UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2010/2011

April – May 2011

MGM 551 – Operations Research
[Penyelidikan Operasi]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of TEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEPULUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instruction : Answer **all nine** [9 questions.

*[**Arahan** : Jawab **semua sembilan** [9] soalan.]*

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1. (a) Standardize the following linear programming (LP) formulation:

Maximize $Z = 12x_1 + 10x_2 - 15x_3$

Subject to
$$\begin{aligned} 6x_1 + 2x_2 - 5x_3 &= 25 \\ 5x_1 - 4x_2 + x_3 &\geq -35 \\ 2x_1 - 3x_2 + 4x_3 &\leq 30 \end{aligned}$$

where $x_1 \geq 0$, x_2 unrestricted and $-5 \leq x_3 \leq 8$

(b) From (a), if the LP remains the same and only the condition on variable x_3 is changed, i.e. $x_3 \geq 0$, show its dual formulation.

[10 marks]

2. Given the following linear programming problem:

Minimize $Z = 4x_1 + 3x_2 + 8x_3$

Subject to
$$\begin{aligned} 2x_1 + x_2 - 4x_3 &\geq 8 \\ x_1 - 2x_2 + x_3 &\geq 6 \end{aligned}$$

where $x_i \geq 0 \ \forall \ i$.

Show the simplex tableau after **one** iteration using the,

(a) M-method.

(b) dual simplex method.

[15 marks]

3. Juruco oil produces two types of petrol, A and B, from two types of crude oil, Crude 1 and Crude 2. Petrol A is allowed to contain up to 4% impurities and Petrol B is allowed to contain up to 3% impurities. Petrol A sells for RM180 per barrel, whereas Petrol B sells for RM220 per barrel. Up to 4,200 barrels of Petrol A and up to 4,300 barrels of Petrol B can be sold. The cost per barrel of each crude, availability, and the level of impurities in each crude are as shown in table below.

Oil	Cost per Barrel (RM)	Impurity Level (%)	Availability (Barrel)
Crude 1	60	10	5.000
Crude 2	80	2	4.500

1. (a) *Piawaikan model pengaturcaraan linear (PL) berikut:*

$$\text{Maksimumkan} \quad Z = 12x_1 + 10x_2 - 15x_3$$

$$\begin{aligned} \text{Terhadap} \quad & 6x_1 + 2x_2 - 5x_3 = 25 \\ & 5x_1 - 4x_2 + x_3 \geq -35 \\ & 2x_1 - 3x_2 + 4x_3 \leq 30 \end{aligned}$$

Dengan $x_1 \geq 0$, x_2 *tak tersekat dan* $-5 \leq x_3 \leq 8$

(b) *Daripada (a,) jika PL tidak berubah kecuali syarat bagi pembolehubah* x_3, iaitu $x_3 \geq 0$, *tunjukkan rumus dualnya.*

[10 markah]

2. *Diberikan rumus pengaturcaraan linear berikut:*

$$\text{Minimumkan} \quad Z = 4x_1 + 3x_2 + 8x_3$$

$$\begin{aligned} \text{Terhadap} \quad & 2x_1 + x_2 - 4x_3 \geq 8 \\ & x_1 - 2x_2 + x_3 \geq 6 \end{aligned}$$

$$\text{dengan} \quad x_i \geq 0 \ \forall \ i.$$

Tunjukkan tablo simpleks setelah ***satu*** *lelaran menggunakan,*

(a) *kaedah M.*

(b) *kaedah simpleks dual.*

[15 markah]

3. *Syarikat Juruco menghasilkan dua jenis petrol, A and B, daripada dua jenis minyak mentah, 'Crude 1' dan 'Crude 2'. Petrol A dibenarkan mengandungi hingga 4% kotoran dan Petrol B dibenarkan mengandungi hingga 3% kotoran. Petrol A berharga RM180 setong, manakala Petrol B berharga RM220 setong. Sebanyak 4,200 tong Petrol A dan sebanyak 4,300 tong Petrol B boleh dijual. Kos setong minyak mentah, kedapatan dan kandungan kotoran setong diberikan di dalam jadual berikut,*

Minyak Mentah	***Kos Setong (RM)***	***Aras Kotoran (%)***	***Kedapatan (Tong)***
Crude 1	*60*	*10*	*5.000*
Crude 2	*80*	*2*	*4.500*

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Before blending the crude oil into gas, any amount of each crude can be "purified" for a cost of RM5.00 per barrel. Purification eliminates half the impurities in the crude oil. Determine how to maximise profit.

