UNIVERSITI SAINS MALAYSIA

First Semester Examination
2010/2011 Academic Session

November 2010

MST 564 – Statistical Reliability
[Kebolehpercayaan Statistik]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FOURTEEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi EMPAT BELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) There are many definitions for Reliability. Give the definitions to the best of your knowledge.

[20 marks]

(b) What is the motivation for the study of Reliability?

[20 marks]

(c) What terms and concepts are needed to describe and evaluate product reliability?

[30 marks]

(d) What types of data a reliability analyst typically obtain from laboratory testing or a customer environment?

[30 marks]

2. (a) Suppose that a population of components is described by the lifetime distribution

$$F(t) = 1 - (1 + 0.001t)^{-1}.$$

(i) What is the probability that a new unit will fail by 1000 hours?

(ii) What is the probability that a new unit will fail by 4000 hours?

(iii) What is the probability that a new unit will fail between 1000 and 4000 hours?

(iv) What proportion of new units will last more than 9000 hours?

(v) If we use 150 of them, how many do we expect to fail in the first 1000 hours?

(vi) If we use 150 of them, how many do we expect to fail in the next 3000 hours?

[40 marks]

(b) Describe and sketch the typical shape for failure rate curves known as the bathtub curve. You can relate it to either humans or manufactured items.

[30 marks]

(c) Every experienced reliability analyst has come across many examples of real failure data exhibiting the shape described in (b). However, of late, the bathtub curve has been disproved as being an oversimplification of the reliability reality. Comment on this new development.

[30 marks]

1. (a) *Terdapat banyak takrif bagi Kebolehpercayaan. Berikan takrif-takrif sebaik mungkin dalam pengetahuan anda.*

[20 markah]

(b) *Apakah motivasi kajian Kebolehpercayaan?*

[20 markah]

(c) *Apakah istilah dan konsep yang diperlukan untuk menghurai dan menilai kebolehpercayaan produk?*

[30 markah]

(d) *Apakah jenis data yang biasa diperolehi oleh seorang penganalisa dari pengujian makmal atau persekitaran pelanggan?*

[30 markah]

2. (a) *Andaikan bahawa sebuah populasi komponen dihurai oleh taburan jangka hayat*

$$F(t) = 1 - (1 + 0.001t)^{-1}.$$

(i) *Apakah kebarangkalian bahawa satu unit baru akan gagal sebelum atau pada 1000 jam?*

(ii) *Apakah kebarangkalian bahawa satu unit baru akan gagal sebelum atau pada 4000 jam?*

(iii) *Apakah kebarangkalian bahawa satu unit baru akan gagal di antara 1000 dan 4000 jam?*

(iv) *Apakah kadaran untuk unit baru akan berfungsi melebihi 9000 jam?*

(v) *Jika 150 unit digunakan, berapa banyak unit yang dijangka akan gagal dalam 1000 jam pertama?*

(vi) *Jika 150 unit digunakan, berapa banyak unit yang dijangka akan gagal dalam selepas 3000 jam?*

[40 markah]

(b) *Huraikan dan lakarkan bentuk yang tipikal untuk lengkungan kadar kegagalan yang dikenali sebagai lengkungan 'bathtub'. Anda boleh kaitkan sama ada dengan manusia atau barangan pengeluaran.*

[30 markah]

(c) *Setiap penganalisa kebolehpercayaan yang berpengalaman pernah menemui banyak contoh data kegagalan yang mempamerkan bentuk seperti ditunjukkan dalam (b). Walau bagaimana pun, kebelakangan ini, lengkungan 'bathtub' telah dibuktikan salah sebagai kejadian terlalu dimudahkan bagi realiti kebolehpercayaan. Komen tentang perkembangan baru ini.*

[30 markah]

3. (a) The following is a dataset from Shouki and Pause (1999) for times (in months) until breast cancer recurrence in 73 female patients (see **Table 3(a)).** Much research has recently been directed at early detection through mass screening. The time to cancer recurrence are recorded. Only 17 observations were uncensored observed times until cancer recurrence and 56 observations were censored.

