UNIVERSITI SAINS MALAYSIA

Second Semester Examination
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MSG 367 – Time Series Analysis
[Analisis Siri Masa]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of TWENTY EIGHT pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi DUA PULUH LAPAN muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Discuss the following, perhaps with example:

(i) Define partial autocorrelation function, pacf. How is the pacf useful in determining the order of time series model?

(ii) What are the main differences between white noise process and a typical stochastic time series process? How the diagnostic checking procedure is related to white noise process?

(iii) What is meant by over-fitting and describe its use in the diagnostic checking process? Briefly explain how over-fitting differs from derivation of a new model?

[45 marks]

(b) Consider two processes defined as:

$$Z_t = \mu + X_t \qquad \text{and} \qquad W_t = \mu t + Y_t$$

where $X_t = \phi X_{t-1} + \varepsilon_t$ and $Y_t = Y_{t-1} + X_t$

such that $|\phi| < 1$ and $\varepsilon_t \sim \mathrm{WN}(0, \sigma_\varepsilon^2)$

(i) Without proving, briefly explain why Z_t is stationary.
State the mean, variance and autocovariance function of Z_t.

(ii) Show that W_t is not stationary.

(iii) Show that $(1-B)W_t = Z_t$.

[35 marks]

(c) Rewrite each of the models below using the backward operator B and state the form of ARIMA(p,d,q) or SARIMA(p,d,q)(P,D,Q). [p, d, q, P, D, and Q are positive finite numbers].

(i) $Y_t = Y_{t-2} + \phi_1 Y_{t-3} + \varepsilon_t - \theta_1(\varepsilon_{t-1} - \varepsilon_{t-3}) - \theta_3 \varepsilon_{t-3}$

(ii) $Y_t = (1+\theta)Y_{t-1} - \theta(1+\theta)Y_{t-2} + \theta^2(1+\theta)Y_{t-3} - \theta^3(1+\theta)Y_{t-4} + \ldots + \varepsilon_t$

(iii) $Y_t = \mu(1 - \phi_{12} + \phi_{24}) + \phi_{12}Y_{t-12} - \phi_{24}Y_{t-24} + \varepsilon_t + \varepsilon_{t-1} - \theta_2\varepsilon_{t-2}$

(iv) $Y_t = \varepsilon_t + (0.7+\phi)\varepsilon_{t-1} + \phi(0.7+\phi)\varepsilon_{t-2} + \phi^2(0.7+\phi)\varepsilon_{t-3} + \ldots$

[20 marks]

1. (a) *Bincangkan yang berikut, mungkin dengan menggunakan contoh:*

(i) *Nyatakan definisi fungsi autokorelasi separa, faks? Bagaimana faks berguna dalam menentukan pangkat bagi model siri masa?*

(ii) *Apakah perbezaan utama di antara proses hingar putih dan proses siri masa stokastik yang biasa? Bagaimana presedur pemeriksaan dianostik berhubung dengan proses hingar putih?*

(iii) *Apakah yang dimaksudkan dengan terlebih-suai dan kegunaannya dalam proses pemeriksaan dianostik? Jelaskan dengan ringkas bagaimana terlebih-suai berbeza daripada penghasilan model terbaru?*

[45 markah]

(b) *Pertimbangkan dua proses yang dinyatakan sebagai:*

$$Z_t = \mu + X_t \qquad \text{dan} \qquad W_t = \mu t + Y_t$$

yang mana $X_t = \phi X_{t-1} + \varepsilon_t$ *dan* $Y_t = Y_{t-1} + X_t$

yang mana $|\phi| < 1$ *dan* $\varepsilon_t \sim \mathrm{WN}(0, \sigma_\varepsilon^2)$

(i) *Tanpa sebarang pembuktian, jelaskan dengan ringkas mengapa* Z_t *adalah pegun.*
Nyatakan min, varians dan fungsi autokovarians bagi Z_t.

(ii) *Tunjukkan bahawa* W_t *adalah tidak pegun.*

(iii) *Tunjukkan bahawa* $(1-B)W_t = Z_t$.

[35 markah]

(c) *Tulis semula setiap model di bawah menggunakan pengoperasi anjak kebelakang B dan nyatakan bentuk ARKPB(p,d,q) atau bermusim ARKPB(p,d,q)(P,D,Q). [p, d, q, P, D dan Q adalah nombor-nombor positif terhingga]*

(i) $Y_t = Y_{t-2} + \phi_1 Y_{t-3} + \varepsilon_t - \theta_1(\varepsilon_{t-1} - \varepsilon_{t-3}) + \theta_3 \varepsilon_{t-3}$

(ii) $Y_t = (1+\theta)Y_{t-1} - \theta(1+\theta)Y_{t-2} + \theta^2(1+\theta)Y_{t-3} - \theta^3(1+\theta)Y_{t-4} + \ldots + \varepsilon_t$

(iii) $Y_t = \mu(1 - \phi_{12} + \phi_{24}) + \phi_{12} Y_{t-12} - \phi_{24} Y_{t-24} + \varepsilon_t + \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$

(iv) $Y_t = \varepsilon_t + (0.7+\phi)\varepsilon_{t-1} + \phi(0.7+\phi)\varepsilon_{t-2} + \phi^2(0.7+\phi)\varepsilon_{t-3} + \ldots$

[20 markah]

2. (a) Assume that a time series X_t comes from an ARMA(2,1) process represented by:

$$(1-\phi^2 B^2)X_t = (1-\theta B)\varepsilon_t$$

(i) What is the condition that the parameters ϕ and θ must satisfy for X_t to be an ARMA(2,1) process?

(ii) Which additional requirement ϕ and θ must satisfy for X_t to be stationary and invertible?

(iii) Show that

$$\gamma_0 = \frac{1+\theta^2}{1-\phi^4}\sigma_\varepsilon^2 \qquad \text{and} \qquad \gamma_1 = \frac{\theta}{1-\phi^2}\sigma_\varepsilon^2$$

(iv) Show that for $\theta = 0$

$$\gamma_{2k} = \frac{\phi^{2k}}{1-\phi^4}\sigma_\varepsilon^2$$

[30 marks]

(b) Given an ARMA(1,3) process:

$$(1-\phi_1 B)Y_t = (1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)\varepsilon_t$$

(i) Show that:

$$\gamma_1 = \phi_1\gamma_0 - \theta_1\sigma_\varepsilon^2 - \theta_2(\phi_1-\theta_1)\sigma_\varepsilon^2 - \theta_3[\phi_1(\phi_1-\theta_1)-\theta_2]\sigma_\varepsilon^2$$

$$\gamma_3 = \phi_1\gamma_2 - \theta_3\sigma_\varepsilon^2$$

$$\rho_k = \phi_1\rho_{k-1} \text{ for } k \geq 4$$

What is the expression for γ_2?

(ii) A daily observation of length 500 was collected and an ARMA(1,3) model has been fitted with the following estimates: $\hat{\phi}_1 = 0.625$, $\hat{\theta}_1 = -0.60$, $\hat{\theta}_2 = 0.40$ and $\hat{\theta}_3 = 0.192$.

Calculate the value of autocorrelation, acf for lag k = 1, 2, 3, 4, 5, and value of partial autocorrelation, pacf for lag k = 1 and 2. What can you say about the calculated values of acf and pacf and its underlying process?
[Given the values of acf for lag 6 through to lag 10 are -0.045, -0.039, -0.002, 0.027 and 0.025 respectively, pacf for lag 3 through to lag 8 are 0.356, -0.273, 0.152, -0.159, 0.116, -0.045 respectively].

[50 marks]

2. (a) *Andaikan suatu siri masa* X_t *datang daripada suatu proses ARPB(2,1) yang diwakili oleh:*

$$(1-\phi^2 B^2)X_t = (1-\theta B)\varepsilon_t$$

(i) *Apakah syarat bagi parameter* ϕ *and* θ *yang perlu dipenuhi untuk* X_t *adalah suatu proses ARPB(2,1)?*

(ii) *Syarat tambahan yang manakah* ϕ *and* θ *perlu dipenuhi untuk* X_t *adalah pegun dan ketaktersongsangkan?*

(iii) *Tunjukkan bahawa*

$$\gamma_0 = \frac{1+\theta^2}{1-\phi^4}\sigma_\varepsilon^2 \qquad \textit{dan} \qquad \gamma_1 = \frac{\theta}{1-\phi^2}\sigma_\varepsilon^2$$

(iv) *Tunjukkan bahawa untuk* $\theta = 0$

$$\gamma_{2k} = \frac{\phi^{2k}}{1-\phi^4}\sigma_\varepsilon^2$$

[30 markah]

(b) *Diberi suatu proses ARPB(1,3):*

$$(1-\phi_1 B)Y_t = (1-\theta_1 B-\theta_2 B^2-\theta_3 B^3)\varepsilon_t$$

(i) *Tunjukkan bahawa:*

$$\gamma_1 = \phi_1\gamma_0 - \theta_1\sigma_\varepsilon^2 - \theta_2(\phi_1-\theta_1)\sigma_\varepsilon^2 - \theta_3[\phi_1(\phi_1-\theta_1)-\theta_2]\sigma_\varepsilon^2$$

$$\gamma_3 = \phi_1\gamma_2 - \theta_3\sigma_\varepsilon^2$$

$$\rho_k = \phi_1\rho_{k-1} \text{ for } k \ge 4$$

Apakah ungkapan bagi γ_2 *?*

(ii) *Suatu cerapan harian dengan panjang 500 telah dikumpul dan suatu model ARPB(1,3) telah disuaikan dengan nilai anggaran berikut:* $\hat{\phi}_1 = 0.625$, $\hat{\theta}_1 = -0.60$, $\hat{\theta}_2 = 0.40$ *and* $\hat{\theta}_3 = 0.192$.

