UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2011/2012

Ogos 2012

MAT 122 – Differential Equations I
[Persamaan Pembezaan I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** (4) questions.

*[**Arahan**: Jawab **semua empat** (4) soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Find the general solution of the differential equation

$$y' = \frac{2x}{y(1+x^2)}.$$

(b) (i) By using the Existence and Uniqueness theorem, find an interval in which the initial value problem

$$xy' + 2y = 4x^2, \quad y(1) = 2,$$

has a unique solution.

(ii) Find the solution of the above initial value problem. Where is the solution valid?

(c) Consider the initial value problem

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, \qquad y(0) = 1.$$

(i) Find two functions that satisfy the initial condition $y(0) = 1$ for the above differential equation.

(ii) Does this fact contradict the Existence and Uniqueness Theorem? Justify your answer using the relevant theorem.

[100 marks]

1. (a) Dapatkan penyelesaian am bagi persamaan pembezaan

$$y' = \frac{2x}{y(1+x^2)}.$$

(b) (i) Dengan menggunakan teorem Kewujudan dan Keunikan, dapatkan suatu selang yang mana masalah nilai awal

$$xy' + 2y = 4x^2, \quad y(1) = 2,$$

mempunyai satu penyelesaian unik.

(ii) Dapatkan penyelesaian bagi masalah nilai awal di atas. Di manakah penyelesaian ini sah?

(c) Pertimbangkan masalah nilai awal

$$\frac{dy}{dx} = \frac{3x^2 + 4x + 2}{2(y-1)}, \qquad y(0) = 1.$$

(i) Dapatkan dua fungsi yang memenuhi syarat awal $y(0) = 1$ bagi persamaan pembezaan di atas.

(ii) Adakah fakta ini bercanggah dengan teorem Kewujudan dan Keunikan? Berikan penjelasan anda dengan disokong oleh teorem yang berkaitan.

[100 markah]

2. (a) Given that $\mu(x,y)=x^m y^n$ is an integrating factor for the following differential equation

$$3y+3y^3+(xy^2-x)\frac{dy}{dx}=0.$$

Find m and n.

(b) (i) Solve the second order homogeneous differential equation $y''-4y'-12y=0$.

(ii) Find the particular solution, y_p for the non-homogeneous differential equation $y''-4y'-12y=3e^{5x}$.

(iii) Hence, find the general solution for $y''-4y'-12y=3e^{5x}$.

(c) Solve the differential equation $y''-6y'+9y=\dfrac{e^{3x}}{x}$.

[100 marks]

2. (a) Diberikan $\mu(x,y)=x^m y^n$ adalah suatu faktor pengamir bagi persamaan pembezaan

$$3y+3y^3+(xy^2-x)\frac{dy}{dx}=0.$$

Dapatkan m dan n.

(b) (i) Selesaikan persamaan pembezaan homogen peringkat kedua $y''-4y'-12y=0$.

(ii) Dapatkan penyelesaian khusus , y_k , bagi persamaan pembezaan tak homogen $y''-4y'-12y=3e^{5x}$.

(iii) Seterusnya, dapatkan penyelesaian am bagi $y''-4y'-12y=3e^{5x}$.

(c) Selesaikan persamaan pembezaan $y''-6y'+9y=\dfrac{e^{3x}}{x}$.

[100 markah]

3. (a) Consider the initial value problem

$$y' = x + y, \quad y(x_0) = y_0 .$$

(i) By using Euler's formula show that

$$y_k = y_{k-1} + h(x_{k-1}^2 + y_{k-1}^2), \quad k = 1, 2, 3, ...$$

(ii) Show that

$$y_n = y_0 + h\sum_{i=0}^{n-1}(x_i^2 + y_i^2), \quad n = 1, 2, 3, ...$$

(iii) Given $x_0 = 0$ and $y_0 = 1$, approximate $y(0.3)$ using $h = 0.1$.

(iv) Obtain a formula for the local formula error e_{n+1} in terms of x and the exact solution ϕ if the Euler's Method is used for the above initial value problem.

(b) Show that $x_0 = 0$ is an ordinary point for the differential equation $y'' - xy - y = 0$. Then, solve the differential equation.

