UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2010/2011 Academic Session

April/May 2011

GTX 102/3 – Mathematics for Radiation Science I
[Matematik Sains Sinaran I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Find the following limits, if they exist:

(i) $\lim\limits_{x \to 0} \dfrac{2x^2 + 7}{\sqrt{3x^4 + 4}}$;

(ii) $\lim\limits_{x \to 1} \dfrac{\sqrt{x} - \sqrt{2x - 1}}{x - 1}$;

(iii) $\lim\limits_{x \to +\infty} \left[\dfrac{2x + 3}{2x + 1} \right]^{x+1}$.

(b) Let $f(x) = \dfrac{x}{x^2 + 1}$.

Find:

(i) $f'(x)$ and $f''(x)$;

(ii) the critical numbers of f;

(iii) the points at which f has a relative extremum;

(iv) the intervals on which f is increasing or decreasing;

(v) the inflection points of the graph of f;

(vi) the vertical and horizontal asymptotes of the graph of f.

Sketch the graph of f.

[100 marks]

1. *(a)* *Cari had berikut, jika wujud:*

(i) $\underset{x\to 0}{\text{had}} \dfrac{2x^2+7}{\sqrt{3x^4+4}}$;

(ii) $\underset{x\to 1}{\text{had}} \dfrac{\sqrt{x}-\sqrt{2x-1}}{x-1}$;

(iii) $\underset{x\to +\infty}{\text{had}} \left[\dfrac{2x+3}{2x+1}\right]^{x+1}$.

(b) *Andaikan* $f(x)=\dfrac{x}{x^2+1}$.

Cari:

(i) $f'(x)$ *dan* $f''(x)$;

(ii) *nombor-nombor genting* f;

(iii) *titik-titik yang padanya* f *mempunyai ekstremum relatif;*

(iv) *selang yang padanya* f *menokok atau menyusut;*

(v) *titik lengkok balas graf* f;

(vi) *asimptot mencancang dan asimptot mengufuk graf* f.

Lakar graf f.

[100 markah]

2. (a) Let three vectors be $\vec{a} = \langle 1,2,3 \rangle, \vec{b} = \langle 4,5,6 \rangle$, and $\vec{c} = \langle 7,8,0 \rangle$.

Find:

(i) $\vec{a} \cdot \vec{b}$, $\vec{a} \times \vec{b}$ and $\vec{c} \cdot (\vec{a} \times \vec{b})$.

(ii) the angle between $\vec{a}$ and $\vec{b}$.

(iii) the area of the parallelogram with two adjacent sides formed by vectors $\vec{a}$ and $\vec{b}$.

(iv) the volume of the parallelepiped with three adjacent edges formed by the three vectors, $\vec{a}$, $\vec{b}$, and $\vec{c}$.

(b) A plane in $\square^3$ contains three points $P(1,2,2)$, $Q(2,-1,4)$ and $R(3,5,-2)$.

Find:

(i) the position vectors corresponding respectively to $\overrightarrow{PQ}, \overrightarrow{PR}$ and $\overrightarrow{QR}$;

(ii) the distance from the point $Q(2,-1,4)$ to the line through the points $P(1,2,2)$ and $R(3,5,-2)$;

(iii) a vector which is orthogonal to both $\overrightarrow{PQ}$ and $\overrightarrow{QR}$;

(iv) a linear equation for the plane.

[100 marks]

3. (a) Solve the following linear system of equation by reducing the augmented matrix into reduced echelon form . Give a justification, if you say that the system has no solution.

(i)
$$x + 2y - 5z = 7$$
$$x + y - 2z = 5$$
$$2x - 3y + 11z = 0$$

(ii)
$$3x - y - 5z = 0$$
$$x - y - z = -1$$
$$2x + 8y + 3z = 10$$

(b) Given z = - 4 - 4 $\bar{}$ and w = 2(cos – + i sin –), find

(i) (ii) Arg z (iii) z^3 (iv) z^4w^2 (v) $z^{-3}w^{-2}$ (vi) —

(ii) Find the six sixth roots of { z : z^6 = -64 }and plot them on the complex plain.