(i) Obtain a summary of the distribution of the data set in **Table 3(a)**.

(ii) Interpret your results. What are your conclusions?

(iii) Find $\hat{S}(120)$. Explain your answer.

(You may use the available computer tools to generate the results. Remember to state where your output is generated from.)

[50 marks]

3. (a) *Berikut adalah satu set data dari Shouki dan Pause (1999) bagi masa (dalam bulan) sehingga berulangnya kanser payudara bagi 73 pesakit wanita (lihat* ***Jadual 3(a)****). Kebelakangan ini banyak kajian telah dijalankan ke arah pengesanan awal melalui saringan beramai-ramai. Masa sehingga kanser berulang dicatat. Hanya 17 cerapan adalah masa tak tertapis dan 56 cerapan adalah tertapis.*

(i) Dapatkan ringkasan taburan set data dalam ***Jadual 3(a).***

(ii) Tafsirkan keputusan anda. Apakah kesimpulan anda?

(iii) Cari $\hat{S}(120)$. *Terangkan jawapan anda.*

(Anda boleh menggunakan alat-alat computer yang sedia ada untuk menjana keputusan. Sila nyatakan sumber output yang dijana.)

[50 markah]

Table 3(a). Time to breast cancer recurrence data for 73 female patients.

Patient No.	Time (month)	Patient No.	Time (month)
1	130+	38	48+
2	136+	39	89+
3	117+	40	95+
4	50	41	91+
5	106+	42	47
6	103+	43	75+
7	86	44	49+
8	63+	45	66+
9	120+	46	65+
10	121+	47	22+
11	108	48	73+
12	121+	49	67+
13	109+	50	75+
14	111+	51	71+
15	60	52	80+
16	106+	53	25+
17	108+	54	67+
18	105+	55	74+
19	9+	56	64
20	108	57	64+
21	62+	58	41
22	106+	59	70
23	95	60	57
24	94+	61	59+
25	19+	62	53+
26	103	63	69+
27	60+	64	55+
28	91+	65	58
29	70+	66	68+
30	65+	67	60+
31	91+	68	126
32	86	69	127+
33	90+	70	126+
34	87+	71	102+
35	89	72	122+
36	89	73	100+
37	92+		

Jadual 3(a). Data masa berulangnya kanser payudara bagi 73 pesakit wanita.

No. Pesakit	*Masa (bulan)*	*No. Pesakit*	*Masa (bulan)*
1	*130+*	*38*	*48+*
2	*136+*	*39*	*89+*
3	*117+*	*40*	*95+*
4	*50*	*41*	*91+*
5	*106+*	*42*	*47*
6	*103+*	*43*	*75+*
7	*86*	*44*	*49+*
8	*63+*	*45*	*66+*
9	*120+*	*46*	*65+*
10	*121+*	*47*	*22+*
11	*108*	*48*	*73+*
12	*121+*	*49*	*67+*
13	*109+*	*50*	*75+*
14	*111+*	*51*	*71+*
15	*60*	*52*	*80+*
16	*106+*	*53*	*25+*
17	*108+*	*54*	*67+*
18	*105+*	*55*	*74+*
19	*9+*	*56*	*64*
20	*108*	*57*	*64+*
21	*62+*	*58*	*41*
22	*106+*	*59*	*70*
23	*95*	*60*	*57*
24	*94+*	*61*	*59+*
25	*19+*	*62*	*53+*
26	*103*	*63*	*69+*
27	*60+*	*64*	*55+*
28	*91+*	*65*	*58*
29	*70+*	*66*	*68+*
30	*65+*	*67*	*60+*
31	*91+*	*68*	*126*
32	*86*	*69*	*127+*
33	*90+*	*70*	*126+*
34	*87+*	*71*	*102+*
35	*89*	*72*	*122+*
36	*89*	*73*	*100+*
37	*92+*		

(b) The following are the survival times (days) of street lights at various locations around the city: where '+' denotes censoring caused by traffic accidents destroying entire lighting units.

$$36, 37, 38, 38+, 78, 111, 112, 114+, 162, 189, 198, 237, 489+$$

(i) Determine an actuarial estimator of the survival function.

