
UNIVERSITI SAINS MALAYSIA

First Semester Examination
2010/2011 Academic Session

November 2010

MAT 251 – Introduction to Operations Research
[Pengantar Penyelidikan Operasi]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of ELEVEN pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi SEBELAS muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all eight** [8] questions.

Arahan: Jawab **semua lapan** [8] soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Kayukom makes tables, chairs and bookshelves. Each table, chair and bookshelf must be made entirely out of either meranti or cengal wood. A total of 300 m^2 of meranti and 400 m^2 of cengal wood of the required thickness are available. The cost for meranti and cengal wood are RM5 and RM4 per m^2 respectively. A table, chair and bookshelf requires 4m^2 , 1.5m^2 and 2.5m^2 of wood respectively. A meranti table sells for RM550 while a cengal table is RM450. A meranti chair sells for RM60 while a cengal chair is RM50. A meranti bookshelf sells for RM250 while a cengal bookshelf is RM180. Formulate an LP that will maximize profit for Kayukom.

[10 marks]

2. Given the following linear programming,

$$\begin{aligned} \text{Maximize} \quad & Z = 25x_1 + 40x_2 \quad [\text{Revenue}] \\ \text{Subject to} \quad & 24x_1 + 15x_2 \leq 120 \quad (\text{Resource 1}) \\ & -20x_1 + 60x_2 \leq 0 \quad (\text{Resource 2}) \\ & 30x_1 - 10x_2 \geq 30 \quad (\text{Resource 3}) \end{aligned}$$

$$\text{where } x_1 \geq 0, x_2 \geq 0$$

- (i) Solve the problem graphically.
- (ii) Determine and list down the binding and non-binding constraints.
- (iii) Determine and list down the status of each resources.
- (iv) What is the shadow price of resource 1?
- (v) Find the range for resource 1 that will not affect the optimum solution found in (i).
- (vi) If the revenue per unit of x_1 changes by δ , find the range for δ that will not affect the optimum solution found in (i).

[20 marks]

1. Kayukom membuat meja, kerusi dan para-para. Setiap meja, kerusi dan para-para diperbuat keseluruhannya daripada sama ada kayu meranti atau cengal. Sebanyak 300 m^2 kayu meranti dan 400 m^2 kayu cengal boleh didapati mengikut ketebalan diperlukan. Kos per m^2 kayu meranti dan cengal masing-masing ialah RM5 dan RM4. Setiap meja, kerusi dan para-para masing-masing memerlukan 4m^2 , 1.5m^2 and 2.5m^2 kayu. Meja meranti dijual dengan harga RM550 sementara meja cengal ialah RM450. Kerusi meranti berharga RM60 sementara kerusi cengal ialah RM50. Para-para meranti berharga RM250 sementara para-para cengal ialah RM180. Rumuskan sebagai suatu PL untuk memaksimumkan keuntungan Kayukom.

[10 markah]

2. Diberikan suatu masalah pengaturcaraan linear seperti berikut:

$$\text{Maksimumkan } Z = 25x_1 + 40x_2 \text{ [Pendapatan]}$$

$$\text{Terhadap } 24x_1 + 15x_2 \leq 120 \text{ (Sumber 1)}$$

$$-20x_1 + 60x_2 \leq 0 \text{ (Sumber 2)}$$

$$30x_1 - 10x_2 \geq 30 \text{ (Sumber 3)}$$

$$\text{dengan } x_1 \geq 0, x_2 \geq 0$$

- (i) Selesaikan masalah ini masalah ini dengan secara graf.
- (ii) Tentukan dan senaraikan kekangan yang terikat dan yang tidak terikat.
- (iii) Tentukan dan senaraikan status setiap sumber.
- (iv) Berapakah harga bayangan bagi sumber 1?
- (v) Dapatkan julat bagi sumber 1 supaya tidak menjejaskan penyelesaian optimum di (i).
- (vi) Sekiranya harga seunit x_1 berubah sebanyak δ , dapatkan julat bagi δ yang tidak akan menjejaskan penyelesaian optimum di (i).

[20 markah]

3. Consider the following LP formulation,

$$\text{Minimize } Z = 5x_1 + 8x_2 + 6x_3$$

$$\begin{aligned} \text{Subject to } \quad 3x_1 + 2x_2 + x_3 &\geq 30 \\ 2x_1 + 4x_2 + 3x_3 &\geq 20 \end{aligned}$$

$$\text{where } x_1 \geq 0, x_2 \geq 0, \text{ and } x_3 \geq 0$$

Perform one iteration of the two-phase method to this problem.

