
UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2010/2011 Academic Session

April/May 2011

MSG 265 – Design and Analysis of Experiments
[Rekabentuk dan Analisis Uji Kaji]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of THIRTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TIGA BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

Arahan: Jawab **semua empat** [4] soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) An experiment is to be conducted to study heat transfer in article molds used in the manufacture of drinking glasses. Three molten-glass temperatures are to be varied in the experiment: 1100, 1200 (the industry standard), and 1300 °C. Two cooling times are also to be varied in the experiment: 15 and 20 seconds. In addition, two types of glass, type A and type B are to be studied. The experiment is repeated twice. Assuming all the factors are fixed, write down the analysis of variance table. Your ANOVA should consists of source of variation, sum of squares, degrees of freedom, mean square and and F_o .

[30 marks]

- (b) Suppose that the experiment described in (a) can be conducted as stated except that no funds are available for repeat tests. The data below represent test results from such an experiment. The response of interest is the mold temperature.

Molten-Glass Temperature (A)	Glass Type (B)			
	A		B	
	Cooling Time (C)		Cooling Time (C)	
	15	20	15	20
1100	98	91	101	97
1200	82	83	84	85
1300	100	99	93	95

- (i) State the linear statistical model for this experiment.
(ii) Given that $SST = 558.667$. Analyze the data and state the conclusion. Use $\alpha = 0.05$.
(iii) Obtain the point estimates of the mean mold temperature at each molten-glass temperature levels.

[70 marks]

1. (a) Satu eksperimen dijalankan bagi mengkaji pemindahan haba dalam acuan yang digunakan bagi pembuatan gelas minuman. Tiga suhu kaca lebur ialah 1100, 1200 (piawaian industri), dan 1300°C. Dua masa penyejukan ialah 15 dan 20 saat. Sebagai tambahan, dua jenis kaca, jenis A dan jenis B turut dikaji. Eksperimen ini diulangi sebanyak dua kali. Andaikan semua faktor adalah tetap, tuliskan jadual analisis varians. Jadual ANOVA anda perlu mengandungi sumber variasi, hasil tambah kuasa dua, darjah kebebasan, min kuasa dua dan F_o .

[30 markah]

- (b) Andaikan eksperimen yang dinyatakan dalam bahagian (a) dilakukan seperti di atas namun tiada sumber kewangan bagi mengulangi ujikaji. Data di bawah menunjukkan keputusan ujian daripada eksperimen tersebut. Pemboleh ubah sambutan yang diminati ialah suhu acuan.

Suhu Kaca Lebur (A)	Jenis Kaca (B)			
	A		B	
	Masa Penyejukan (C)		Masa Penyejukan (C)	
	15	20	15	20
1100	98	91	101	97
1200	82	83	84	85
1300	100	99	93	95

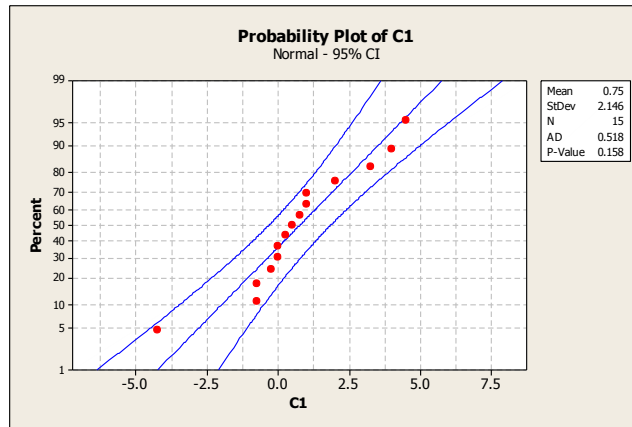
- (i) Nyatakan model statistik linear bagi eksperimen di atas.
(ii) Diberikan bahawa $SST = 558.667$. Analisa data dan berikan kesimpulan. Guna $\alpha = 0.05$.
(iii) Dapatkan anggaran titik bagi min suhu acuan pada setiap tahap suhu kaca lebur.

