UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2011/2012

Ogos 2012

MAT 102 – Advanced Calculus
[Kalkulus Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>FOUR</u> pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>EMPAT</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

<u>Instructions</u> : Answer **<u>all ten</u>** [10] questions.

*[**<u>Arahan</u>** : Jawab **<u>semua sepuluh</u>** [10] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Find $\lim\limits_{x\to\infty} x\sin \pi/x$. [5 marks]

1. *Dapatkan had $\lim\limits_{x\to\infty} x\sin \pi/x$.* [5 markah]

2. Prove that if $\lim\limits_{n\to\infty} a_n = 0$ and b_n is bounded, then $\lim\limits_{n\to\infty} a_n b_n = 0$.

    [7 marks]

2. *Buktikan bahawa jika had $\lim\limits_{n\to\infty} a_n = 0$ dan b_n terbatas, maka had $\lim\limits_{n\to\infty} a_n b_n = 0$.*

    [7 markah]

3. Test the convergence of the series.

    (a) $\displaystyle\sum_{k=1}^{\infty} \frac{-3^{k+1}}{2^{3k}}$

    (b) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k^2-1}}{k^3+2k^2+5}$

    [10 marks]

3. *Uji penumpuan siri.*

    (a) $\displaystyle\sum_{k=1}^{\infty} \frac{-3^{k+1}}{2^{3k}}$

    (b) $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k^2-1}}{k^3+2k^2+5}$

    [10 markah]

4. Discuss the continuity of f if

    $$f(x,y) = \begin{cases} \dfrac{x^2 y^2}{x^2+y^2} & , \quad (x,y) \neq (0,0) \\ 0 & , \quad (x,y) = (0,0) \end{cases}$$

    [15 marks]

4. *Bincangkan keselanjaran bagi f jika*

    $$f(x,y) = \begin{cases} \dfrac{x^2 y^2}{x^2+y^2} & , \quad (x,y) \neq (0,0) \\ 0 & , \quad (x,y) = (0,0) \end{cases}$$

    [15 markah]

5. Let $f(x) = x^{5/2}$. Find

    (a) $T_3(x)$ at $a = 4$.

    (b) the error between $f(x)$ and $T_3(x)$ for $2 \le x \le 6$.

    [12 marks]

5. *Biar $f(x) = x^{5/2}$. Dapatkan*

    (a) $T_3(x)$ pada $a = 4$.

    (b) ralat antara $f(x)$ dan $T_3(x)$ bagi $2 \le x \le 6$.

    [12 markah]

6. Determine whether the integral $\int_{-2}^{\infty} \dfrac{dx}{(x+1)^{3/5}}$ is convergent or divergent.

    [8 marks]

6. *Tentukan samada kamiran $\int_{-2}^{\infty} \dfrac{dx}{(x+1)^{3/5}}$ menumpu atau mencapah.*

    [8 markah]

7. If $z = xy + f(x^2 + y^2)$ where f is differentiable, show that

    $$x\left(\frac{\partial z}{\partial y} - x\right) = y\left(\frac{\partial z}{\partial x} - y\right).$$

    [7 marks]

7. *Jika $z = xy + f(x^2 + y^2)$ yang mana f terbezakan, tunjukkan bahawa*

    $$x\left(\frac{\partial z}{\partial y} - x\right) = y\left(\frac{\partial z}{\partial x} - y\right).$$

    [7 markah]

8.   Determine whether there exists a direction $\underset{\sim}{u}$ in which the rate of change(degrees Celsius/feet) of the temperature function $T\ (x, y, z) = 2xy + yz$ at $P\ (1, -1, 1)$ is -3.

[6 marks]

8.   *Tentukan samada wujud arah $\underset{\sim}{u}$ yang mana kadar perubahan (darjah Celsius per kaki) fungsi suhu $T\ (x, y, z) = 2xy + yz$ pada $P\ (1, -1, 1)$ adalah -3.*

[6 markah]

9.   Show that the maximum value of $f\ (x, y, z) = x^2 y^2 z^2$ on the sphere $x^2 + y^2 + z^2 = r^2$ is $\left(\dfrac{r^2}{3}\right)^3$.

[15 marks]

9.   *Tunjukkan bahawa nilai maksimum $f\ (x, y, z) = x^2 y^2 z^2$ di atas sfera $x^2 + y^2 + z^2 = r^2$ adalah $\left(\dfrac{r^2}{3}\right)^3$.*

[15 markah]

10.   (a)   Evaluate $\displaystyle\int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{0}^{x} (x^2 + y^2)\ dz\,dx\,dy$ by changing to cylindrical coordinates.

(b)   Find the volume of the solid under the parabolic cylinder $z = x^2$ and above the region enclosed by the parabola $y = 6 - x^2$ and the line $y = x$ in the *xy*-plane.

[15 marks]

10.   (a)   *Nilaikan $\displaystyle\int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{0}^{x} (x^2 + y^2)\ dz\,dx\,dy$ dengan menukar ke koordinat silinder.*

(b)   *Dapatkan isipadu pepejal di bawah silinder parabolik $z = x^2$ dan atas rantau yang dibatasi oleh parabola $y = 6 - x^2$ dan garis $y = x$ dalam satah- xy.*

[15 markah]

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