Hitung nilai autokorelasi, fak untuk susulan k = 1, 2, 3, 4, 5, dan nilai autokorelasi separa, faks untuk susulan k = 1 dan 2. Apakah yang boleh anda katakan terhadap nilai fak dan faks yang dihitung dan juga proses yang diwakilkan?
[Diberi nilai fak bagi susulan 6 hingga susulan 10 masing-masing adalah -0.045, -0.039, -0.002, 0.027 dan 0.025, faks untuk susulan 3 hingga susulan 8 masing-masing adalah 0.356, -0.273, 0.152, -0.159, 0.116, -0.045].

[50 markah]

(c) Given a monthly seasonal process as below:

$$Y_t = Y_{t-12} + \varepsilon_t - \theta_1\varepsilon_{t-1} - \theta_{12}\varepsilon_{t-12} + \theta_1\theta_{12}\varepsilon_{t-13}$$

Identify the order of SARIMA(p,d,q,P,D,Q) process.

Show that for a seasonally differenced process, $S_t = Y_t - Y_{t-12}$, the autocovariance and autocorrelation functions are given as the following:

$\gamma_1 = -\theta_1(1+\theta_{12}^2)\sigma_\varepsilon^2$ $\quad\quad$ $\rho_k = 0$ for $k = 2, \ldots, 10, 14, \ldots$

$$\rho_{11} = \frac{\theta_1\theta_{12}}{1+\theta_1^2+\theta_{12}^2+\theta_1^2\theta_{12}^2} \quad\quad \rho_{12} = \frac{-\theta_{12}\left(1+\theta_1^2\right)}{1+\theta_1^2+\theta_{12}^2+\theta_1^2\theta_{12}^2}$$

What is the expression for ρ_{13}?

[20 marks]

(c) *Diberi suatu proses bermusim bulanan seperti di bawah:*

$$Y_t = Y_{t-12} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_{12}\varepsilon_{t-12} + \theta_1\theta_{12}\varepsilon_{t-13}$$

Kenalpasti peringkat bagi proses bermusim ARKPB(p,d,q,P,D,Q).

Tunjukkan bahawa bagi proses yang telah dibezakan secara bermusim, $S_t = Y_t - Y_{t-12}$ *, fungsi autokovarians dan autokorelasi adalah diberikan seperti berikut:*

$\gamma_1 = -\theta_1(1+\theta_{12}^2)\sigma_\varepsilon^2$ $\qquad\qquad$ $\rho_k = 0$ *untuk k = 2, ..., 10, 14, ...*

$$\rho_{11} = \frac{\theta_1\theta_{12}}{1+\theta_1^2+\theta_{12}^2+\theta_1^2\theta_{12}^2} \qquad\qquad \rho_{12} = \frac{-\theta_{12}(1+\theta_1^2)}{1+\theta_1^2+\theta_{12}^2+\theta_1^2\theta_{12}^2}$$

Apakah ungkapan bagi ρ_{13}*?*

[20 markah]

3. (a) Consider an ARMA(1,1) process below:

$$(1-\phi_1 B)Y_t = (1-\theta_1 B)\varepsilon_t$$

(i) Using method of moments, show that the estimates of ϕ_1 is given by $\hat{\phi}_1 = \dfrac{\hat{\rho}_2}{\hat{\rho}_1}$ while the estimate of θ_1 can be obtained by solving the following quadratic equation:

$$\hat{\rho}_1 = \frac{(1-\hat{\phi}_1\theta_1)(\hat{\phi}_1-\theta_1)}{1-2\hat{\phi}_1\theta_1+\theta_1^2}$$

(ii) Show that the standard error of the estimates, SE are given by:

$$\text{SE}(\hat{\phi}_1) = \sqrt{\frac{1-\hat{\phi}_1^2}{n}\left(\frac{1-\hat{\phi}_1\hat{\theta}_1}{\hat{\phi}_1-\hat{\theta}_1}\right)^2} \quad \text{and} \quad \text{SE}(\hat{\theta}_1) = \sqrt{\frac{1-\hat{\theta}_1^2}{n}\left(\frac{1-\hat{\phi}_1\hat{\theta}_1}{\hat{\phi}_1-\hat{\theta}_1}\right)^2}$$

(iii) A series of 250 observations was collected and produces estimated acf and pacf as given in Output A1 in Appendix A. Briefly explain why it would be sensible to fit an ARMA(1,1) model to the series.

Output A2 in Appendix A gives the EViews output of the fitted ARMA(1,1) model. Based on acf and pacf, find the estimate for A, B, C, D and E.

[40 marks]

(b) Mr. Chua, who has just been employed as an analyst by a research company based in Putrajaya has been told that stock prices and its returns are very much changes in random pattern. Undeterred, he is interested to know how stock prices in Malaysia have changed since the ringgit was floated back in the currency market at the end of July 2005. He also intends to find a model to help him to forecast future returns. A weekly data of Kuala Lumpur Composite Index has been obtained for the period from August 2005 to December 2010.

Analyses have been conducted and the outputs are given in Appendix B. Explain with reason each of the steps taken by Mr. Chua.

Write down the equation of the final model obtained and explain the practical interpretation.
[Note that the general equation of ARMA model used by EViews is given by: $(1-\phi_1 B-\ldots-\phi_p B^p)Y_t = (1+\theta_1 B+\ldots+\theta_q B^q)\varepsilon_t$]

[30 marks]

3. (a) *Pertimbangkan suatu proses ARPB(1,1) seperti di bawah:*

$$(1-\phi_1 B)Y_t = (1-\theta_1 B)\varepsilon_t$$

(i) *Menggunakan kaedah momen, tunjukkan bahawa anggaran bagi* ϕ_1 *diberikan oleh* $\hat{\phi}_1 = \dfrac{\hat{\rho}_2}{\hat{\rho}_1}$ *sementara anggaran bagi* θ_1 *boleh diperoleh dengan menyelesaikan persamaan kuadratik yang berikut:*

$$\hat{\rho}_1 = \frac{(1-\hat{\phi}_1\theta_1)(\hat{\phi}_1-\theta_1)}{1-2\hat{\phi}_1\theta_1+\theta_1^2}$$

(ii) *Tunjukkan bahawa sisihan piawai bagi anggaran, SE adalah diberikan oleh:*

$$\mathrm{SE}(\hat{\phi}_1) = \sqrt{\frac{1-\hat{\phi}_1^2}{n}\left(\frac{1-\hat{\phi}_1\hat{\theta}_1}{\hat{\phi}_1-\hat{\theta}_1}\right)^2} \quad dan \quad \mathrm{SE}(\hat{\theta}_1) = \sqrt{\frac{1-\hat{\theta}_1^2}{n}\left(\frac{1-\hat{\phi}_1\hat{\theta}_1}{\hat{\phi}_1-\hat{\theta}_1}\right)^2}$$

(iii) *Suatu siri dengan 250 cerapan telah dikumpul dan menghasilkan anggaran fak dan faks seperti dalam Output A1 di Lampiran A. Terangkan dengan ringkas mengapa adalah munasabah untuk menyuaikan suatu model ARPB(1,1) terhadap siri tersebut.*

Output A2 di Lampiran A menunjukkan output EViews bagi model ARMA(1,1) yang telah disuaikan. Berdasarkan fak dan faks, cari nilai anggaran untuk A, B, C, D dan E.

[40 markah]

(b) *En. Chua, yang baru diambil bekerja sebagai penganalisa oleh syarikat penyelidikan yang berpusat di Putrajaya telah diberitahu bahawa harga saham dan pulangannya kebanyakannya berubah secara rawak. Tidak berputus asa, beliau berminat untuk mengetahui bagaimana harga saham di Malaysia telah berubah sejak ringgit kembali diapungkan di pasaran tukaran wang pada hujung bulan Julai 2005. Beliau juga merancang untuk mencari suatu model bagi membantu beliau untuk meramalkan nilai pulangan pada masa akan datang. Suatu set data mingguan bagi Indeks Komposit Kuala Lumpur telah diperoleh bagi jangkamasa dari Ogos 2005 hingga Disember 2010.*

Analisis telah dilakukan dan output diberikan dalam Lampiran B. Terangkan dengan alasan bagi setiap langkah yang diambil oleh En. Chua.

