UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2010/2011 Academic Session

April/May 2011

MSS 212 - Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all seven** [7] questions.

*[**Arahan**: Jawab **semua tujuh** [7] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Given that

$$\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 6 \quad \text{and} \quad \det\begin{pmatrix} a & b & c \\ j & k & l \\ g & h & i \end{pmatrix} = -1$$

Find

(i) $\det\begin{pmatrix} d & e & f \\ g & h & i \\ a & b & c \end{pmatrix}$ [20 marks]

(ii) $\det\begin{pmatrix} 3a & b & c \\ 3d & e & f \\ 3g-12d & h-4e & i-4f \end{pmatrix}$ [30 marks]

(iii) $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^{-1}$ [10 marks]

(iv) $\det\begin{pmatrix} a & b & c \\ d+j & e+k & f+\ell \\ g & h & i \end{pmatrix}$ [20 marks]

(b) Let $A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ and $B = \begin{pmatrix} d & e \\ 0 & f \end{pmatrix}$

Show that $AB = BA$ if and only if $\det\begin{pmatrix} b & a-c \\ e & d-f \end{pmatrix} = 0$

[30 marks]

2. (a) Let P_2 (□) be a vector space over □ and $\alpha = 1, 2x, x+1 \subseteq P_2$ (□).

(i) Is α linearly independent over □ ? [15 marks]

(ii) Find a basis of P_2 (□) say β such that $|\beta \cap \alpha| = 2$ [50 marks]

(b) Let M_{2x2} (□) be a vector space over □ and $W = A \in M_{2x2}$ (□) $|\det A = 0$. Is W a subspace of M_{2x2} (□) ?

[35 marks]

1. (a) *Diberi*

$$\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 6 \quad dan \quad \det\begin{pmatrix} a & b & c \\ j & k & l \\ g & h & i \end{pmatrix} = -1$$

Cari

(i) $\det\begin{pmatrix} d & e & f \\ g & h & i \\ a & b & c \end{pmatrix}$ [20 markah]

(ii) $\det\begin{pmatrix} 3a & b & c \\ 3d & e & f \\ 3g-12d & h-4e & i-4f \end{pmatrix}$ [30 markah]

(iii) $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^{-1}$ [10 markah]

(iv) $\det\begin{pmatrix} a & b & c \\ d+j & e+k & f+\ell \\ g & h & i \end{pmatrix}$ [20 markah]

(b) *Biar* $A=\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$ *dan* $B=\begin{pmatrix} d & e \\ 0 & f \end{pmatrix}$

Tunjukkan $AB = BA$ *jika dan hanya jika* $\det\begin{pmatrix} b & a-c \\ e & d-f \end{pmatrix} = 0$

[30 markah]

2. (a) *Biar* P_2 □ *ialah suatu ruang vector atas* □ *dan* $\alpha = \ 1, 2x, x+1 \ \subseteq P_2$ □ .

(i) *Adakah* α *tak bersandar linear atas* □ *?* [15 markah]

(ii) *Cari suatu asas bagi* P_2 □ , *katakan* β *sedemikian hingga* $|\beta \cap \alpha| = 2$

[50 markah]

(b) *Biar* M_{2x2} □ *ialah suatu ruang vector atas* □ *dan*

$W = \ A \in M_{2x2}$ □ $|\det A = 0$. *Adakah* W *suatu subruang bagi* M_{2x2} □ *?*

[35 markah]

3. (a) Let T be a linear transformation form $P_2(\mathbb{R})$ to $M_{2x2}(\mathbb{R})$. Let $A = [T]_{\alpha,\beta}$ where $\alpha = \{1, x, x^2\}$ and $\beta = \{e_{13} e_{12} e_{21} e_{22}\}$.

If $A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 1 \\ 0 & 2 & 1 & 0 \end{pmatrix}$, find the definition of T.

[50 marks]

(b) Using linear extention method, show that

(i) $P_2(\mathbb{R})$ will never be isomorphic to $Fun(S, \mathbb{R})$, $|S| = 2$. [20 marks]

(ii) $M_{2x2}(\mathbb{R})$ will never be isomorphic to $\mathbb{R}^5$. [20 marks]

4. Find $\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}^{101}$

[100 marks]

5. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$. Find $J(A)$. Justify your answer

[50 marks]

6. Let $W = L_{\mathbb{R}}(\{(1,0,-1), (0,1,1)\})$

(i) Find $W^{\perp}$. [20 marks]

(ii) Show that $\mathbb{R}^3 = W \oplus W^{\perp}$ [30 marks]

7. Let $T : M_{2\times 2}(IR) \to M_{2\times 2}(IR)$ be a linear transformation such that $(A)T = A^T$.

(i) Let $A = (a_{ij}), B = (b_{ij}) \in M_{2\times 2}(\mathbb{R})$. Show that

$$< A, B >= a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22} = T_r(AB^T)$$

is an inner product of $M_{2\times 2}(IR)$ [20 marks]

(ii) Show that T is a self-adjoint linear transformation [20 marks]

(iii) Find a basis β of $M_{2\times 2}(IR)$ such that $[T]_{\beta,\beta}$ is a diagonal matrix [60 marks]

3. (a) *Biar* T *ialah suatu transformasi linear dari* $P_2(\mathbb{R})$ *ke* $M_{2x2}(\mathbb{R})$. *Biar* $A = T_{\alpha,\beta}$ *sedemikian hingga* $\alpha = \{1, x, x^2\}$ *dan* $\beta = \{e_{13} e_{12} e_{21} e_{22}\}$.

Jika $A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & -1 & 1 \\ 0 & 2 & 1 & 0 \end{pmatrix}$, *cari definasi bagi* T.

[50 markah]

(b) *Dengan menggunakan kaedah sambungan linear, tunjukkan bahawa*

(i) $P_2(\mathbb{R})$ *tidak akan berisomorfis dengan* $Fun(S, \mathbb{R}), |S| = 2$.

[20 markah]

(ii) $M_{2x2}(\mathbb{R})$ *tidak akan berisomorfis dengan* $\mathbb{R}^5$. *[20 markah]*

4. *Cari* $\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}^{101}$

[100 markah]

5. *Biar* $A = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$. *Cari* $J(A)$. *Justifikasikan jawapan anda.*

[50 markah]

6. *Biar* $W = L_{\mathbb{R}}\{(1,0,-1), (0,1,1)\}$

(i) *Cari* $W^{\perp}$. *[20 markah]*

(ii) *Tunjukkan bahawa* $\mathbb{R}^3 = W \oplus W^{\perp}$ *[30 markah]*

7. *Biar* $T : M_{2\times2}(IR) \to M_{2\times2}(IR)$ *ialah suatu transformasi linear sedemikian hingga* $A(T) = A^T$.

(i) *Biar* $A = (a_{ij}), B = (b_{ij}) \in M_{2\times2}(\mathbb{R})$. *Tunjukkan bahawa*

$$< A, B >= a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22} = T_r(AB^T)$$

ialah suatu hasil darab terkedalaman bagi $M_{2\times2}(IR)$ *[20 markah]*

(ii) *Tunjukkan bahawa* T *adalah suatu transformasi linear yang swaadjoin*

[20 markah]

(iii) *Cari suatu asas* β *bagi* $M_{2\times2}(IR)$ *sedemikian hingga* $T_{\beta,\beta}$ *adalah suatu matrik pepenjuru.* *[60 markah]*

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