UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2010/2011 Academic Session

April/May 2011

MSG 389 – Engineering Computation II
[Pengiraan Kejuruteraan II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Use Heun's method to find $y(0.5)$ for the following problem:

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=e^{-t},\ y(0)=1, \frac{dy}{dt}(0)=2,$$

[75 marks]

(b) Given

$$\frac{d^2y}{dx^2}=6x-0.5x^2, y(0)=0, y(12)=0$$

Compute the value of y(4) using the finite difference method with step size of *h=4*.

[25 marks]

2. (a) A cascade of two tanks is set up where tank 1, initially contains 200 gallons of corn syrup and tank 2 initially contains 100 gallons of water. Suppose that water flows into tank 1 at a rate of 1 gallon per minute and the solution in tank 1 flows from tank 1 into tank 2 at a rate also 1 gallon per minute. The solution in tank 2 exits tank 2 at a rate of 1 gallon per minute. Assume that the solutions are completely mixed instantaneously.

(i) Find the amount of corn syrup in tank 1 at time $t>0$.

(ii) Find the amount of corn syrup in tank 2 at time $t>0$.

(iii) What is the maximum amount of corn syrup which can contains in tank 2?

[50 marks]

(b) A certain chemical reaction takes place such that the time-rate of change of the amount of the unconverted substance Q is equal to -2Q, $\frac{dq}{dt}=-2q$. If the initial mass is 50 grams, use the 4th order Runge-Kutta method to estimate the amount of unconverted substance at *t=0.8* sec. Use *h=0.8.*

[30 marks]

(c) Find the solution of the initial value problem

$$ty'=\frac{1}{y+1}, \quad y(1)=0$$

and for what *t*-interval is the solution defined?

[20 marks]

1. (a) *Gunakan Kaedah Heun untuk menyelesaikan*

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = e^{-t}, y(0) = 1, \frac{dy}{dt}(0) = 2,$$

dapatkan nilai $y(0.5)$.

[75 markah]

(b) *Diberikan*

$$\frac{d^2y}{dx^2} = 6x - 0.5x^2, y(0) = 0, y(12) = 0$$

Dapatkan nilai y(4) dengan menggunakan kaedah beza terhingga dengan saiz langkah, h=4.

[25 markah]

2. (a) *Sebuah tangki buatan dibina dengan dua tangki di mana tangki 1 mengandungi 200 gelan sirap jagung dan tangki 2 mengandungi 100 gelan air. Andaikan air mengalir ke dalam tangki 1 dengan kadar 1 gelan setiap minit dan campuran dalam tangki 1 mengalir dari tangki 1 ke tangki 2 dengan kadar 1 gelan setiap minit. Campuran dalam tangki 2 keluar dengan kadar 1 gelan setiap minit. Andaikan campuran adalah sekata.*

(i) Dapatkan jumlah sirap jagung dalam tank 1 pada masa, $t > 0$.

(ii) Dapatkan jumlah sirap jagung dalam tank 2 pada masa $t > 0$.

(iii) Apakah jumlah maksima sirap jagung yang boleh dicapai dalam tangki 2?

[50 markah]

(b) *Satu tindakbalas kimia berlaku dengan masa-kadar baki bahan tertinggal, Q bersamaan dengan -2Q,* $\frac{dq}{dt} = -2q$ *. Sekiranya jisim awal adalah 50 gram, gunakan kaedah Runge-Kutta tertib 4 untuk menganggarkan jumlah bahan yang tertinggal pada t=0.8 saat. Gunakan saiz langkah, h=0.8.*

[30 markah]

(c) *Selesaikan masalah nilai awal*

$$ty' = \frac{1}{y+1}, \quad y(1) = 0$$

dan dapatkan nilai selang untuk t di mana penyelesaian tertakrif.

[20 markah]

3. (a) Using $[x_1, x_2, x_3] = [1, 3, 5]$ as the initial guesses, compute the values of $[x_1, x_2, x_3]$ after three iterations in the Gauss-Seidel method for

$$\begin{pmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 6 \end{pmatrix}$$

[30 marks]

(b) Consider the linear boundary value problem with Dirichlet boundary conditions

$$-u'' + \pi^2 u = 2\pi^2 \sin(\pi x)$$
$$u(0) = u(1) = 0$$

Solve the boundary value problem using shooting method with *h=0.5.*

[70 marks]

3. (a) *Dengan menggunakan* $[x_1, x_2, x_3] = [1, 3, 5]$ *sebagai nilai awal, dapatkan nilai* $[x_1, x_2, x_3]$ *selepas tiga lelaran dengan menggunakan kaedah Gauss-Seidel*

$$\begin{pmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 6 \end{pmatrix}$$

[30 markah]

(b) *Pertimbangkan masalah nilai sempadan linear dengan syarat sempadan Dirichlet*