Tuliskan persamaan bagi model terakhir yang dihasilkan dan terangkan interpretasi praktikalnya.
[Persamaan umum bagi model ARPB yang digunakan oleh EViews adalah diberikan seperti berikut: $(1-\phi_1 B-\ldots-\phi_p B^p)Y_t = (1+\theta_1 B+\ldots+\theta_q B^q)\varepsilon_t$ *]*

[30 markah]

(c) Ms. Saliah has been working with the Ministry of Tourism Malaysia since graduating in July 2010. She has been asked by her superior to investigate the growth of number of tourists visiting Malaysia. In particular, she is required to investigate the prospect of European tourists in the future. She was given monthly data from January 1999 to December 2009. She was informed that following the 97/98 Asian Financial Crisis and the outbreak of Nipah virus, the Malaysian Government has put a lot of effort to make the tourism industry as one of the main source of national income.

Looking at the time series plot in Appendix C, Ms. Saliah has decided to analyze the data starting only from January 2002 as there was a large drop in the number of tourists following the terrorist attack on the World Trade Centre, in New York city.

Describe each of the steps that have been taken by Ms. Saliah in finding the appropriate model for the given data. Is there significant evidence of continual growth in the number of tourists from the European countries? If so, estimate the increase in the number of tourists per year.

[30 marks]

(c) *Cik Saliah telah bekerja di Kementerian Pelancongan Malaysia sejak menerima ijazah pada bulan Julai 2010. Beliau telah diarahkan oleh ketuanya untuk mengkaji pertumbuhan bilangan pelancong yang datang ke Malaysia. Terutamanya, beliau dikehendaki mengkaji prospek pelancong dari negara-negara Eropah pada masa akan datang. Beliau telah diberikan data bulanan dari Januari 1999 hingga Disember 2009. Beliau telah diberitahu bahawa berikutan daripada krisis Kewangan Asia 97/98 dan wabak kuman Nipah, kerajaan Malaysia telah berusaha keras untuk menjadikan industri pelancongan sebagai salah satu sumber utama bagi pendapatan negara.*

Melihat pada plot siri masa di Lampiran C, Cik Saliah mengambil keputusan untuk menganalisa data hanya bermula dari Januari 2002 disebabkan terdapatnya penurunan besar pada bilangan pelancong berikutan daripada serangan penganas di Pusat Dagangan Dunia, di bandaraya New York.

Jelaskan setiap langkah yang diambil oleh Cik Saliah untuk mencari model yang sesuai bagi data yang diberikan. Adakah terdapat bukti yang signifikan bagi pertumbuhan berterusan dalam bilangan pelancong Negara Eropah? Jika benar, anggarkan peningkatan dalam bilangan pelancong setiap tahun.

[30 markah]

4. (a) Consider an ARMA(2,1) model for a series with non-zero mean:

$$(1-\phi_1^2 B^2)(Y_t-\mu) = (1-\theta_1 B)\varepsilon_t$$

(i) Show that the MA coefficients are given by:

$$\phi_k = \begin{cases} -\theta_1\phi_1^{2k-1} & \text{when } k \text{ is odd} \\ \\ \phi_1^k & \text{when } k \text{ is even} \end{cases}$$

(ii) Show that the m-step ahead forecasts made at time $t = n$ is given by:

$$\hat{Y}_n(m) = \mu(1-\phi_1^2) + \phi_1^2 \hat{Y}_n(m-2) \quad \text{for} \quad m \geq 3$$

and that it can be rewritten as:

$$\hat{Y}_n(m) = \begin{cases} \mu(1-\phi_1^{m+1}) + \phi_1^{m+1} Y_{n-1} - \phi_1^{m-1}\theta_1\varepsilon_n & \text{when } m \text{ is odd} \\ \\ \mu(1-\phi_1^m) + \phi_1^m Y_n & \text{when } m \text{ is even} \end{cases}$$

(iii) Show that, for m is even and such that $m \geq 4$, the variance of forecast error is given by:

$$Var[e_n(m)] = (1+\theta_1^2)\left(\frac{1-\phi_1^{2m}}{1-\phi_1^4}\right)\sigma_\varepsilon^2$$

(iv) Consider $n = 250$. If estimated values for the coefficients are $\hat{\phi}_1 = 0.8$, $\hat{\theta}_1 = -0.6$, $\hat{\mu} = 300$, $s_\varepsilon^2 = 12$ with $Y_{250} = 266$, $Y_{249} = 282$, $\hat{\varepsilon}_{250} = 14$ and $\hat{\varepsilon}_{249} = 12$ obtain value of $\hat{Y}_{250}(m)$ for $m = 1, 2, \ldots, 6$ and the corresponding 95% forecast intervals. Comment on the six forecast values obtained.

(v) It is now observed at time $t = 251$ that a new observation is noted as $Y_{251} = 308$. Calculate the updated forecasts for $Y_{252}, \ldots Y_{256}$. Compare these new forecasts with those calculated in (iv) above and discuss.

[60 marks]

4. (a) *Pertimbangkan suatu model ARPB(2,1) model bagi suatu siri dengan min bukan sifar:*

$$(1-\phi_1^2 B^2)(Y_t - \mu) = (1-\theta_1 B)\varepsilon_t$$

(i) *Tunjukkan bahawa koefisien bagi PB adalah diberikan oleh:*

$$\phi_k = \begin{cases} -\theta_1 \phi_1^{2k-1} & \text{untuk } k \text{ ganjil} \\ \phi_1^k & \text{untuk } k \text{ genap} \end{cases}$$

(ii) *Tunjukkan bahawa telahan m-langkah kehadapan yang dibuat pada waktu t = n adalah diberikan oleh:*

$$\hat{Y}_n(m) = \mu(1-\phi_1^2) + \phi_1^2 \hat{Y}_n(m-2) \quad \text{untuk } m \geq 3$$

dan ia boleh ditulis semula sebagai:

$$\hat{Y}_n(m) = \begin{cases} \mu(1-\phi_1^{m+1}) + \phi_1^{m+1} Y_{n-1} - \phi_1^{m-1}\theta_1 \varepsilon_n & \text{untuk } m \text{ ganjil} \\ \mu(1-\phi_1^m) + \phi_1^m Y_n & \text{untuk } m \text{ genap} \end{cases}$$

(iii) *Tunjukkan bahawa, untuk m genap dan* $m \geq 4$*, varians bagi rlat telahan adalah diberikan oleh:*

$$Var[e_n(m)] = (1+\theta_1^2)\left(\frac{1-\phi_1^{2m}}{1-\phi_1^4}\right)\sigma_\varepsilon^2$$

(iv) *Pertimbangkan n = 250. Jika nilai teranggar bagi koefisien adalah* $\hat{\phi}_1 = 0.8$, $\hat{\theta}_1 = -0.6$, $\hat{\mu} = 300$, $s_\varepsilon^2 = 12$ *dengan* $Y_{250} = 266$, $Y_{249} = 282$, $\hat{\varepsilon}_{250} = 14$, $\hat{\varepsilon}_{249} = 12$ *dapatkan nilai bagi* $\hat{Y}_{250}(m)$ *untuk m = 1, 2, ..., 6 dan selang telahan 95% yang sepadan. Komen terhadap enam nilai telahan yang diperoleh.*

(v) *Pada waktu t = 251 satu cerapan terbaru dicatat sebagai* $Y_{251} = 308$ *. Hitung telahan kemaskini bagi* $Y_{252}, \ldots Y_{256}$ *. Bandingkan nilai telahan terbaru ini dengan telahan yang diperoleh dalam (iv) di atas dan bincangkan.*

[60 markah]

(b) Consider the ARIMA(0,1,1) process below:

$$(1-B)Y_t = \theta_0 + (1-\theta_1 B)\varepsilon_t$$

(i) Show that : $\hat{Y}_n(m) = Y_n - \theta_1\varepsilon_n + m\theta_0$ for $m \ge 1$

(ii) Show that: $\upsilon_n(m) = Y_{n+m} - \hat{Y}_n(m) = \sum_{j=0}^{m-1} \varphi_j \varepsilon_{n+m-j}$

where $\varphi_j = (1-\theta_1)$ for $j \ge 1$

and that: $Var[\upsilon_n(m)] = \{1 + (m-1)(1-\theta_1)^2\}\sigma_\varepsilon^2$

What can you say about the variance of the forecast?