[70 markah]

2. In a process development study on yield, four factors were studied, each at two levels: time (*A*: 2.5 hours , 3.0 hours), concentration (*B*: 14% , 18%), pressure (*C*: 60 psi , 80 psi), and temperature (*D*: 225°C , 250°C). A single replicate of a 2^4 design was run, and the effect estimates are given as follows:

$$\begin{array}{llll}
 A = 4.50 & AB = -0.75 & BD = 0.00 & ACD = -0.25 \\
 B = 0.50 & AC = -4.25 & CD = 0.00 & BCD = -0.75 \\
 C = 2.00 & AD = 4.00 & ABC = 1.00 & ABCD = 1.00 \\
 D = 3.25 & BC = 0.25 & ABD = 0.75 &
 \end{array}$$

- (a) Given that $SST = 291.75$. Conduct an analysis of variance using the following normal probability plot of the effect estimates for guidance in forming an error term. What are your conclusions?



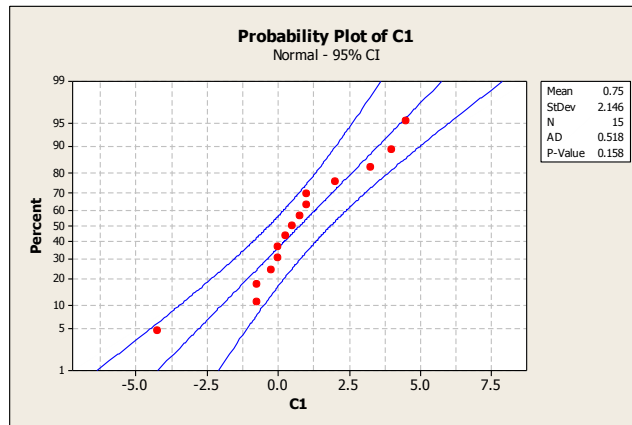
- (b) Write down a regression model relating yield to the important process variables.
- (c) Analyse the residuals from this experiment. Are there any indications of model inadequacy? (Refer to Appendix B)
- (d) Can this design be collapsed into a 2^3 design with two replicates? If so, sketch the design with the average yield shown at each point in the cube.
- (e) Suppose that two blocks are required in this experiment. Construct a design with $ABCD$ confounded with blocks. Analyse the data from this design.

[100 marks]

2. Dalam satu kajian proses pembangunan ke atas hasil, empat faktor dikaji, setiap satu faktor mempunyai dua tahap: masa (A: 2.5 jam , 3.0 jam), kepekatan (B: 14%, 18%), tekanan (C: 60 psi , 80 psi), dan suhu (D: 225°C , 250°C). Rekabentuk 2^4 dengan replika tunggal dijalankan, dan anggaran kesan diberikan seperti berikut:

$A = 4.50$	$AB = -0.75$	$BD = 0.00$	$ACD = -0.25$
$B = 0.50$	$AC = -4.25$	$CD = 0.00$	$BCD = -0.75$
$C = 2.00$	$AD = 4.00$	$ABC = 1.00$	$ABCD = 1.00$
$D = 3.25$	$BC = 0.25$	$ABD = 0.75$	

(a) Diberikan bahawa $SST = 291.75$. Jalankan analisis varians menggunakan plot kebarangkalian normal bagi anggaran kesan yang diberikan sebagai panduan untuk membentuk ralat. Apakah kesimpulan anda?



- (b) Tuliskan model regresi yang menghubungkan hasil dengan pemboleh ubah proses yang penting.
- (c) Analisa ralat daripada eksperimen ini. Adakan terdapat sebarang petunjuk bagi ketidacukupan model? (Rujuk lampiran B)
- (d) Bolehkah rekabentuk ini diturunkan kepada rekabentuk 2^3 dengan dua replika? Sekiranya ya, lakarkan rekabentuk tersebut dengan menunjukkan nilai purata hasil pada setiap bucu kiub.
- (e) Andaikan dua blok diperlukan dalam eksperimen ini. Bina suatu rekabentuk pembauran dengan blok ABCD. Analisa data bagi rekabentuk tersebut.