[10 marks]

4. Consider the following linear programming problem,

$$\text{Maximize} \quad Z = 2x_1 + 5x_2 + x_3 + 4x_4$$

$$\begin{aligned} \text{Subject to} \quad & 3x_2 + 2x_3 - x_4 \leq 5 \quad \text{(Resource 1)} \\ & -x_1 + 2x_2 + 2x_3 \leq 6 \quad \text{(Resource 2)} \\ & x_1 + x_2 + x_3 + x_4 \leq 8 \quad \text{(Resource 3)} \end{aligned}$$

$$x_i \geq 0 \ , \ i = 1,2,3,4.$$

Assume x_5, x_6 and x_7 are the slack variables for constraints 1, 2 dan 3 respectively. The optimal tableau is as follows:

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Solution
Z	1.5	0	4	0	0	0.5	4	35
x_2	-0.5	1	1	0	0	0.5	0	3
x_4	1.5	0	0	1	0	-0.5	1	5
x_5	3	0	-1	0	1	-2	1	1

(i) What is the optimal solution to this problem?
(ii) State the status of each resource.
(iii) What are the shadow prices of each resource?
(iv) Find the range for resource 2 so that the current optimal basis remains.
(v) If the limit of resource 2 is now 10, what is the new optimal solution?
(vi) Find the range for c_4 (the objective function coefficient of x_4) so that the current optimal basis remains.

[15 marks]

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Sebelum mengadun minyak mentah menjadi petrol, setiap amaun minyak mentah boleh disuling dengan kos RM5.00 setong. Penyulingan menyahkan separuh kekotoran yang wujud dalam miyak mentah. Tentukan bagaimana hendak memaksimumkan keuntungan.

[10 markah]

4. *Pertimbangkan masalah pengaturan linear berikut:*

Maksimumkan $Z = 2x_1 + 5x_2 + x_3 + 4x_4$

Terhadap

$$\begin{aligned} 3x_2 + 2x_3 - x_4 &\leq 5 \quad \text{(Sumber 1)} \\ -x_1 + 2x_2 + 2x_3 &\leq 6 \quad \text{(Sumber 2)} \\ x_1 + x_2 + x_3 + x_4 &\leq 8 \quad \text{(Sumber 3)} \end{aligned}$$

$$x_i \geq 0 \ , \ i = 1,2,3,4.$$

Andaikan x_5, x_6 dan x_7 masing-masing adalah pembolehubah lalai bagi kekangan 1, 2 dan 3. Tablo optimumnya adalah seperti berikut:

Asas	x_1	x_2	x_3	x_4	x_5	x_6	x_7	*Penyelesaian*
Z	*1.5*	*0*	*4*	*0*	*0*	*0.5*	*4*	*35*
x_2	*-0.5*	*1*	*1*	*0*	*0*	*0.5*	*0*	*3*
x_4	*1.5*	*0*	*0*	*1*	*0*	*-0.5*	*1*	*5*
x_5	*3*	*0*	*-1*	*0*	*1*	*-2*	*1*	*1*

(i) Apakah penyelesaian optimum masalah ini?
(ii) Nyatakan status setiap sumber.
(iii) Berapakah nilai harga bayangan setiap sumber?
(iv) Dapatkan julat bagi sumber 2 supaya penyelesaian asas optimum semasa tidak berubah.
(v) Jika had sumber 2 berubah kepada 10, apakah penyelesaian optimum yang baru?
(vi) Dapatkan julat bagi c_4 *(pekali fungsi matlamat bagi pembolehubah x_4) supaya penyelesaian asas optimum semasa tidak berubah.*

[15 markah]

5. A furniture parts manufacturer seeks to reduce the time producing a chair arm. The producing of chair arm involves 12 activities whose precedence relations are identified with their node numbers given below:

