UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2011/2012

Ogos 2012

MSS 212 – Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all five** [5] questions.

*[**Arahan:** Jawab **semua lima** [5] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Explain why a non-singular $n \times n$ matrix will have minimally n many non-zero entries.

[30 marks]

(b) Find $\det \begin{pmatrix} 1 & a & c & e \\ 0 & w & c & e \\ -1 & -a & v & e \\ -1 & -a & -c & t \end{pmatrix}$.

[30 marks]

(c) Solve the following system of linear equations by using Cramer's rule

$$\begin{aligned} x \quad + \frac{z}{2} + w &= 1 \\ x + y \quad + w &= 0 \\ x + y + z \quad &= 2 \\ y + z + w &= 2 \end{aligned}$$

[60 marks]

1. (a) Jelaskan mengapa suatu matriks $n \times n$ yang tak singular akan ada sekurang-kurangmya n banyak pemasukan yang bukan kosong.

[30 markah]

(b) Cari $\det \begin{pmatrix} 1 & a & c & e \\ 0 & w & c & e \\ -1 & -a & v & e \\ -1 & -a & -c & t \end{pmatrix}$.

[30 markah]

(c) Selesaikan sistem persamaan linear berikut dengan menggunakan petua Cramer

$$\begin{aligned} x \quad + \frac{z}{2} + w &= 1 \\ x + y \quad + w &= 0 \\ x + y + z \quad &= 2 \\ y + z + w &= 2 \end{aligned}$$

[60 markah]

2. Let $W = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \middle| a, b, c \in \square \right\}$

(a) Show that W is a subspace of $M_{2\times 2}\ \square$ over $\square$.

[40 marks]

(b) Find a basis of W over $\square$ and thus gives its dimension over $\square$.

[40 marks]

(c) Find a basis of $M_{2\times 2}\ \square$ over $\square$ by extending or enlarging the basis of W over $\square$ that was found in part (b). Justify your answer.

[40 marks]

2. Biar $W = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \middle| a, b, c \in \square \right\}$

(a) Tunjukkan bahawa W ialah subruang bagi $M_{2\times 2}\ \square$ *atas* $\square$.

[40 markah]

(b) Cari satu asas bagi W atas $\square$ *dan seterusnya berikan dimensi W atas* $\square$.

[40 markah]

(c) Cari satu asas bagi $M_{2\times 2}\ \square$ *atas* $\square$ *dengan melanjutkan asas W atas* $\square$ *yang diperolehi dari bahagian (b). Jelaskan jawapan anda.*

[40 markah]

3. (a) Define $T : M_{2x3}\ \square \longrightarrow P_3\ \square$ such that

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} T = a + \ b - f \ \ x + \ c - d \ \ x^3 .$$

(i) Is T a linear transformation over $\square$?

[20 marks]

(ii) Find $\ker\ T$ and $\operatorname{Im}\ T$

[20 marks]

(iii) Is T onto? Why?

[20 marks]

(iv) Is T one to one? Why?

[20 marks]

(b) Construct an isomorphism over $\mathbb{R}$ from $M_{2x2}\ \mathbb{R}$ to $P_3\ \mathbb{R}$

[30 marks]

(c) Construct an onto linear transformation over $\mathbb{R}$ from $M_{2x2}\ \mathbb{R}$ to $\mathbb{R}^2$

[30 marks]

3. *(a)* *Takrifkan* $T : M_{2x3}\ \mathbb{R} \longrightarrow P_3\ \mathbb{R}$ *sedemikian hingga*

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} T = a + b - f\ x + c - d\ x^3.$$

(i) *Adakah T satu transformasi linear atas* $\mathbb{R}$ *?*

[20 markah]

(ii) *Cari* $\ker T$ *dan* $\operatorname{Im} T$

[20 markah]

(iii) *Adakah T keseluruhan? Mengapa?*

[20 markah]

(iv) *Adakah T satu ke satu? Mengapa?*

[20 markah]

(b) *Bina satu isomorfisma atas* $\mathbb{R}$ *dari* $M_{2x2}\ \mathbb{R}$ *ke* $P_3\ \mathbb{R}$.

[30 markah]

(c) *Bina satu transformasi linear atas* $\mathbb{R}$ *yang keseluruhan dari* $M_{2x2}\ \mathbb{R}$ *ke* $\mathbb{R}^2$.

[30 markah]

4. Let $A = \begin{pmatrix} 2 & 2 & -1 \\ -1 & -1 & 1 \\ -1 & -2 & 2 \end{pmatrix}$. Find A^{100}.

[100 marks]

4. *Biar* $A = \begin{pmatrix} 2 & 2 & -1 \\ -1 & -1 & 1 \\ -1 & -2 & 2 \end{pmatrix}$. *Cari* A^{100}.

[100 markah]

5. (a) Show that $\mathbb{C}^n$ is an inner product space over $\mathbb{C}$

[30 marks]

(b) Give an orthonormal basis of $\mathbb{C}^n$ over $\mathbb{C}$. Justify your answer.

[40 marks]

(c) Let $T:\mathbb{C}^5 \to \mathbb{C}^5$ be a linear transformation such that

$$x, y, z, v, w\ T = \ 2x+y, 2y+x, 2z+v+w, z+2v+w, z+v+2w\ .$$

By using part (b) or otherwise, show that T is a self-adjoint linear transformation.

[40 marks]

(d) Can T be diagonalised? Give your reason.

[10 marks]

5. (a) Tunjukkan bahawa $\mathbb{C}^n$ ialah suatu ruang hasildarab terkedalam atas $\mathbb{C}$.

[30 markah]

(b) Beri satu asas ortonormal bagi $\mathbb{C}^n$ atas $\mathbb{C}$. Justifikasikan jawapan anda.

[40 markah]

(c) Biar $T:\mathbb{C}^5 \to \mathbb{C}^5$ satu transformasi linear sedemikian hingga

$$x, y, z, v, w\ T = \ 2x+y, 2y+x, 2z+v+w, z+2v+w, z+v+2w\ .$$

Gunakan bahagian (b) atau cara lain untuk menunjukkan T ialah suatu transformasi linear yang swaadjoin.

[40 markah]

(d) Bolehkah T dipepenjurukan? Beri alasan anda.

[10 markah]

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