(iii) Consider n = 250. If estimated values for the coefficients are $\hat{\theta}_0 = 10$, $\hat{\theta}_1 = -0.6$, $s_\varepsilon^2 = 12$ with $Y_{250} = 300$ and $\varepsilon_{250} = 4$, obtain value of $\hat{Y}_{250}(m)$ for m = 1, 2, …, 6 and the corresponding 95% forecast intervals. Comment on the six forecast values obtained. What can you say about forecast values from a process containing deterministic trend?

[40 marks]

(b) *Pertimbangkan proses ARIMA(0,1,1) di bawah:*

$$(1-B)Y_t = \theta_0 + (1-\theta_1 B)\varepsilon_t$$

(i) *Tunjukkan bahawa:* $\hat{Y}_n(m) = Y_n - \theta_1\varepsilon_n + m\theta_0$ *untuk* $m \geq 1$

(ii) *Tunjukkan bahawa:* $\upsilon_n(m) = Y_{n+m} - \hat{Y}_n(m) = \sum_{j=0}^{m-1} \varphi_j \varepsilon_{n+m-j}$

yang mana $\varphi_j = (1-\theta_1)$ *for* $j \geq 1$

dan bahawa: $Var[\upsilon_n(m)] = \{1 + (m-1)(1-\theta_1)^2\}\sigma_\varepsilon^2$

Apakah yang boleh dikatakan tentang varians bagi telahan?

(iii) *Pertimbangkan n = 250. Jika nilai teranggar bagi koefisien adalah* $\hat{\theta}_0 = 10$, $\hat{\theta}_1 = -0.6$, $s_\varepsilon^2 = 12$ *dengan* $Y_{250} = 300$ *dan* $\varepsilon_{250} = 4$, *dapatkan nilai bagi* $\hat{Y}_{250}(m)$ *untuk m = 1, 2, ..., 6 dan selang telahan 95% yang sepadan. Komen terhadap enam nilai telahan yang diperoleh. Apakah yang boleh diperkatakan tentang nilai telahan dari process yang mengandungi tren deterministik?*

[40 markah]

APPENDIX/LAMPIRAN A

Output A1

Sample: 1 250
Included observations: 250

	AC	PAC	Q-Stat	Prob
1	0.379	0.379	36.354	0.000
2	0.298	0.180	58.933	0.000
3	0.201	0.048	69.219	0.000
4	0.170	0.050	76.613	0.000
5	0.160	0.059	83.190	0.000
6	0.166	0.068	90.344	0.000
7	0.155	0.044	96.577	0.000
8	0.092	-0.030	98.801	0.000
9	0.108	0.035	101.87	0.000
10	0.093	0.019	104.15	0.000
12	0.104	0.037	109.20	0.000
15	0.170	0.024	134.24	0.000
18	0.103	-0.011	147.63	0.000
24	0.092	0.025	155.98	0.000

Output A2

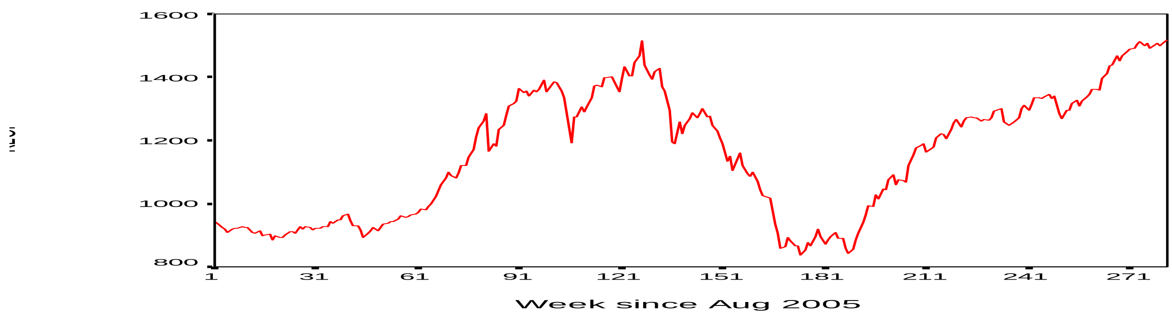
Dependent Variable: AA
Sample (adjusted): 2 250
Included observations: 249 after adjustments

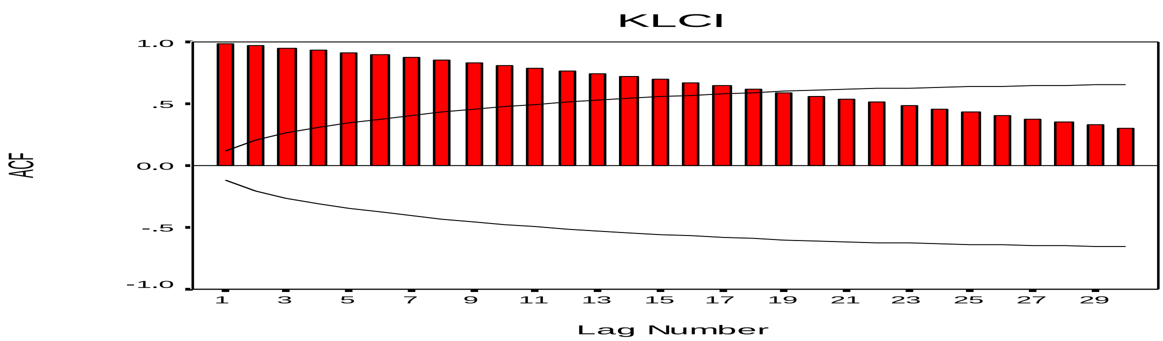
	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	**A**	**B**	A / B	0.0000
MA(1)	**C**	**D**	C / D	0.0000

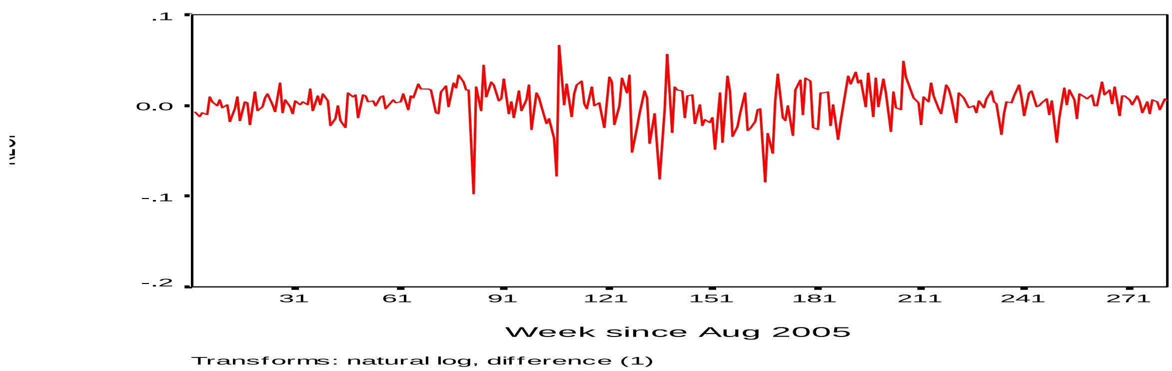
R-squared	0.174419	Mean dependent var	0.331830
Adjusted R-squared	0.171077	S.D. dependent var	2.406347
S.E. of regression	**E**	Akaike info criterion	4.414468
Log likelihood	-547.6013	Schwarz criterion	4.442721