[10 marks]

4. Consider the following LP,

$$\text{Maximize } Z = 4x_1 + 6x_2 + 7x_3 + 8x_4 \quad (\text{profit (RM)})$$

$$\begin{aligned} \text{Subject to } \quad x_1 + x_2 + x_3 + x_4 &\leq 950 && (\text{Resource 1}) \\ 2x_1 + 3x_2 + 4x_3 + 7x_4 &\leq 4600 && (\text{Resource 2}) \\ 3x_1 + 4x_2 + 5x_3 + 6x_4 &\leq 5000 && (\text{Resource 3}) \\ &&& x_4 \geq 400 && (\text{Resource 4}) \end{aligned}$$

$$\text{where } x_1, x_2, x_3 \text{ and } x_4 \geq 0$$

The optimal tableau is:

Basic	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	Solution
Z	1	0	0	0	3	1	0	2	6650
x_2	2	1	0	0	4	-1	0	-3	400
x_3	-1	0	1	0	-3	1	0	4	150
s_3	0	0	0	0	-1	-1	1	-2	250
x_4	0	0	0	1	0	0	0	-1	400

where $s_1, s_2, s_3,$ and $s_4,$ are the slack variables for the constraints respectively.

- (i) Write the optimal solution and the value of Z.
- (ii) What are the binding and non-binding constraints?
- (iii) Get the dual price of each resources.
- (iv) How much can Resource 1 change without affecting the current optimal solution?
- (v) Find the range of the unit profit of x_1 that will keep the current basis optimal.
- (vi) If the unit profit of x_2 increases by 50 sen, what is the new optimal solution? Get the range for it.

[15 marks]

3. Pertimbangkan perumusan PL berikut,

$$\text{Minimumkan } Z = 5x_1 + 8x_2 + 6x_3$$

$$\begin{aligned} \text{Terhadap } 3x_1 + 2x_2 + x_3 &\geq 30 \\ 2x_1 + 4x_2 + 3x_3 &\geq 20 \end{aligned}$$

$$\text{dengan } x_1 \geq 0, x_2 \geq 0, \text{ dan } x_3 \geq 0$$

Lakukan satu lelaran teknik dua-fasa terhadap masalah ini.

[10 markah]

4. Pertimbangkan masalah PL berikut,

$$\text{Maksimumkan } Z = 4x_1 + 6x_2 + 7x_3 + 8x_4 \text{ (keuntungan (RM))}$$

$$\begin{aligned} \text{Terhadap } x_1 + x_2 + x_3 + x_4 &\leq 950 && \text{Sumber 1} \\ 2x_1 + 3x_2 + 4x_3 + 7x_4 &\leq 4600 && \text{Sumber 2} \\ 3x_1 + 4x_2 + 5x_3 + 6x_4 &\leq 5000 && \text{Sumber 3} \\ x_4 &\geq 400 && \text{Sumber 4} \end{aligned}$$

$$\text{dengan } x_1, x_2, x_3 \text{ dan } x_4 \geq 0$$

Tablo optimumnya ialah,

Asas	x_1	x_2	x_3	x_4	s_1	s_2	s_3	s_4	Penyelesaian
Z	1	0	0	0	3	1	0	2	6650
x_2	2	1	0	0	4	-1	0	-3	400
x_3	-1	0	1	0	-3	1	0	4	150
s_3	0	0	0	0	-1	-1	1	-2	250
x_4	0	0	0	1	0	0	0	-1	400

Dengan $s_1, s_2, s_3,$ and $s_4,$ masing-masing ialah pembolehubah lalai kekangan.

- (i) Tuliskan penyelesaian optimum dan nilai Z.
- (ii) Apakah kekangan yang terikat dan yang tidak terikat?
- (iii) Berikan harga dual setiap sumber.
- (iv) Berapa banyakkah sumber 1 boleh berubah tanpa menjejaskan penyelesaian optimum semasa?
- (v) Dapatkan julat keuntungan seunit produk x_1 dengan mengekalkan penyelesaian asas semasa optimum.
- (vi) Jika keuntungan seunit bagi x_2 meningkat sebanyak 50 sen, apakah penyelesaian optimum baru? Dapatkan julat baginya.

[15 markah]

5. Using the transportation method we obtained the following tableau,

Source	Destination				Supply
	1	2	3	4	
A	60 50	40	28	25 130	180
B	50 50	30	25 100	35	150
C	43 0	20 120	20	30	120
Demand	100	120	100	130	

(C_{ij} is the unit cost of transportation from source I, to destination ,j and it is located in the upper right hand corner of each cell).

- Continue solving and list out the optimal solution.
- Find the range for C_{12} so that the solution found in (a) remains optimal.
- Find the range for C_{31} , so that the solution found in (a) remains optimal.
- Get the new solution if both the supply from Source C and demand at Destination 1 increase by 20 units.
- Get the new solution if both the supply from Source A and demand at Destination 1 increase by 30 units.

[15 marks]

6. A company has four workers. On a particular day, five jobs are scheduled to be completed. The estimated time for each worker to complete each job and are shown in the following table. Un-assignable worker to job is given as a '- '.

Worker	Job				
	1	2	3	4	5
A	7	5	3	6	5
B	4	-	7	5	9
C	8	5	8	9	3
D	5	6	4	-	4

- Determine the assignment of worker to job that minimizes the total time.
- Which job is not assigned to any worker?

[10 marks]

5. Menggunakan kaedah pengangkutan kita dapat tablo berikut,

		Destinasi				Bekalan
		1	2	3	4	
Punca	A	60 50	40	28	25 130	180
	B	50 50	30	25 100	35	150
C	43 0	20 120	20	30	120	
Permintaan		100	120	100	130	

(C_{ij} ialah kos pengangkutan seunit dari punca, i ke destinasi, j dan ia terletak di penjuru kanan atas setiap sel).