[100 markah]

3. An experiment was run in a semiconductor fabrication plant in an effort to increase yield. Five factors, each at two levels were studied. The factors (and levels) were A (small, large), B (20% below nominal, 20% above nominal), C (30s, 45s), D (small, large) and E (14.5min, 15.5min). The unreplicated 2^5 design shown below was run.

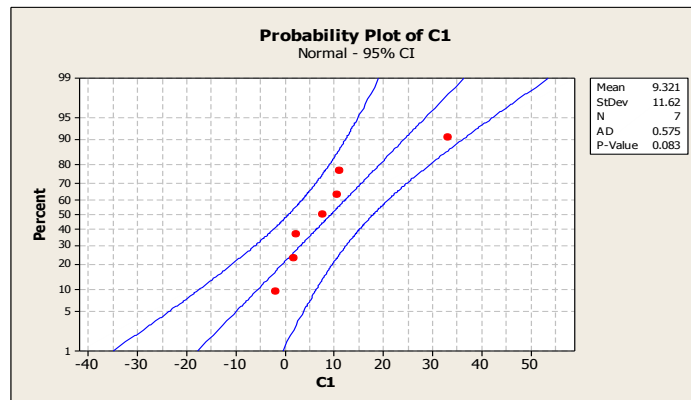
(1) = 7	d = 8	e = 8	de = 6
a = 9	ad = 10	ae = 12	ade = 10
b = 34	bd = 32	be = 35	bde = 30
ab = 55	abd = 50	abe = 52	abde = 53
c = 16	cd = 18	ce = 15	cde = 15
ac = 20	acd = 21	ace = 22	acde = 20
bc = 40	bcd = 44	bce = 45	bcde = 41
abc = 60	abcd = 61	abce = 65	abcde = 63

However, only one-quarter fraction could be run.

- (a) What type of design did the experimenter use?
- (b) State the complete defining relation and the aliases for this design. Choose $I = ABD$ and $I = ACE$ as the design generators. Which treatment combinations will be run in this experiment?
- (c) Estimate the factor effects. Given that:

$$\begin{aligned} \text{contrast}_A &= 45 & \text{contrast}_E &= 9 \\ \text{contrast}_B &= 133 & \text{contrast}_{BC} &= -7 \\ \text{contrast}_C &= 43 & \text{contrast}_{BE} &= 7 \\ \text{contrast}_D &= 31 \end{aligned}$$

Based on the estimates of the factor effects and its normal probability plot, which factors can be identified as important?



- (d) Write down a tentative regression model based on the results obtained in (c).
- (e) What term would you use as the “experimental error” to test the hypothesis?

[100 marks]

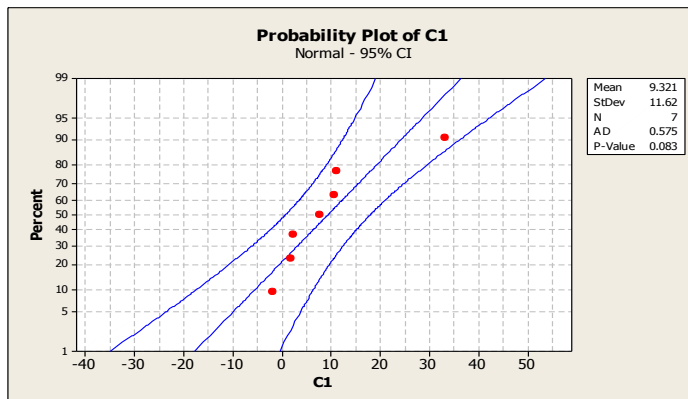
3. Satu eksperimen dalam semikonduktor tanaman fabrik dijalankan sebagai usaha bagi meningkatkan hasil. Lima faktor, setiapnya mempunyai dua tahap dikaji. Faktor-faktor tersebut (dengan tahap) adalah A (kecil, besar), B (20% di bawah nominal, 20% di atas nominal), C (30s, 45s), D (kecil, besar) dan E (14.5min, 15.5min). Larian bagi rekabentuk 2^5 replika tunggal diberikan di bawah.

