &= 26
\end{aligned}
$$

[40 marks]

1. (a) Selesaikan sistem berikut dengan menggunakan kaedah penguraian LU

$$
\begin{aligned}
x_1 + 2x_2 + 3x_3 &= 5 \\
2x_1 - 4x_2 + 6x_3 &= 18 \\
3x_1 - 9x_2 - 3x_3 &= 6
\end{aligned}
$$

[50 markah]

(b) Apakah yang dimaksudkan dengan set persamaan linear $Ax = b$ masalah berkeadaan sakit?

[10 markah]

(c) Selesaikan sistem berikut dengan menggunakan kaedah pangsi separa

$$
\begin{aligned}
8x_2 + 2x_3 &= -7 \\
3x_1 + 5x_2 + 2x_3 &= 8 \\
6x_1 + 2x_2 + 8x_3 &= 26
\end{aligned}
$$

[40 markah]

2. (a) By using Lagrange polynomials, determine the coefficients a_i of the quadratic curve

$$y = a_2 x^2 + a_1 x + a_0$$

which interpolates the points (1.1,7.1), (4.2,12.3), and (6.8,16.8). Hence estimate $y(5.3)$ and the value of x for which $y(x) = 10$.

[55 marks]

(b) Write down the divided difference table for e^x using the following values

x	e^x
0.0	1.00000
0.4	1.49182
0.9	2.45960
1.5	4.48169
1.8	6.04965

Estimate $e^{1.2}$ using

(i) cubic interpolation with $x_0 = 0.0$

(ii) cubic interpolation with $x_0 = 0.4$

(iii) quartic interpolation with $x_0 = 0.0$

[45 marks]

2. *(a) Dengan menggunakan interpolasi Lagrange, tentukan pemalar a_i dalam persamaan kuadratik*

$$y = a_2 x^2 + a_1 x + a_0$$

yang interpolasi titik-titik (1.1,7.1), (4.2,12.3), dan (6.8,16.8). Dengan itu anggarkan $y(5.3)$ dan nilai x untuk $y(x) = 10$.

[55 markah]

(b) Tuliskan jadual beza terhingga untuk e^x menggunakan nilai-nilai

x	e^x
0.0	*1.00000*
0.4	*1.49182*
0.9	*2.45960*
1.5	*4.48169*
1.8	*6.04965*

Anggarkan $e^{1.2}$ menggunakan

(i) interpolasi kubik dengan $x_0 = 0.0$

(ii) interpolasi kubik dengan $x_0 = 0.4$

(iii) interpolasi kuartik dengan $x_0 = 0.0$

[45 markah]

3. (a) Try the power method on

$$A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$$

with $x_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $x_0 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Explain and justify the results. [30 marks]

(b) By using subintervals, $n = 4$ approximate the value of the following integral

$$\int_0^2 e^{x^2} dx$$

with
(i) Midpoint Rule
(ii) Trapezoid Rule
(iii) Simpson's 1/3 Rule

[30 marks]

(b) The error in Simpson's rule for approximating the integral $I = \int_a^b f(x)dx$ is;

$$-\frac{(b-a)^5}{2880} f^{(4)}(t_s), a < t_s < b$$

By assuming that the fourth derivative of *f(x)* is continuous in [a, b], show that the error in the Composite Simpson's Rule for integrating $f(x)$ from *a* to *b* is

$$-\frac{(b-a)}{180} h^4 f^{(4)}(\Theta), a < \Theta < b$$

where *h* is the step size.

[40 marks]

3. (a) *Cuba kaedah kuasa ke atas*

$$A = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$$

dengan $x_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ *dan* $x_0 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Terangkan dan huraikan jawapan anda.

[30 markah]

(b) *Dengan menggunakan sub selang,* $n = 4$ *anggarkan nilai kamiran berikut*

$$\int_0^2 e^{x^2} dx$$

dengan

(i) *Kaedah Titik Tengah*
(ii) *Kaedah Trapezoid*
(iii) *Kaedah 1/3 Simpson*

[30 markah]

(c) *Ralat untuk kaedah Simpson bagi menganggarkan kamiran* $I = \int_a^b f(x)dx$

diberikan sebagai;

$$-\frac{(b-a)^5}{2880} f^{(4)}(t_s), a < t_s < b$$

Dengan membuat andaian bahawa terbitan keempat untuk f(x) adalah selanjar dalam selang [a, b], tunjukkan bahawa ralat untuk kaedah Komposit Simpson bagi mengamirkan $f(x)$ *dari a ke b adalah*

$$-\frac{(b-a)}{180} h^4 f^{(4)}(\Theta), a < \Theta < b$$

di mana h adalah saiz langkah.

[40 markah]

4. (a) Use the Newton-Raphson method to find the smallest positive roots for $f(x) = x - \tan(x)$ with $x_0 = 4.5$ up to 2 iterations.

[30 marks]

(b) Approximate the solutions of the following system of linear equations

$$5x_1 - 2x_2 + 3x_3 = -1$$
$$-3x_1 + 9x_2 + x_3 = 2$$
$$2x_1 - x_2 - 7x_3 = 3$$

by using

(i) Jacobi method;

(ii) Gauss-Seidel method

Continue the iterations until two successive approximations are identical when rounded to three significant digits. Use initial values $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$.
Which method is better? Explain your answer.

[40 marks]

(c) Complete six iterations of the power method to approximate the dominant eigenvector of $A = \begin{pmatrix} 2 & -12 \\ 1 & -5 \end{pmatrix}$ with an initial nonzero approximations of $x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

[30 marks]

4. (a) *Gunakan kaedah Newton-Raphson untuk mencari punca positif yang terkecil bagi* $f(x) = x - \tan(x)$ *dengan* $x_0 = 4.5$ *sehingga dua lelaran.*

[30 markah]

(b) *Anggarkan penyelesaian untuk sistem linear berikut*

$$5x_1 - 2x_2 + 3x_3 = -1$$
$$-3x_1 + 9x_2 + x_3 = 2$$
$$2x_1 - x_2 - 7x_3 = 3$$

menggunakan

(i) *Kaedah Jacobi;*

(ii) *Kaedah Gauss-Seidel*

Teruskan lelaran sehingga dua lelaran berturutan mempunyai nilai serupa apabila dibundarkan kepada tiga angka bererti.

Gunakan nilai awalan $x_1^{(0)} = x_2^{(0)} = x_3^{(0)} = 0$.

Kaedah yang mana lebih baik? Jelaskan jawapan anda.

[40 markah]

(c) *Guna Kaedah Kuasa untuk 6 lelaran bagi menganggarkan eigen vektor dominan bagi* $A = \begin{pmatrix} 2 & -12 \\ 1 & -5 \end{pmatrix}$ *dengan nilai awalan* $x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

[30 markah]

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