Activity	Node	Duration (days)	Activity	Nodes	Duration (days)
A	1-2	10	G	3-7	11
B	1-3	5	H	4-5	14
C	1-4	7	I	5-6	5
D	2-3	6	J	6-7	5
E	2-5	11	K	6-8	3
F	2-6	8	L	7-8	7

(a) Draw a network diagram to represent the production activities
(b) Find the critical path and production duration
(c) Calculate Earliest Start Time (EST), Latest Event Time (LET), Latest Start Time (LST), Latest Finish Time (LFT) for all the activities and total float

[10 marks]

6. A small project involves the following activities:

Activity	Precedence Activities	Duration (days)		
		Optimistic time	Most Likely time	Pessimistic time
A	-	2	5	8
B	-	9	12	28
C	A	6	11	15
D	B	4	5	20
E	B	4	5	9
F	D	4	6	7
G	E	6	7	12
H	F, G	3	5	11
I	C	2	4	9

(a) Draw a network diagram using the above data.
(b) Calculate the activity times for all activities (give your answers in round numbers) and find the critical path.
(c) Find the probability of completing the project
(i) before 35 days.
(ii) after 41 days.
Give your answers to two decimal places.
(d) What impact do you think will be on the critical path if the most likely time of activity G is changed from 7 to 12 days?

[10 marks]

5. *Pengeluar bahagian perabot berusaha untuk mengurangkan masa menghasilkan lengan kerusi. Penghasilan lengan kerusi melibatkan 12 kegiatan yang hubungannya dikenalpasti dengan nombor nod diberi seperti berikut:*

Kegiatan	*Nod*	*Jangkamasa (Hari)*	*Kegiatan*	*Nod*	*Jangkamasa (Hari)*
A	*1-2*	*10*	*G*	*3-7*	*11*
B	*1-3*	*5*	*H*	*4-5*	*14*
C	*1-4*	*7*	*I*	*5-6*	*5*
D	*2-3*	*6*	*J*	*6-7*	*5*
E	*2-5*	*11*	*K*	*6-8*	*3*
F	*2-6*	*8*	*L*	*7-8*	*7*

(a) Lakarkan gambarajah rangkaian untuk mewakili kegiatan pengeluaran tersebut.

(b) Cari lintasan genting dan jangkamasa pengeluaran.

(c) Kira masa mula terawal, masa peristiwa terlewat, masa mula terlewat, masa tamat terlewat bagi semua kegiatan dan jumlah apungan.

[10 markah]

6. Sebuah projek kecil melibatkan kegiatan seperti berikut:

Kegiatan	*Kegiatan Pendahulu*	*Jangkamasa (Hari)*		
		Masa Optimistik	*Masa Paling boleh jadi*	*Masa Pesimistik*
A	-	*2*	*5*	*8*
B	-	*9*	*12*	*28*
C	*A*	*6*	*11*	*15*
D	*B*	*4*	*5*	*20*
E	*B*	*4*	*5*	*9*
F	*D*	*4*	*6*	*7*
G	*E*	*6*	*7*	*12*
H	*F, G*	*3*	*5*	*11*
I	*C*	*2*	*4*	*9*

(a) Lakarkan gambarajah rangkaian berdasarkan data di atas.

(b) Kira jangkamasa bagi semua kegiatan (beri jawapan anda dalam nombor bulat) dan dapatkan lintasan genting.

(c) Cari kebarangkalian untuk menyiapkan projek

(i) sebelum 35 hari.

(ii) selepas 41 hari.

Beri jawapan anda dalam dua tempat perpuluhan.

(d) Pada pendapat anda, apakah impak yang terdapat pada lintasan genting jika masa paling bolehjadi untuk kegiatan G ditukar dari 7 ke 12 hari?

[10 markah]

7. Consider the following two-person game:

		Player B		
		B1	B2	B3
Player A	A1	5	6	9
	A2	7	5	7
	A3	5	3	5

(a) Why the pure strategies cannot be used here to solve the game? Explain the reason.

(b) Apply the dominance method to give the reduced matrix.