[100 marks]

2. (a) *Andaikan tiga vektor* $\vec{a} = \langle 1,2,3 \rangle, \vec{b} = \langle 4,5,6 \rangle,$ dan $\vec{c} = \langle 7,8,0 \rangle.$

Cari:

(i) $\vec{a} \cdot \vec{b}, \quad \vec{a} \times \vec{b}$ dan $\vec{c} \cdot (\vec{a} \times \vec{b});$

(ii) sudut di antara $\vec{a}$ dan $\vec{b};$

(iii) luas segi empat selari dengan dua sisi sebelah dibentuk oleh vektor $\vec{a}$ dan $\vec{b};$

(iv) isipadu paralelepiped dengan tiga tepi sebelah dibentuk oleh tiga vektor, $\vec{a}, \quad \vec{b},$ dan $\vec{c}.$

(b) *Suatu satah dalam* $\square^3$ *mengandungi tiga titik, P(1,2,2), Q(2,-1,4) dan R(3,5,-2).*

Cari:

(i) vektor kedudukan masing-masingnya sepadan dengan $\overrightarrow{PQ}, \overrightarrow{PR}$ dan $\overrightarrow{QR};$

(ii) jarak dari titik Q(2,-1,4) ke garis yang melalui titik P(1,2,2) dan R(3,5,-2);

(iii) vektor yang berortogon kepada kedua-dua $\overrightarrow{PQ}$ dan $\overrightarrow{QR};$

(iv) persamaan linear bagi satah tersebut.

[100 markah]

3. (a) *Selesaikan sistem persamaan linear berikut dengan menurunkan matriks diperluas kepada bentuk eselon terturun. Berikan justifikasi, jika anda menyatakan bahawa sistem tidak mempunyai penyelesaian.*

(i)
$$x + 2y - 5z = 7$$
$$x + y - 2z = 5$$
$$2x - 3y + 11z = 0$$

(ii)
$$3x - y - 5z = 0$$
$$x - y - z = -1$$
$$2x + 8y + 3z = 10$$

(b) *Diberi* $z = -4 - 4$ *dan* $w = 2(\cos - + i \sin -)$, *cari*

(i) *(ii) Arg z* *(iii)* z^3 *(iv)* $z^4 w^2$ *(v)* $z^{-3} w^{-2}$ *(vi)* —

(c) *Cari enam punca keenam* $\{ z : z^6 = -64 \}$ *dan plot di atas satah kompleks.*

[100 markah]

4. (a) Identify each of the conic section below. Whereever applicable, find vertices,foci, axes, directrices, centre and asymptotes. Then, sketch the graph

(i) $x^2 - 6x + 4y + 15 = 0$

(ii) $4x^2 + 9y^2 + 24x - 36y + 36 = 0$

(b) Find the volume of solid formed when the region bounded by the parabolas with equation $y = x^2$ and $y = 8 - x^2$ is rotated about (i) the x-axis (ii) the y-axis.

(c) Solve

(i)

(ii) ———

(iii) ————

[100 marks]

4. (a) *Kenalpasti setiap keratan kon di bawah. Di mana sesuai cari bucu, fokus, paksi, direktriks, pusat dan asimptot. Seterusnya, lakarkan graf tersebut.*

(i) $x^2 - 6x + 4y + 15 = 0$

(ii) $4x^2 + 9y^2 + 24x - 36y + 36 = 0$

(b) *Cari isipadu yang terjana apabila rantau yang dibatasi parabola* $y = x^2$ *dan* $y = 8 - x^2$ *diputarkan di sekitar (i) paksi-x (ii) paksi-y*

(c) *Selesaikan*

(i)

(ii) _____

(iii) _____

[100 markah]

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