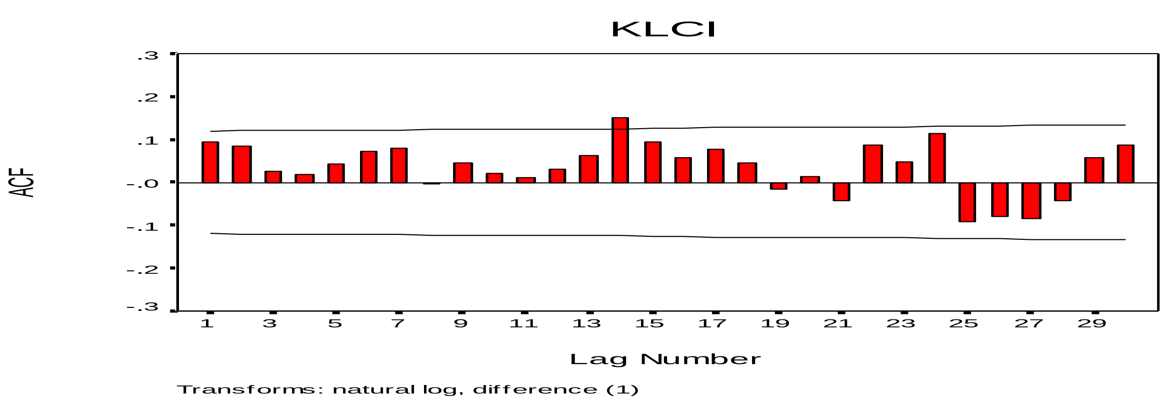
APPENDIX/LAMPIRAN B

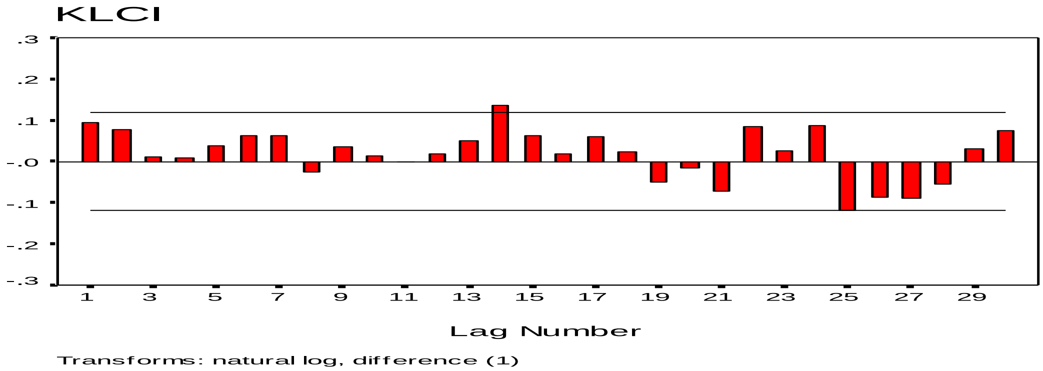
Step 1











Step 2a

Dependent Variable: RKLCI (return KLCI)
Method: Least Squares
Sample (adjusted): 3 282

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.099136	0.059568	1.664248	0.0972

R-squared	0.003149	S.D. dependent var	2.103893
S.E. of regression	2.100578	Schwarz criterion	4.338848
Log likelihood	-604.6214	Akaike info criterion	4.325867

Step 2b

Dependent Variable: RKLCI
Method: Least Squares
Sample (adjusted): 3 282

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.946896	0.054892	17.25026	0.0000
MA(1)	-0.893349	0.076446	-11.68607	0.0000

R-squared	0.021105	S.D. dependent var	2.103893
S.E. of regression	2.085313	Schwarz criterion	4.340795
Log likelihood	-602.0765	Akaike info criterion	4.314832

Correlation matrix

	AR(1)	MA(1)
AR(1)	0.003013	-0.003927
MA(1)	-0.003927	0.005844

Step 2c

Dependent Variable: RKLCI
Method: Least Squares
Sample (adjusted): 2 282

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	0.085089	0.059547	1.428937	0.1541

R-squared	0.001940	S.D. dependent var	2.100744
S.E. of regression	2.098705	Schwarz criterion	4.337019
Log likelihood	-606.5319	Akaike info criterion	4.324071

Step 3: Residual Analysis from Step 2b

	AC	PAC	Q-Stat	Prob
1	0.020	0.020	0.1119	
2	0.016	0.016	0.1839	
3	-0.044	-0.045	0.7460	0.388
4	-0.047	-0.045	1.3657	0.505
5	-0.017	-0.014	1.4527	0.693
6	0.017	0.017	1.5384	0.820
8	-0.058	-0.063	2.7530	0.839
10	-0.028	-0.023	2.9885	0.935
12	-0.012	-0.015	3.3791	0.971
15	0.060	0.055	8.9586	0.776
18	0.017	0.032	10.007	0.866
24	0.109	0.108	17.736	0.722

ARCH Test: LAG = 1

Obs*R-squared	4.437642	Probability	0.035155

ARCH Test: LAG = 3

Obs*R-squared	9.447467	Probability	0.023897

ARCH Test: LAG = 6

Obs*R-squared	9.208775	Probability	0.162173

Step 4

Dependent Variable: RKLCI
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.676674	0.313858	2.155987	0.0311
MA(1)	-0.641712	0.338015	-1.898471	0.0576
		Variance Equation		
C	0.067384	0.037812	1.782075	0.0747
RESID(-1)^2	0.102799	0.028981	3.547154	0.0004
GARCH(-1)	0.890438	0.026180	34.01203	0.0000

R-squared	0.006173	S.D. dependent var	2.103893
S.E. of regression	2.112587	Schwarz criterion	4.212141
Log likelihood	-575.6128	Akaike info criterion	4.147234

Residual Analysis

	AC	PAC	Q-Stat	Prob
1	0.058	0.058	0.9685	
2	0.065	0.061	2.1569	
3	0.039	0.032	2.5853	0.108
4	-0.019	-0.027	2.6847	0.261
5	0.020	0.018	2.7963	0.424
6	0.014	0.014	2.8548	0.582
8	-0.005	-0.016	4.0933	0.664
10	0.022	0.015	4.8027	0.778
12	0.062	0.054	6.0346	0.812
15	0.054	0.039	10.337	0.666
24	0.135	0.118	19.514	0.613

ARCH Test: LAG = 1

Obs*R-squared	0.060037	Probability	0.806437

ARCH Test: LAG = 3

Obs*R-squared	0.184944	Probability	0.979982

Correlation matrix

	AR(1)	MA(1)
AR(1)	0.098507	-0.105665
MA(1)	-0.105665	0.114254

Step 5a

Dependent Variable: RKLCI
Sample (adjusted): 4 282
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.929104	0.136544	6.804447	0.0000
AR(2)	-0.000731	0.077427	-0.009438	0.9925
MA(1)	-0.909581	0.105449	-8.625757	0.0000
Variance Equation				
C	0.066887	0.042049	1.590675	0.1117
RESID(-1)^2	0.103515	0.029041	3.564395	0.0004
GARCH(-1)	0.890207	0.026567	33.50741	0.0000

R-squared	0.009974	S.D. dependent var	2.105976
S.E. of regression	2.114550	Schwarz criterion	4.241119
Log likelihood	-574.7424	Akaike info criterion	4.163028

Inverted AR Roots	.93	.00
Inverted MA Roots	.91	

Step 5b

Dependent Variable: RKLCI
Sample (adjusted): 3 282
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.659811	0.318104	2.074201	0.0381
MA(1)	-0.642715	0.327558	-1.962140	0.0497
MA(2)	0.023828	0.079961	0.297994	0.7657
Variance Equation				
C	0.067187	0.037980	1.769032	0.0769
RESID(-1)^2	0.104080	0.029678	3.506983	0.0005
GARCH(-1)	0.889470	0.026647	33.37958	0.0000

R-squared	0.005701	S.D. dependent var	2.103893
S.E. of regression	2.116942	Schwarz criterion	4.231786
Log likelihood	-575.5457	Akaike info criterion	4.153898

Inverted AR Roots	.66	
Inverted MA Roots	.60	.04

Step 5c

Dependent Variable: RKLCI
Sample (adjusted): 4 282
GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

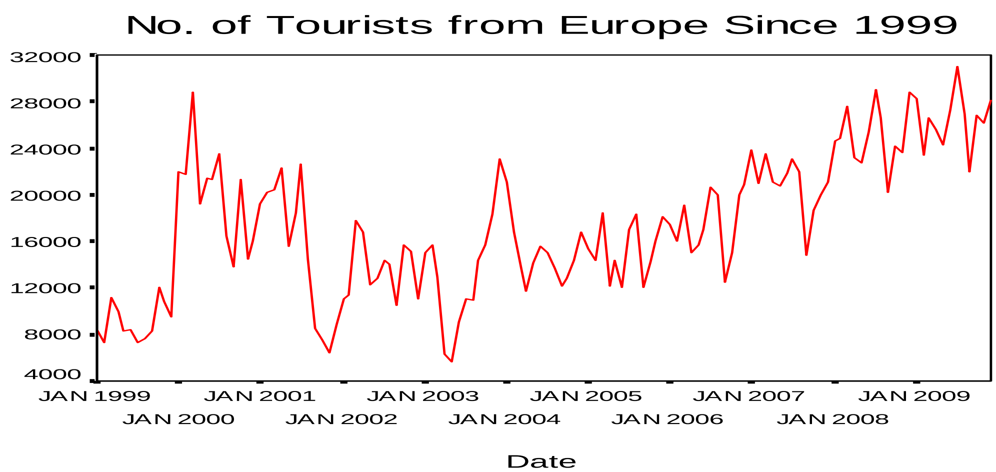
	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.033162	0.062638	0.529430	0.5965
AR(2)	0.051064	0.068808	0.742131	0.4580
		Variance Equation		
C	0.062430	0.038290	1.630462	0.1030
RESID(-1)^2	0.102409	0.028672	3.571702	0.0004
GARCH(-1)	0.891979	0.025885	34.45962	0.0000
R-squared	0.004712	S.D. dependent var		2.105976
S.E. of regression	2.116289	Schwarz criterion		4.223815
Log likelihood	-575.1442	Akaike info criterion		4.158740
Inverted AR Roots	.24	-.21		