(1) = 7	d = 8	e = 8	de = 6
a = 9	ad = 10	ae = 12	ade = 10
b = 34	bd = 32	be = 35	bde = 30
ab = 55	abd = 50	abe = 52	abde = 53
c = 16	cd = 18	ce = 15	cde = 15
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abc = 60	abcd = 61	abce = 65	abcde = 63

Walau bagaimanapun, hanya satu per empat larian boleh dilakukan.

- (a) Apakah jenis reka bentuk yang digunakan oleh pengujikaji?
- (b) Nyatakan hubungan pentakrif yang lengkap dan struktur alias bagi rekabentuk ini. Pilih $I = ABD$ dan $I = ACE$ sebagai penjana. Rawatan manakah yang akan dilarikan dalam eksperimen ini?
- (c) Anggarkan kesan faktor. Diberikan bahawa:
- $kontras_A = 45$ $kontras_E = 9$
 $kontras_B = 133$ $kontras_{BC} = -7$
 $kontras_C = 43$ $kontras_{BE} = 7$
 $kontras_D = 31$

Berdasarkan anggaran kesan faktor yang telah dikira dan plot kebarangkalian normal, faktor manakah yang dikenalpasti sebagai penting?



- (d) Tuliskan model regresi tentatif berdasarkan keputusan yang diperolehi dalam bahagian (c).
- (e) Terma apakah yang akan digunakan sebagai “ralat eksperimen” bagi menguji hipotesis?

[100 markah]

4. (a) (i) List down three basic principles of experimental design.
(ii) Give two important properties of replication. [30 marks]
- (b) An agricultural experiment, to test the effectiveness of certain crop stimulants on certain crops is carried out as follows. Three 100 hectare plots of land are selected and randomly labeled as 1, 2 and 3. In plot 1, wheat is grown. In plot 2, rye is grown and in plot 3, canola is grown. Then each plot is divided into four 25 hectare fields and the four possible stimulants are applied, one to a field, in random order. Each replicate of a factorial experiment requires 12 observations, and the experimenter has decided to run four replicates. However, due to time constraint, the experimenter is only capable of making 12 runs per day. So, the experimenter decides to run one replicate on each of the four days and to consider the days or replicates as blocks.
- (i) What is the name of the design being used here?
(ii) Write down the linear model which you would use to analyze these data. Define all the parameters involved in this model.
(iii) Give the general analysis of variance for the data described in this experiment. Your ANOVA table should consist of source of variation, degrees of freedom, expected mean squares and F_o .

[70 marks]

4. (a) (i) Nyatakan tiga prinsip asas bagi analisis ujikaji.
(ii) Berikan dua ciri penting pereplikaan.

[30 markah]

- (b) Suatu eksperimen pertanian bagi menguji keberkesanan perangsang tanaman tertentu ke atas beberapa tanaman dilakukan seperti berikut. Tiga plot tanah 100 hektar dipilih dan dilabelkan 1,2 dan 3 secara rawak. Dalam plot 1, gandum ditanam. Manakala dalam plot 2, 'rye' ditanam dan plot 3, kanola ditanam. Seterusnya, setiap plot dibahagikan kepada empat padang seluas 25 hektar dan empat perangsang diletakkan secara rawak di setiap padang. Setiap replika bagi eksperimen faktorial memerlukan 12 cerapan, dan pengujikaji menetapkan untuk melakukan empat replika. Walau bagaimanapun, disebabkan oleh kekangan masa, pengujikaji hanya boleh membuat 12 larian per hari. Maka, pengujikaji memutuskan untuk menjalankan satu replika setiap hari dan menganggap hari atau replika sebagai blok.