(ii) Determine the Product Limit (PL) estimator of the survival function.

(iii) When do you use actuarial and PL estimates in survival analysis? Explain.

[50 marks]

4. (a) John Day, a proactive maintenance management advocate, titled his vision to maintenance as:

STRATEGY TO ACHIEVE WORLD-CLASS PRODUCTION THROUGH RELIABILITY

(i) Are maintenance and reliability related? Explain.

(ii) What is maintenance?

(iii) What are maintenance approaches?

(iv) What is the difference between the old and new definition of failure? Illustrate.

[40 marks]

(b) In a study carried out at a Health Sciences Centre, data were obtained on the survival times of 36 patients with a malignant tumour in the kidney. The patients had all been treated with a combination of chemotherapy and immunotherapy, but additionally a nephrectomy, the surgical removal of the kidney, had been carried out on some of the patients. Of particular interest is whether the survival time of the patients depends on their age at the time of diagnosis and on whether or not they had received a nephrectomy. The data obtained in the study were given in Lee (1992). In the data set, the age of a patient has been classified according to whether the patient is less than 60, between 60 and 70 or greater than 70. **Table 4(b)** gives the survival times of the patients in months, where '+' denotes a censored observation.

(b) *Berikut adalah jangka hayat (hari) bagi lampu jalan di berbagai lokasi di sekitar bandar raya: yang mana '+' menandakan penapisan yang disebabkan oleh kemalangan jalan raya yang merosakkan seluruh unit lampu.*

$$36, 37, 38, 38+, 78, 111, 112, 114+, 162, 189, 198, 237, 489+$$

(i) Tentukan penganggar aktuari bagi fungsi hayat.

(ii) Tentukan penganggar Had Produk (PL) bagi fungsi hayat.

(iii) Bilakah anda gunakan anggaran aktuari dan PL dalam analisis hayat? Terangkan.

[50 markah]

4. (a) *John Day, seorang penyokong pengurusan penyelenggaraan proaktif, memberi tajuk kepada visi penyelenggaraannya sebagai:*

STRATEGI UNTUK MENCAPAI PENGHASILAN KELAS-DUNIA MELALUI KEBOLEHPERCAYAAN

(i) Adakah penyelenggaraan dan kebolehpercayaan berkaitan? Terangkan.

(ii) Apakah penyelenggaraan?

(iii) Apakah pendekatan penyelenggaraan?

(iv) Apakah bezanya antara takrif lama dan baru bagi kegagalan? Illustrasikan.

[40 markah]

(b) *Dalam satu kajian yang dijalankan di sebuah Pusat Sains Kesihatan, data tentang jangka hayat bagi 36 orang pesakit dengan tumor yang berbahaya di dalam ginjal diperoleh. Semua pesakit telah dirawat dengan kombinasi kemoterapi dan immunoterapi, tetapi tambahan pula, suatu prosedur pembedahan pembuangan ginja (iaitu, nephrektomi) dilakukan pada beberapa pesakit. Yang menarik adalah sama ada jangka hayat pesakit bergantung kepada usia mereka pada masa diagnosis dan pada sama ada atau tidak mereka telah menerima nephrektomi. Data yang diperoleh dalam kajian ini diberikan dalam Lee (1992). Dalam set data, usia pesakit telah dikelaskan mengikut sama ada usia pesakit kurang dari 60, antara 60 dan 70 atau melebihi 70. Jadual 4(b) memberikan jangka hayat pesakit dalam bulan, yang mana '+' menunjukkan pemerhatian tertapis.*

Table 4(b). Survival times of 36 patients classified according to age-group and whether or not they had a nephrectomy.

No Nephrectomy			Nephrectomy		
<60	**60 - 70**	**>70**	**<60**	**60 - 70**	**>70**
9	15	12	104+	108+	10
6	8		9	26	9
21	17		56	14	18
			35	115	6
			52	52	
			68	5+	
			77+	18	
			84	36	
			8	9	
			38		
			72		
			36		
			48		
			26		
			108		
			5		

(i) How do you model the survival data in **Table 4(b)**? Give the steps.