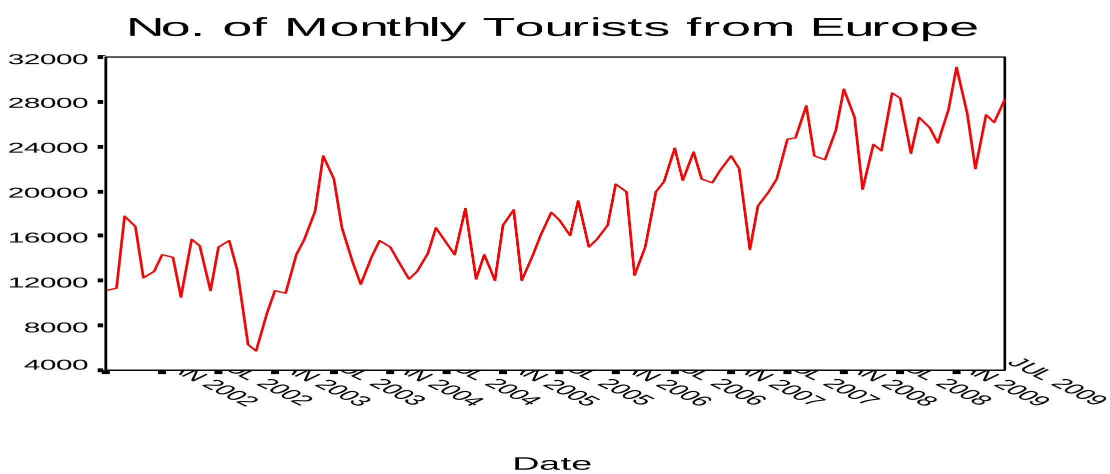
Step 6

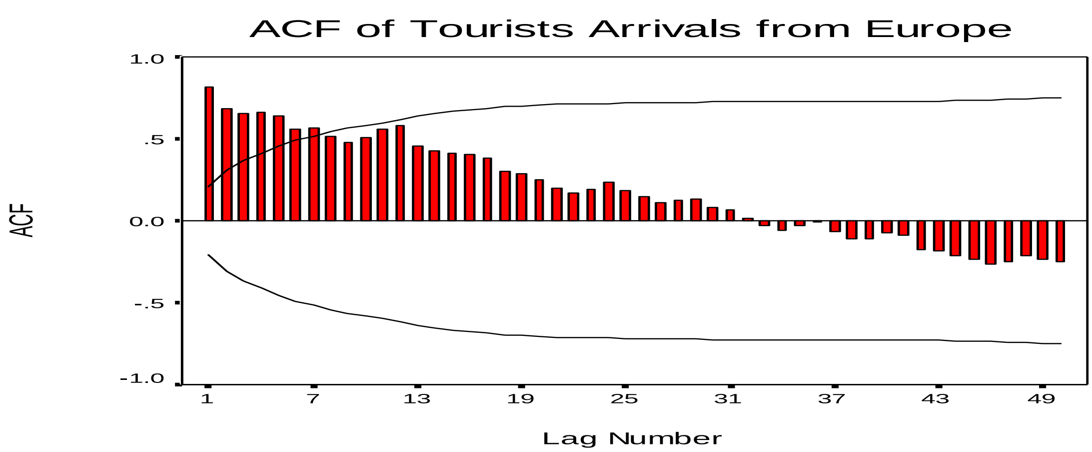
Dependent Variable: RKLCI
Sample (adjusted): 3 282
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) +
C(5)*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

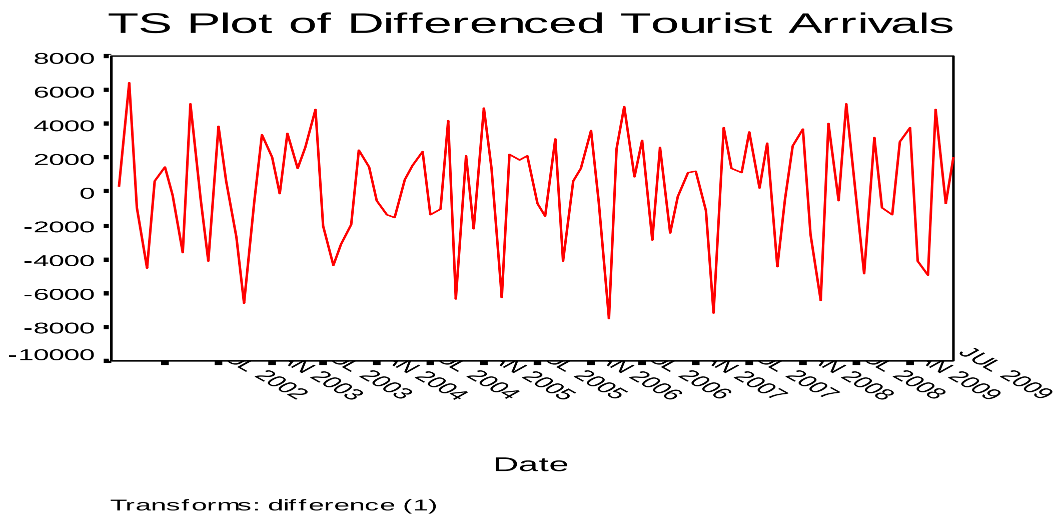
	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.896263	0.100104	8.953289	0.0000
MA(1)	-0.868366	0.112233	-7.737208	0.0000
		Variance Equation		
C(3)	-0.120736	0.036462	-3.311255	0.0009
C(4)	0.215281	0.047884	4.495840	0.0000
C(5)	-0.015478	0.027457	-0.563701	0.5730
C(6)	0.969396	0.018371	52.76775	0.0000
R-squared	0.012909	S.D. dependent var		2.103893
S.E. of regression	2.109255	Schwarz criterion		4.215301
Log likelihood	-573.2377	Akaike info criterion		4.137412
Inverted AR Roots	.90			
Inverted MA Roots	.87			

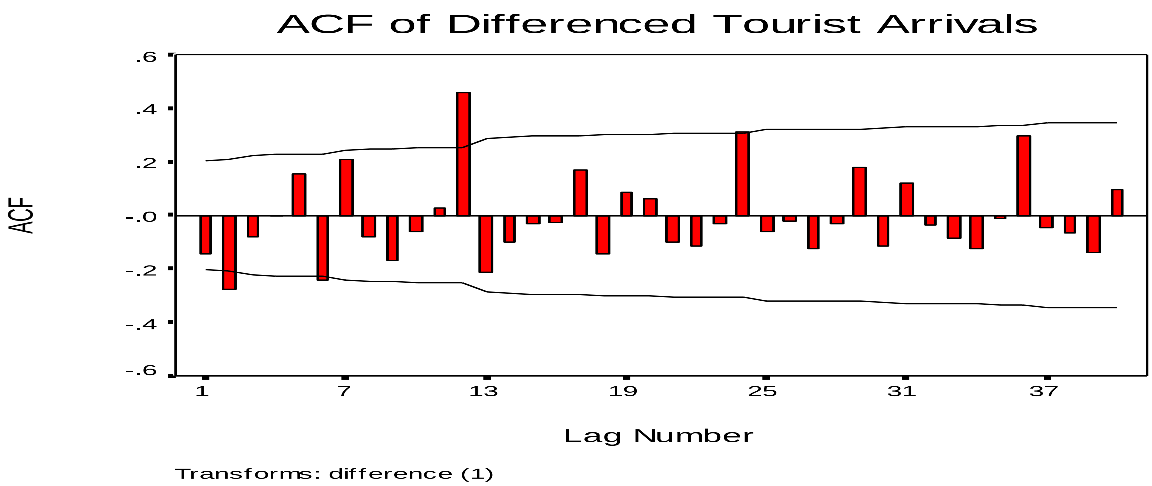
APPENDIX/LAMPIRAN C

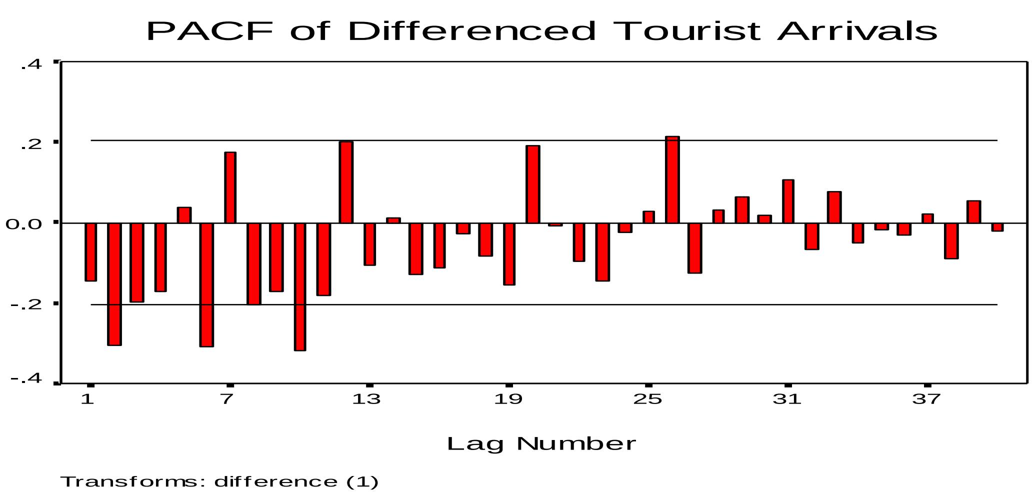







TS Plot of Differenced Tourist Arrivals
8000
6000
4000
2000
0
-2000
-4000
-6000
-8000
-10000
JUL 2009
Date
Transforms: difference (1)


