UNIVERSITI SAINS MALAYSIA

First Semester Examination
2010/2011 Academic Session

November 2010

MAT 222 – Differential Equations II
[Persamaan Pembezaan II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Consider the system of differential equations below where x, y and z are functions in t:

$$x' = -x + y + z\,,\ \ y' = 4y + 5z\,,\ \ z' = -6x - z\,.$$

(i) Write the system above as a matrix equation for a suitable vector $\underset{\sim}{u}$.

(ii) Obtain the eigen values and corresponding eigen vectors for the 3×3 matrix obtained above.

(iii) Hence, obtain the general solution for x.

(iv) Solve for x given $x(0) = 5$ and $x'(0) = 7$.

(v) Solve the system completely given $z(0) = 4$.

[25 marks]

2. Solve the second order partial differential equation $u_{xx} + 2xu_x = 7e^{-x^2} + 3x + 2x\cos y$ where $u = u(x, y)$ given $u(0, y) = -\dfrac{7}{2} + \sin y$ and $u_x(0, y) = \dfrac{3}{2} + \cos y$. Subsequently, verify that your answer satisfies the given equation as well as the two initial conditions.

[15 marks]

3. Find the Fourier series for the function below:

$$f(x) = \frac{1}{2}x^2 \text{ where } -\pi < x < \pi\,.$$

Hence, use the value of $f\left(\dfrac{\pi}{2}\right)$ as determined by the Fourier series to obtain a series for π^2. Using the first 18 terms of this expansion, estimate the value of π to 2 decimal places.

[20 marks]

1. *Pertimbangkan sistem persamaan pembezaan berikut di mana x, y dan z merupakan fungsi-fungsi dalam t :*

$$x' = -x + y + z\,,\;\; y' = 4y + 5z\,,\;\; z' = -6x - z\,.$$

(i) *Tuliskan persamaan di atas sebagai suatu persamaan matriks bagi suatu vektor $\underset{\sim}{u}$ yang sesuai.*

(ii) *Dapatkan nilai-nilai eigen dan vektor eigen yang separan bagi matriks* 3×3 yang diperoleh *di atas.*

(iii) *Dengan itu, dapatkan penyelesaian am bagi x .*

(iv) *Dapatkan penyelesaian bagi x diberikan $x(0) = 5$ dan $x'(0) = 7$.*

(v) *Dapatkan penyelesaian lengkap bagi persamaan pembezaan jika $z(0) = 4$.*

[25 markah]

2. *Selesaikan persamaan pembezaan separa peringkat kedua*

$u_{xx} + 2xu_x = 7e^{-x^2} + 3x + 2x\cos y$ *di mana* $u = u(x, y)$, *diberikan* $u(0, y) = -\dfrac{7}{2} + \sin y$ dan $u_x(0, y) = \dfrac{3}{2} + \cos y$ .. *Seterusnya, tentusahkan bahawa jawapan anda memenuhi persamaan pembezaan serta syarat-syarat awal yang diberi.*

[15 markah]

3. *Dapatkan siri Fourier bagi fungsi berikut:*

$$f(x) = \frac{1}{2}x^2 \text{ dengan } -\pi < x < \pi\,.$$

Dengan itu, gunakan nilai $f\left(\dfrac{\pi}{2}\right)$ yang ditentukan oleh siri Fourier tersebut untuk mendapatkan suatu kembangan bersiri untuk nilai π^2. Gunakan 18 sebutan pertama dalam kembangan ini untuk menganggarkan nilai π betul sehingga 2 titik perpuluhan.

[20 markah]

4. Consider the following one-dimensional wave equation for $u = u(x,t)$:

$$9u_{xx} = u_{tt},\ 0 < x < \pi,\ t > 0,$$

with the initial conditions

$$u(0,t) = u(\pi,t) = 0,\ t > 0 \text{, and}$$

$$u(x,0) = f(x),\ u_t(x,0) = 0,\ 0 < x < \pi.$$

(i) Solve the given problem using the method of separation of variables.

(ii) Hence, simplify your solution given that

$$f(x) = \begin{cases} \frac{1}{4}x, & 0 < x \le \frac{\pi}{2} \\ \frac{1}{4}(\pi - x), & \frac{\pi}{2} < x < \pi \end{cases}.$$

[40 marks]

4. *Pertimbangkan persamaan gelombang satu dimensi bagi* $u = u(x,t)$ *yang berikut:*

$$u_{xx} = c^{-2}u_{tt}\,,\ 0 < x < \pi,\ t > 0\,,$$

dengan syarat awal

$$u(0,t) = u(\pi,t) = 0,\ t > 0\ ,\ \textit{dan}$$

$$u(x,0) = f(x),\ u_t(x,0) = 0,\ 0 < x < \pi\ .$$

(i) *Selesaikan masalah di atas dengan menggunakan kaedah pemisahan pembolehubah.*

(ii) *Dengan demikian, ringkaskan penyelesaian anda diberikan*

$$f(x) = \begin{cases} \dfrac{1}{4}x, & 0 < x \le \dfrac{\pi}{2} \\ \dfrac{1}{4}(\pi - x), & \dfrac{\pi}{2} < x < \pi \end{cases}.$$

[40 markah]

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