- (i) Apakah nama rekabentuk yang digunakan?
(ii) Nyatakan model linear yang akan anda gunakan bagi menganalisis data tersebut. Takrifkan semua parameter yang terlibat dalam model ini.
(iii) Berikan jadual analisis varians umum bagi data yang digunakan dalam eksperimen ini. Jadual ANOVA anda perlu mengandungi sumber variasi, darjah kebebasan, jangkakan min kuasa dua dan F_o .

[70 markah]

Appendix A (Formula)

Appendix A (Formula)

1. Two-factor Factorial Design

$$SS_T = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^n y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$SS_A = \sum_{i=1}^a \frac{y_{i..}^2}{bn} - \frac{y_{...}^2}{abn}$$

$$SS_B = \sum_{j=1}^b \frac{y_{.j.}^2}{an} - \frac{y_{...}^2}{abn}$$

$$SS_{Subtotal} = \sum_{i=1}^a \sum_{j=1}^b \frac{y_{ij.}^2}{n} - \frac{y_{...}^2}{abn}$$

$$SS_{AB} = SS_{Subtotal} - SS_A - SS_B$$

$$SS_E = SS_T - SS_{Subtotal} \quad \text{atau} \quad SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

Expected Mean Square

Random Effects Model:

$$E[MS_A] = \sigma^2 + n\sigma_{\tau\beta}^2 + bn\sigma_{\tau}^2$$

$$E[MS_B] = \sigma^2 + n\sigma_{\tau\beta}^2 + an\sigma_{\beta}^2$$

$$E[MS_{AB}] = \sigma^2 + n\sigma_{\tau\beta}^2$$

$$E[MS_E] = \sigma^2$$

Expected Mean Square

Mixed Model (A: fixed, B: random):

$$E[MS_A] = \sigma^2 + n\sigma_{\tau\beta}^2 + \frac{bn \sum_{i=1}^a \tau_i^2}{a-1}$$

$$E[MS_B] = \sigma^2 + an\sigma_{\beta}^2$$

$$E[MS_{AB}] = \sigma^2 + n\sigma_{\tau\beta}^2$$

$$E[MS_E] = \sigma^2$$

2. Three-factor Factorial Design

$$SS_T = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^c \sum_{l=1}^n y_{ijkl}^2 - \frac{y_{\dots}^2}{abcn}$$

$$SS_A = \sum_{i=1}^a \frac{y_{i\dots}^2}{bcn} - \frac{y_{\dots}^2}{abcn}$$

$$SS_B = \sum_{j=1}^b \frac{y_{\cdot j \cdot \cdot}^2}{acn} - \frac{y_{\dots}^2}{abcn}$$

$$SS_C = \sum_{k=1}^c \frac{y_{\cdot \cdot k \cdot}^2}{abn} - \frac{y_{\dots}^2}{abcn}$$

$$SS_{Subtotal(AB)} = \sum_{i=1}^a \sum_{j=1}^b \frac{y_{ij\dots}^2}{cn} - \frac{y_{\dots}^2}{abcn}$$

$$SS_{AB} = SS_{Subtotal(AB)} - SS_A - SS_B$$

$$SS_{Subtotal(AC)} = \sum_{i=1}^a \sum_{k=1}^c \frac{y_{i \cdot k \cdot}^2}{bn} - \frac{y_{\dots}^2}{abcn}$$

$$SS_{AC} = SS_{Subtotal(AC)} - SS_A - SS_C$$

$$SS_{Subtotal(BC)} = \sum_{j=1}^b \sum_{k=1}^c \frac{y_{\cdot jk \cdot}^2}{an} - \frac{y_{\dots}^2}{abcn}$$

$$SS_{BC} = SS_{Subtotal(BC)} - SS_B - SS_C$$

$$SS_{Subtotal(ABC)} = \sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^c \frac{y_{i \cdot j \cdot k \cdot}^2}{bn} - \frac{y_{\dots}^2}{abcn}$$

$$SS_E = SS_T - SS_{Subtotal} \quad \text{atau} \quad SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