ACF of Differenced Tourist Arrivals
ACF
.6
.4
.2
-.0
-.2
-.4
-.6
1
7
13
19
25
31
37
Lag Number
Transforms: difference (1)

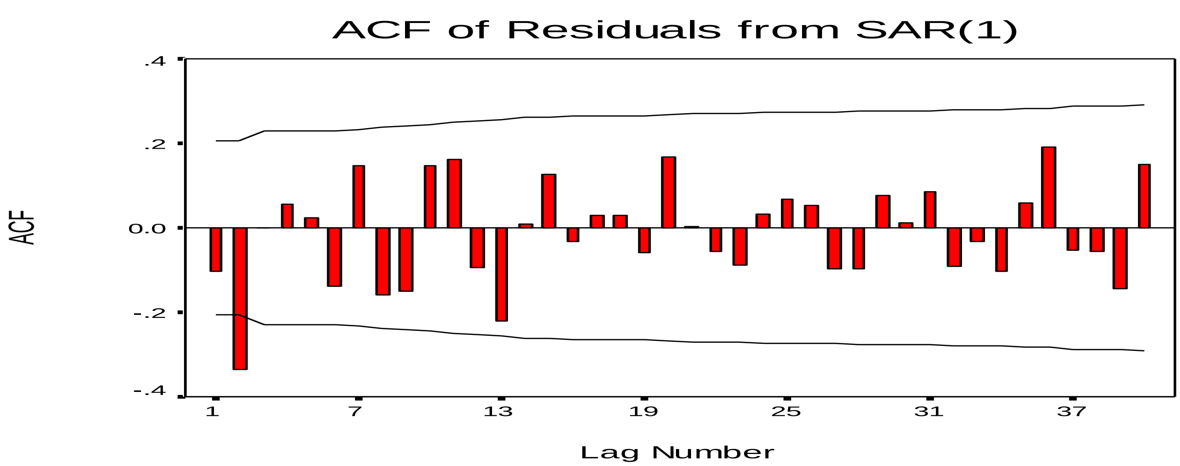

PACF of Differenced Tourist Arrivals
.4
.2
0.0
-.2
-.4
1
7
13
19
25
31
37
Lag Number
Transforms: difference (1)

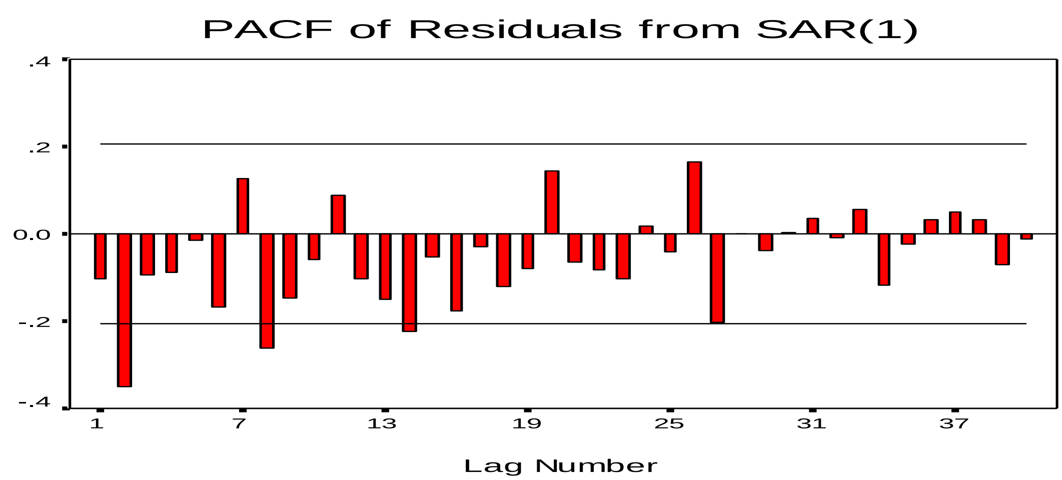
Step 2

```
Variable:   ARRIVALS
Non-seasonal differencing: 1
    Seasonal differencing: 0
Length of Seasonal Cycle: 12

FINAL PARAMETERS:
Standard error        2727.6674      Log likelihood      -887.41807
AIC                   1778.8361      SBC                  1783.9439

           Variables in the Model:
                     B           SEB      T-RATIO   APPROX. PROB.
SAR1            .53840        .08970    6.0019995       .00000004
CONSTANT     169.55763     532.82251     .3182253       .75102751
```





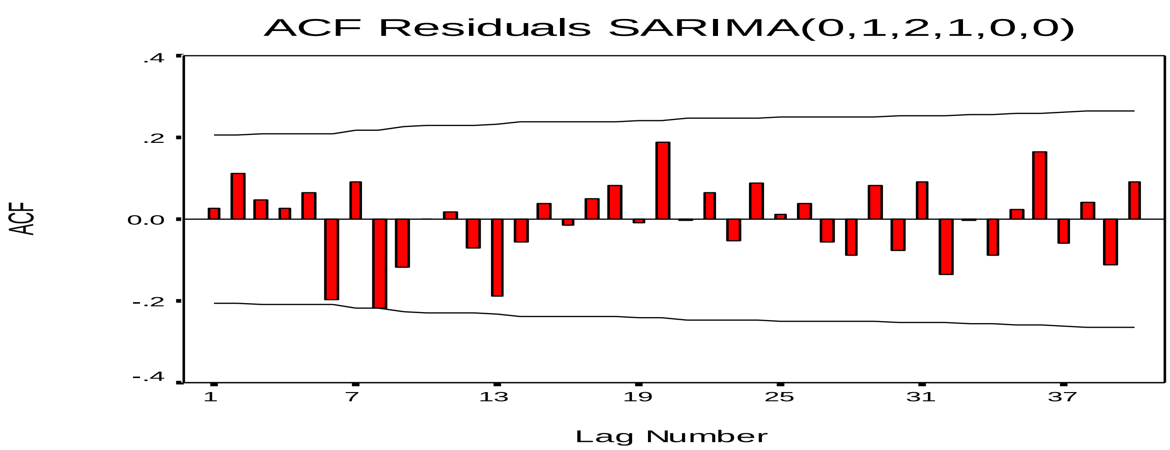
Step 3a

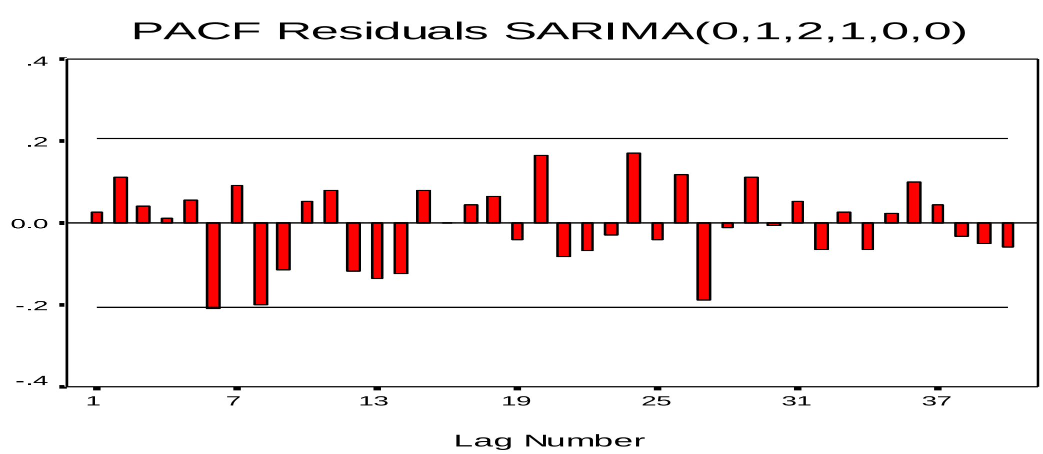
```
Variable:   ARRIVALS

FINAL PARAMETERS:
Standard error        2472.7238       Log likelihood       -877.15297
AIC                   1762.3059       SBC                  1772.5215

           Variables in the Model:
                        B          SEB      T-RATIO   APPROX. PROB.
MA1              .29673      .084241    3.5223829       .00067093
MA2              .58574      .083981    6.9746932       .00000000
SAR1             .48331      .091895    5.2593407       .00000095
CONSTANT      147.32636    58.497154    2.5185218       .01353160

Correlation Matrix:
                MA1           MA2          SAR1
MA1        1.0000000    -.7216529     .0222727
MA2        -.7216529    1.0000000     .0660413
SAR1        .0222727     .0660413    1.0000000
```