(c) Based on the answer obtain in part (b), calculate the game value and probabilities of strategies for Player A and B using the odds method.

[10 marks]

8. In a two-person game, Player A has four strategies and Player B has two strategies. The pay-off matrix of the game is given as:

		Player B	
		1	2
Player A	1	-6	-2
	2	-3	-4
	3	-2	-9
	4	-7	-1

(a) Write the expected pay-off for Player B when Player A adopts strategies. Assume that Player B uses mixed strategies of 1 and 2 with probabilities of q and (1- q) respectively.

(b) Plot the pay-off equations in a graph and identify the minimax point.

(c) Calculate the game value and probabilities of strategies for Player A and B.

[10 marks]

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7. *Pertimbangkan permaian dua orang berikut:*

		Pemain B		
		B1	*B2*	*B3*
Pemain A	*A1*	*5*	*6*	*9*
	A2	*7*	*5*	*7*
	A3	*5*	*3*	*5*

(a) *Kenapa strategi tulen tidak boleh digunakan di sini untuk menyelesaikan permainan? Jelaskan sebabnya.*

(b) *Gunakan kaedah dominan untuk mendapat matriks dikurangkan.*

(c) *Berdasarkan jawapan yang didapati di bahagian (b), kira nilai permainan dan kebarangkalian strategi untuk pemain A dan B menggunakan kaedah odds.*

[10 markah]

8. *Dalam permainan dua orang, Pemain A mempunyai empat strategi manakala Pemain B mempunyai dua strategi. Matriks pembayaran untuk permainan ini diberi seperti berikut:*

		Pemain B	
		1	*2*
Pemain A	*1*	*-6*	*-2*
	2	*-3*	*-4*
	3	*-2*	*-9*
	4	*-7*	*-1*

(a) *Tulis anggaran pembayaran untuk Pemain B apabila Pemain A menggunakan strategi. Andaikan Pemain B menggunakan strategi campuran 1 dan 2 dengan kebarangkalian q dan (1- q) masing-masing.*

(b) *Plot persamaan pembayaran dalam graf dan tentukan titik minimaks.*

(c) *Kira nilai permainan dan kebarangkalian strategi untuk Pemain A dan B.*

[10 markah]

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9. A printer machine dealer has customers from five main cities. A salesperson is sent to handle the deliveries. The salesperson has to move from the store centre in city A to deliver orders to customers in these five cities. The distances (in km) among these cities are summarized in the following table.

	Distance between cities				
	A	B	C	D	E
A	0	120	220	150	210
B	120	0	80	110	130
C	220	80	0	160	185
D	150	110	160	0	190
E	210	130	185	190	0

The salesperson seeks to minimize the total traveling distance.

(a) Solve the above problem using the nearest-neighbor heuristic starting at city E.

(b) Solve the above problem using the reversal heuristic starting with the best nearest-neighbor tour. Can you find better a solution to the initial tour?

[10 marks]

9. Pengedar mesin pencetak mempunyai pelanggan dari lima buah bandar utama. Seorang penjual diarah untuk mengendalikan kerja penghantaran. Penjual tersebut perlu bertolak dari pusat stor di bandar A untuk menghantar tempahan kepada pelanggan di lima buah bandar ini. Jarak (dalam km) antara bandar ini diringkaskan dalam jadual berikut:

	Jarak antara bandar				
	A	*B*	*C*	*D*	*E*
A	*0*	*120*	*220*	*150*	*210*
B	*120*	*0*	*80*	*110*	*130*
C	*220*	*80*	*0*	*160*	*185*
D	*150*	*110*	*160*	*0*	*190*
E	*210*	*130*	*185*	*190*	*0*

Penjual tersebut berusaha untuk meminimumkan jumlah jarak perjalanannya.

(a) Selesaikan masalah di atas menggunakan heuristik jiran terdekat bermula dengan bandar E.

(b) Selesaikan masalah di atas menggunakan heuristik pembalikan bermula dengan perjalanan jiran terdekat. Bolehkah anda mencari penyelesaian yang lebih baik kepada perjalanan asal?

[10 markah]

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Appendix

Z score table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

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