[100 marks]

3. (a) Pertimbangkan masalah nilai awal

$$y' = x + y, \quad y(x_0) = y_0 .$$

(i) Dengan menggunakan rumus Euler, tunjukkan bahawa

$$y_k = y_{k-1} + h(x_{k-1}^2 + y_{k-1}^2), \quad k = 1, 2, 3, ...$$

(ii) Tunjukkan bahawa

$$y_n = y_0 + h\sum_{i=0}^{n-1}(x_i^2 + y_i^2), \quad n = 1, 2, 3, ...$$

(iii) Diberikan $x_0 = 0$ dan $y_0 = 1$, anggarkan $y(0.3)$ dengan menggunakan $h = 0.1$.

(iv) Dapatkan suatu rumus bagi ralat rumus setempat e_{n+1} dalam sebutan x dan penyelesaian tepat ϕ jika kaedah Euler digunakan untuk menyelesaikan masalah nilai awal tersebut.

(b) Tunjukkan bahawa $x_0 = 0$ adalah suatu titik biasa bagi persamaan pembezaan $y'' - xy - y = 0$. Seterusnya, selesaikan persamaan pembezaan tersebut.

[100 markah]

4. (a) Solve the initial value problem

$$\mathbf{x}' = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 2 & 1 \\ 4 & 1 & -3 \end{pmatrix}\mathbf{x}, \quad \mathbf{x}(1) = \begin{pmatrix} 1 \\ 4 \\ -7 \end{pmatrix}.$$

(b) Two spaces A_1 and A_2, with volumes 10 litres each, are segregated with a membrane. Let $y_1(t), y_2(t)$ be the amount of nutrient solution in A_1 and A_2 at time *t* respectively. Then,

$$P_1(t) = \frac{y_1(t)}{10}, \quad P_2(t) = \frac{y_2(t)}{10}$$

represent the concentrations of the solution in A_1 and A_2, respectively. The nutrient solution diffuses through the membrane from one space to another where the rate of diffusion is proportional to the difference in the concentration of the solution in both spaces, that is,

$$y_1'(t) = k(P_2(t) - P_1(t))$$
$$y_2'(t) = k(P_1(t) - P_2(t))$$

where *k* is the proportional constant (*k* > 0).

(i) Show that

$$P_1''(t) + \frac{k}{5}P_1'(t) = 0$$

and

$$P_1'(t) = P_1(0) - \frac{1}{2}[P_2(0) - P_1(0)](e^{-kt/5} - 1).$$

(ii) The equilibrium situation is obtained in an infinite time. What is the concentration $P_1(t)$ at that time?

[100 marks]

4. (a) Selesaikan masalah nilai awal

$$\mathbf{x}' = \begin{pmatrix} 3 & 1 & -2 \\ -1 & 2 & 1 \\ 4 & 1 & -3 \end{pmatrix}\mathbf{x}, \quad \mathbf{x}(1) = \begin{pmatrix} 1 \\ 4 \\ -7 \end{pmatrix}.$$

(b) Dua ruang A_1 dan A_2, masing-masing dengan isipadu 10 liter, diasingkan dengan suatu selaput. Katakan $y_1(t), y_2(t)$ ialah kuantiti larutan zat masing-masing di dalam A_1 and A_2 pada masa t. Maka,

$$P_1(t) = \frac{y_1(t)}{10}, \quad P_2(t) = \frac{y_2(t)}{10}$$

merupakan kepekatan larutan masing-masing di dalam A_1 dan A_2. Larutan zat dapat meresap melalui selaput dari suatu ruang ke ruang yang satu lagi dengan kadar resapan berkadaran dengan perbezaan di antara kepekatan larutan di dalam kedua-dua ruang, iaitu

$$y_1'(t) = k(P_2(t) - P_1(t))$$
$$y_2'(t) = k(P_1(t) - P_2(t))$$

dengan k sebagai pemalar berkadaran(k > 0).

(i) *Tunjukkan bahawa*

$$P_1''(t) + \frac{k}{5} P_1'(t) = 0$$

dan

$$P_1'(t) = P_1(0) - \frac{1}{2}[P_2(0) - P_1(0)](e^{-kt/5} - 1).$$

(ii) *Keadaan keseimbangan dicapai selepas suatu masa yang terlalu panjang. Apakah nilai kepekatan* $P_1(t)$ *ketika itu?*

[100 markah]

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