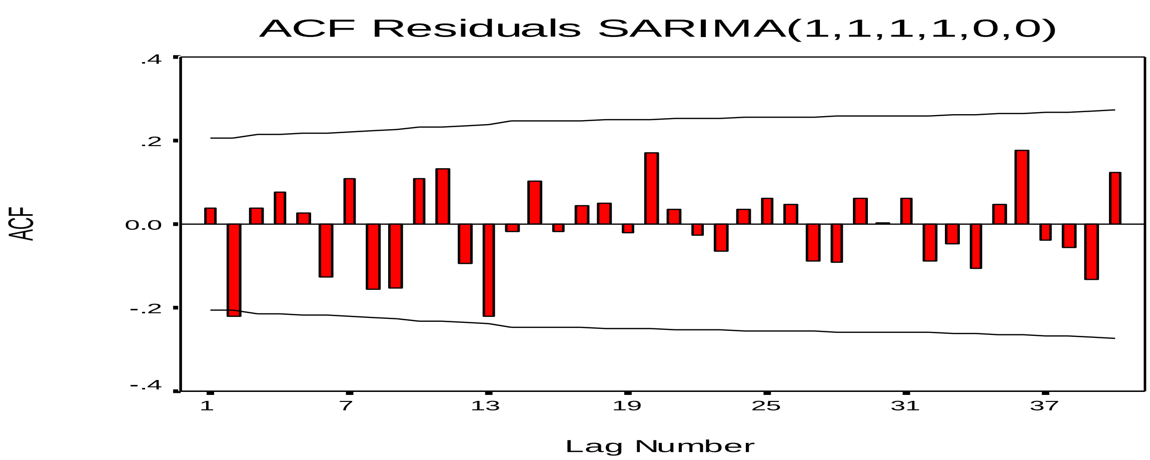
Step 3b

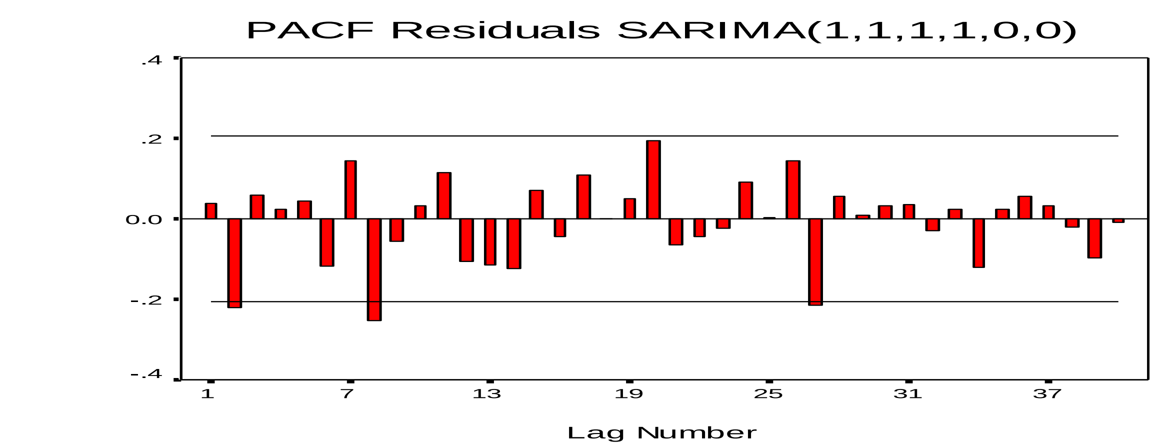
```
Variable:   ARRIVALS

FINAL PARAMETERS:
Standard error        2469.7791      Log likelihood       -877.5269
AIC                   1763.0538      SBC                  1773.2693

           Variables in the Model:
                       B          SEB      T-RATIO   APPROX. PROB.
AR1               .62604      .091051    6.8756938       .00000000
MA1               .99306      .106376    9.3354017       .00000000
SAR1              .53046      .092911    5.7093645       .00000014
CONSTANT       159.87225    35.879849    4.4557672       .00002375

Correlation Matrix:
                  AR1          MA1          SAR1
AR1         1.0000000     .3911368     .0814330
MA1          .3911368    1.0000000     .1567493
SAR1         .0814330     .1567493    1.0000000
```





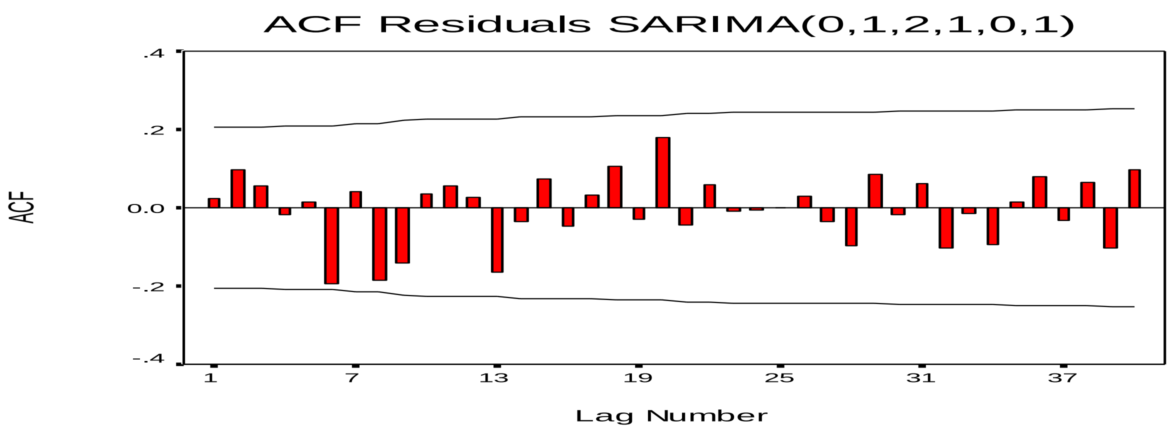
Step 4

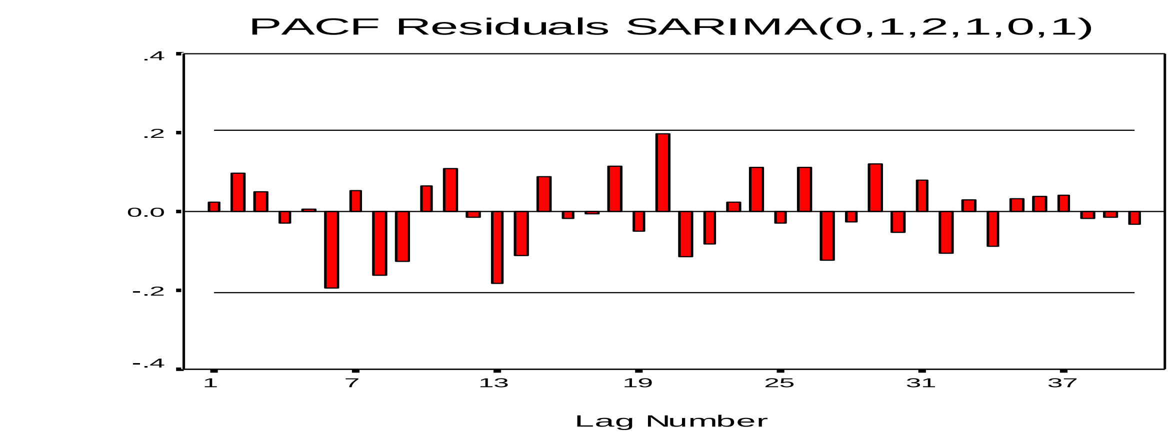
```
Variable:   ARRIVALS

FINAL PARAMETERS:
Standard error        2414.3273      Log likelihood       -875.27315
AIC                   1760.5463      SBC                  1773.3157

           Variables in the Model:
                      B          SEB       T-RATIO    APPROX. PROB.
MA1              .27447      .085305     3.2175057        .00179791
MA2              .56643      .084786     6.6807548        .00000000
SAR1             .85759      .146102     5.8698110        .00000007
SMA1             .54428      .234015     2.3258219        .02227528
CONSTANT      153.59323    88.691899     1.7317617        .08674225

Correlation Matrix:
                    MA1          MA2          SAR1          SMA1
MA1           1.0000000    -.6401939     -.0487334     -.0573398
MA2           -.6401939    1.0000000     -.0713010     -.0884986
SAR1          -.0487334    -.0713010     1.0000000      .9353874
SMA1          -.0573398    -.0884986      .9353874     1.0000000
```





-ooo000ooo-