3. Two-stage Nested Design

$$SS_T = \sum_i \sum_j \sum_k y_{ijk}^2 - \frac{y_{\dots}^2}{abn}$$

$$SS_A = \sum_{i=1}^a \frac{y_{i\dots}^2}{bn} - \frac{y_{\dots}^2}{abn}$$

$$SS_{B(A)} = \sum_{i=1}^a \sum_{j=1}^b \frac{y_{ij\dots}^2}{n} - \sum_{i=1}^a \frac{y_{i\dots}^2}{bn}$$

$$SS_E = SS_T - SS_A - SS_{B(A)}$$

**4. Expected Mean Square
Split-Plot Design**

$$\tau_i : \sigma^2 + ab\sigma_\tau^2$$

$$\beta_j : \sigma^2 + b\sigma_{\tau\beta}^2 + \frac{rb \sum \beta_j^2}{a-1}$$

$$\gamma_k : \sigma^2 + a\sigma_{\tau\gamma}^2 + \frac{ra \sum \gamma_k^2}{b-1}$$

$$\tau\beta_{ij} : \sigma^2 + b\sigma_{\tau\beta}^2$$

$$\tau\gamma_{ik} : \sigma^2 + a\sigma_{\tau\gamma}^2$$

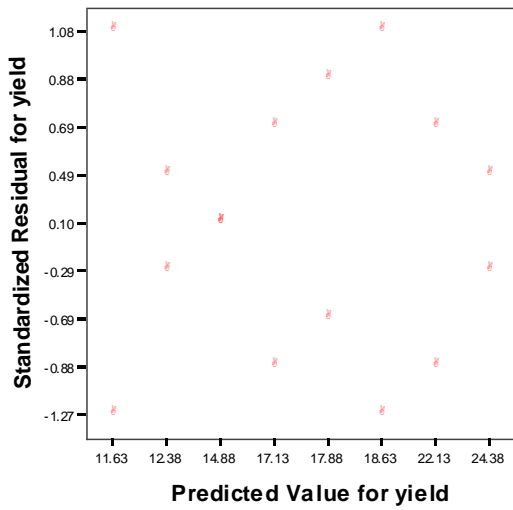
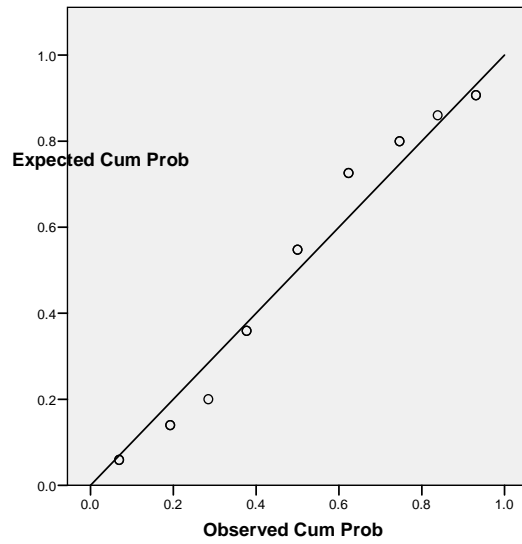
$$\beta\gamma_{jk} : \sigma^2 + \sigma_{\tau\beta\gamma}^2 + \frac{r \sum \sum \beta\gamma_{jk}^2}{a-1 \quad b-1}$$

$$\tau\beta\gamma_{ijk} : \sigma^2 + \sigma_{\tau\beta\gamma}^2$$

$$\varepsilon_{ijkh} : \sigma^2$$

Appendix B

Normal P-P Plot of Residual for yield



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