(ii) Find the appropriate model.

(iii) Examine the fit of the model.

(iv) Interpret the model.

(v) What can you conclude?

(You may use the available computer tools to generate the results. Remember to state where your output is generated from.)

[60 marks]

Jadual 4(b). Jangka hayat bagi 36 orang pesakit yang dikelaskan mengikut kumpulan-umur dan sama ada Nephrektomi dilaksanakan.

Tiada Nephrektomi			***Nephrektomi***		
<60	***60 - 70***	***>70***	***<60***	***60 - 70***	***>70***
9	*15*	*12*	*104+*	*108+*	*10*
6	*8*		*9*	*26*	*9*
21	*17*		*56*	*14*	*18*
			35	*115*	*6*
			52	*52*	
			68	*5+*	
			77+	*18*	
			84	*36*	
			8	*9*	
			38		
			72		
			36		
			48		
			26		
			108		
			5		

(i) *Bagaimanakah anda memodel data hayat dalam **Jadual 4(b)**? Berikan langkah-langkahnya.*

(ii) *Cari model yang bersesuaian.*

(iii) *Periksa kesesuaian model ini.*

(iv) *Tafsirkan model ini.*

(v) *Apakah kesimpulan anda?*

(Anda boleh menggunakan alat-alat computer yang sedia ada untuk menjana keputusan. Sila nyatakan sumber output yang dijana.)

[60 markah]

APPENDIX

Summary of Reliability Formulae

$$F(t) = \int_0^t f(t)dt$$

$$R(t) = 1 - F(t)$$

$$f(t) = \frac{dF(t)}{dt} = -\frac{dR(t)}{dt}$$

$$h(t) = \frac{f(t)}{R(t)}$$

$$H(t) = \int_0^t h(t)dt$$

$$R(t) = e^{-H(t)}$$

$$H(t) = -\ln R(t)$$

$$MTTF = \int_0^\infty tf(t)dt = \int_0^\infty R(t)dt$$

APPENDIX (contd)

Summary of Reliability Formulae (contd)

Lifetime following an **Exponential Distribution:**

$$f(t) = \lambda e^{-\lambda t}$$
$$F(t) = 1 - e^{-\lambda t}$$
$$R(t) = e^{-\lambda t}$$
$$h(t) = \lambda$$
$$H(t) = \lambda t$$
$$MTTF = \frac{1}{\lambda}$$

Lifetime following a **Weibull Distribution:**

$$f(t) = \beta \alpha^{-\beta} t^{\beta-1} \exp\left[-\left(\frac{t}{\alpha} \right)^{\beta} \right]$$

$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}}$$

$$h(t) = \beta \alpha^{-\beta} t^{\beta-1}$$

$$H(t) = \left(\frac{t}{\alpha} \right)^{\beta}$$

$$MTTF = \alpha \Gamma\left(1 + \frac{1}{\beta} \right)$$

$$\text{Design Life} = t_R = \alpha(-\ln R)^{1/\beta}$$

APPENDIX (contd)

Summary of Reliability Formulae (contd)

Lifetime following a **Normal Distribution:**

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2} \frac{(t-\mu)^2}{\sigma^2} \right]$$

$$F(t) = \Phi\left(\frac{t-\mu}{\sigma} \right)$$

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma} \right)$$

$$h(t) = \frac{f(t)}{1 - \Phi\left(\frac{t-\mu}{\sigma} \right)}$$

Lifetime following a **Lognormal Distribution:**

$$f(t) = \frac{1}{\sqrt{2\pi}st} \exp\left[-\frac{1}{2s^2} \left(\ln \frac{t}{t_{median}} \right)^2 \right]$$

$$F(t) = \Phi\left(\frac{1}{s} \ln \frac{t}{t_{median}} \right)$$

$$R(t) = 1 - \Phi\left(\frac{1}{s} \ln \frac{t}{t_{median}} \right)$$

$$MTTF = t_{median} \exp\left(\frac{s^2}{2} \right)$$

$$t_R = t_{median} \exp\ sz_{1-R}$$

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