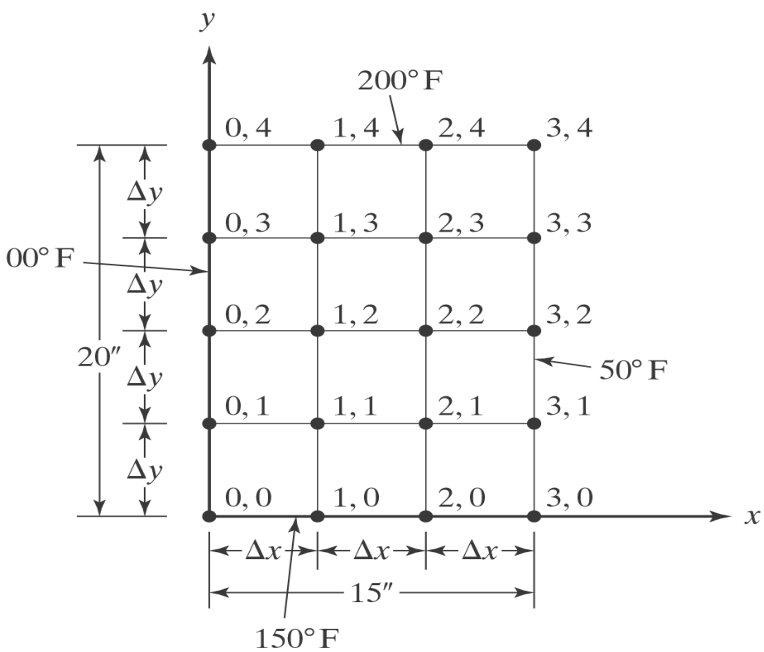
$$-u'' + \pi^2 u = 2\pi^2 \sin(\pi x)$$
$$u(0) = u(1) = 0$$

Selesaikan masalah ini dengan kaedah tembakan dengan saiz langkah, h=0.5.

[70 markah]

4. (a) Determine the steady-state temperature distribution in a rectangular plate of size $15'' \times 20''$ by solving the Laplace equation using $\Delta x = \Delta y = 5''$. The temperatures on the four sides of the plate are specified in the diagram below. The boundary conditions are

$$u(x,0) = 150^o F$$
$$u(0,y) = 0^o F$$
$$u(3,y) = 50^o F$$
$$u(x,4) = 200^o F$$



[75 marks]

(b) Apply finite difference methods to the equation

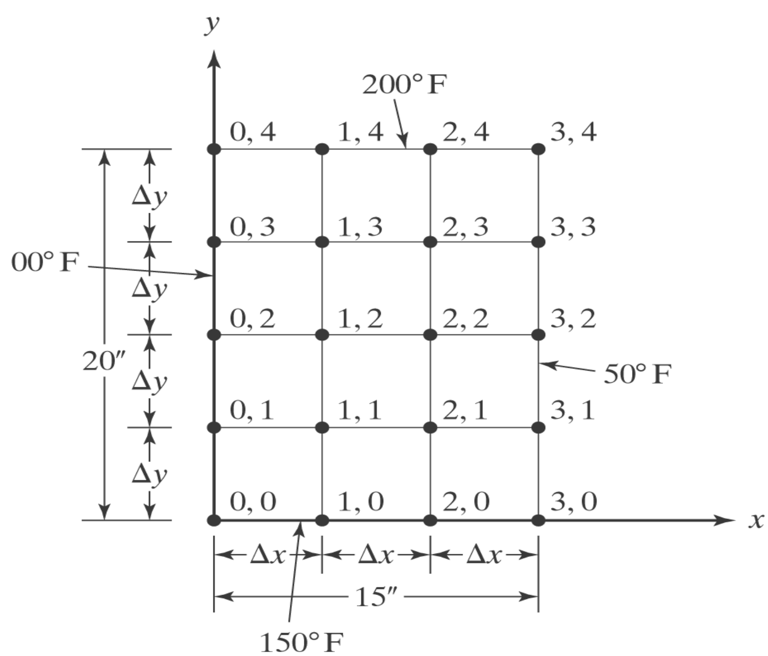
$$\frac{\partial^2 U}{\partial t^2} - \frac{\partial^2 U}{\partial x^2} = F[t, x, U, U_t, U_x] \qquad -\infty < x < \infty, t \geq 0$$

with initial conditions $U(x,0) = f(x), \quad U_t(x,0) = g(x)$

[25 marks]

4. (a) *Dapatkan nilai stabil bagi taburan suhudalam satu satah segi empat bersaiz* $15'' \times 20''$ *dengan persamaan Laplace menggunakan* $\Delta x = \Delta y = 5''$. *Nilai suhu di keempat-empat sisi satah ditunjukkan dalan rajah di bawah. Syarat-syarat sempadan adalah*

$$u(x,0) = 150^o F$$
$$u(0,y) = 0^o F$$
$$u(3,y) = 50^o F$$
$$u(x,4) = 200^o F$$



[75 markah]

(b) *Aplikasi kaedah beza terhingga kepada persamaan*

$$\frac{\partial^2 U}{\partial t^2} - \frac{\partial^2 U}{\partial x^2} = F[t, x, U, U_t, U_x] \qquad -\infty < x < \infty, t \ge 0$$

dengan syarat awal $U(x,0) = f(x), \quad U_t(x,0) = g(x)$

[25 markah]

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