UNIVERSITI SAINS MALAYSIA

Second Semester Examination
Academic Session 2010/2011

April/May 2011

MSS 211 – Modern Algebra
[Aljabar Moden]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of THREE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TIGA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions : Answer **all five** [5] questions.

*[**Arahan** : Jawab **semua lima** [5] soalan.]*

1. Given A, B and C are sets, prove or disprove the following:

   (a) $A \cap B = A \cap C \Leftrightarrow B = C$.

   (b) $A \cap B \subseteq A \cup B \Leftrightarrow A \subseteq B$ or $B \subseteq A$.

[10 marks]

2. Given the functions $f : \mathbb{R} - \{-3\} \to \mathbb{R} - \{0\}$ and $g : \mathbb{R} - \{0\} \to \mathbb{R} - \{0\}$ where $(x)f = \dfrac{2}{3+x}$ and $(x)g = \dfrac{1}{x}$.

   (i) Show that f and g are bijective functions.

   (ii) Find $f \circ g$ and discuss whether it is also bijective.

   (iii) Evaluate $-\frac{1}{3}\ g$ and state the reason why $g \circ f$ cannot be defined.

   (iv) Find the inverse functions of f, g and $f \circ g$.

   (v) Verify that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

[25 marks]

3. Given the elements $f, g, h, k \in S_n$ with

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 5 & 8 & 6 & 4 & 9 & 7 & 2 & 1 \end{pmatrix},\ g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 4 & 5 & 6 & 7 & 8 & 9 & 2 & 1 \end{pmatrix},$$

$$h = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 5 & 1 & 6 & 2 & 4 & 7 & 9 & 8 \end{pmatrix},\ k = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 5 & 6 & 4 & 3 & 9 & 2 & 1 \end{pmatrix}$$

   (i) Write the value of n and hence determine the value of $|S_n|$.

   (ii) Write each of f, g, h and k as a cycle or product of disjoint cycles.

   (iii) Write each element as a product of transpositions and hence determine which of these belong to A_n.

   (iv) Find the order of each element f, g, h and k.

   (v) Find the inverse of each element f, g, h and k, hence write the elements g^{19} and k^{11}.

[25 marks]

4. Given that $\langle G, * \rangle$ is a group where $G = \{e, a, b, c\}$ and e is its identity element.

   (i) Show that $x^4 = e\ \forall x \in G$.

   (ii) Given that $|a| = 4$, obtain the two possible Cayley table for $\langle G, * \rangle$ and show that they are both isomorphic.

   (iii) On the other hand, if $x^2 = e\ \forall x \in G$, obtain the Cayley table for $\langle G, * \rangle$.

   (iv) Discuss the possible non-isomorphic cases for $\langle G, * \rangle$.

[20 marks]

5. List all the elements of the alternating group A_4 and show that there exists only one subgroup of order 4 in it. Show that this is a normal subgroup of A_4. Is A_4 isomorphic to the group $S_3 \times S_2$?

[20 marks]

1. *Diberikan* A, B *dan* C *adalah set, buktikan atau sangkalkan yang berikut:*

   (a) $A \cap B = A \cap C \Leftrightarrow B = C$.

   (b) $A \cap B \subseteq A \cup B \Leftrightarrow A \subseteq B$ *atau* $B \subseteq A$.

[10 markah]

2. *Diberikan fungsi* $f : \mathbb{R} - \{-3\} \to \mathbb{R} - \{0\}$ *dan* $g : \mathbb{R} - \{0\} \to \mathbb{R} - \{0\}$ *dengan* $(x)f = \dfrac{2}{3+x}$ *dan* $(x)g = \dfrac{1}{x}$

   (i) *Tunjukkan bahawa* f dan g *merupakan fungsi bijektif.*

   (ii) *Cari* $f \circ g$ *dan bincangkan kenapa ia juga bijektif.*

   (iii) *Nilaikan* $-\tfrac{1}{3}$ g *nyatakan kenapa* $g \circ f$ *tidak tertakrif.*

   (iv) *Cari songsangan setiap fungsi* f, g dan $f \circ g$.

   (v) *Tentusahkan bahawa* $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.

[25 markah]

3. *Diberikan* $f, g, h, k \in S_n$ *dengan*

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 5 & 8 & 6 & 4 & 9 & 7 & 2 & 1 \end{pmatrix}, \; g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 4 & 5 & 6 & 7 & 8 & 9 & 2 & 1 \end{pmatrix},$$

$$h = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 5 & 1 & 6 & 2 & 4 & 7 & 9 & 8 \end{pmatrix}, \; k = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 5 & 6 & 4 & 3 & 9 & 2 & 1 \end{pmatrix}$$

   (i) *Tuliskan nilai* n *dan dengan demikian tentukan nilai* $|S_n|$.

   (ii) *Tuliskan setiap* f, g, h dan k *sebagai kitar atau hasil darab kitar tak-terkait.*

   (iii) *Tuliskan setiap unsur tersebut sebagai hasidarab transposisi dan dengan demikian tentukan yang mana berada dalam* A_n.

   (iv) *Cari peringkat* f, g, h dan k.

   (v) *Cari songsangan bagi* f, g, h dan k, *dengan demikian tuliskan* g^{19} *and* k^{11}.

[25 markah]

4. *Diberikan bahawa* $\langle G, * \rangle$ *merupakan suatu kumpulan dengan* $G = \{e, a, b, c\}$ *dan* e *merupakan unsur identitinya.*

   (i) *Buktikan* $x^4 = e \; \forall x \in G$.

   (ii) *Diberi* $|a| = 4$, *dapatkan dua sifir Cayley yang mungkin bagi* $\langle G, * \rangle$ *dan buktikan bahawa kedua-duanya berisomorfik.*

   (iii) *Sebaliknya, jika* $x^2 = e \; \forall x \in G$, *dapatkan sifir Cayley bagi* $\langle G, * \rangle$.

   (iv) *Bicangkan kes-kes tak berisomorfik yang mungkin bagi* $\langle G, * \rangle$.

[20 markah]

5. *Senaraikan semua unsur kumpulan selang-seli* A_4 *dan buktikan kewujudan hanya satu subkumpulan berperingkat 4 di dalamnya. Tunjukkan bahawa ini merupakan suatu subkumpulan normal bagi* A_4. *Adakah* A_4 *berisomorfik dengan kumpulan* $S_3 \times S_2$ *?*

[20 markah]

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