- Teruskan menyelesaikan dan senaraikan penyelesaian optimum.
- Dapatkan julat bagi C_{12} supaya penyelesaian didapati di (a) kekal optimum.
- Dapatkan julat bagi C_{31} supaya penyelesaian didapati di (a) kekal optimum.
- Dapatkan penyelesaian baru jika bekalan dari punca C dan permintaan di destinasi 1 meningkat kedua-duanya sebanyak 20 unit.
- Dapatkan penyelesaian baru jika bekalan dari punca A dan permintaan di destinasi 1 meningkat kedua-duanya sebanyak 30 unit.

[15 markah]

6. Sebuah syarikat mempunyai empat orang pekerja. Pada suatu hari, lima tugas perlu dilaksanakan. Masa bagi setiap pekerja menyiapkan tugas telah dianggarkan dan ia ditunjukkan di dalam jadual berikut. Tugas yang tidak boleh dilaksanakan oleh pekerja ditandakan '-'.

Pekerja	Tugas				
	1	2	3	4	5
A	7	5	3	6	5
B	4	-	7	5	9
C	8	5	8	9	3
D	5	6	4	-	4

- Tentukan suatu umpukan pekerja ke tugas yang akan meminimumkan jumlah masa.
- Tugas manakah yang tidak diberikan kepada sebarang pekerja?

[10 markah]

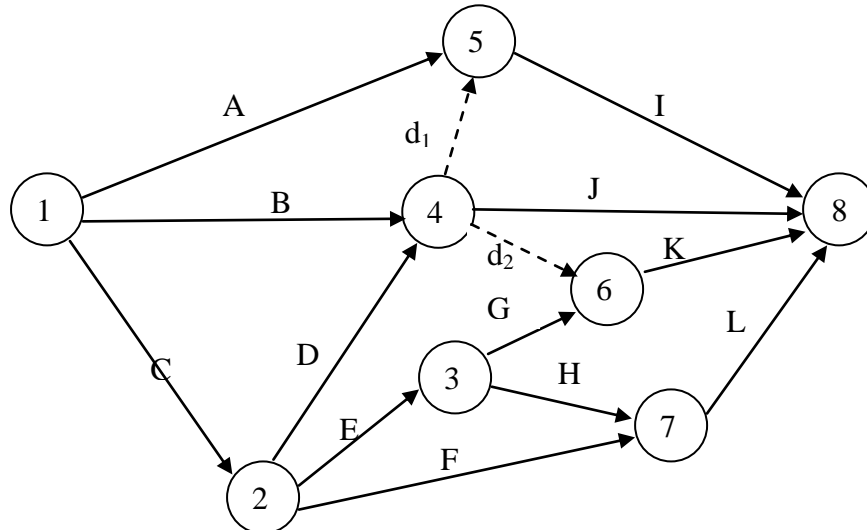
7. Consider the list of activities and the precedence activities of a project.

Activity	Precedence Activities	Duration (days)
A	-	6
B	-	8
C	-	7
D	B	5
E	A	4
F	A	7
G	C	6
H	A	3
I	G, H	2
J	D, E, F	4
K	E	5

- Draw the project network diagram.
- Show the critical path of the project and the shortest time to complete it.
- List the critical activities.

[10 marks]

8. The network diagram below represents a project.



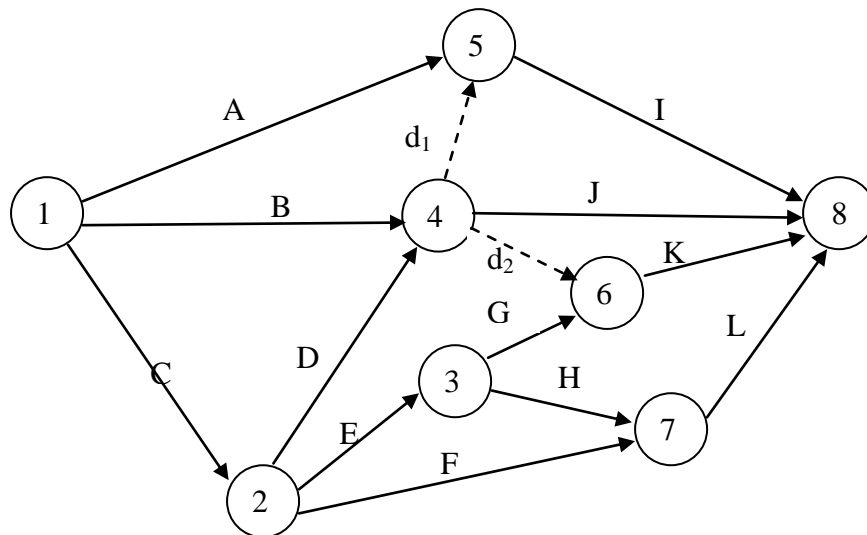
7. Pertimbangkan senarai kegiatan dan kegiatan pendahulu bagi suatu projek.

Kegiatan	Kegiatan Pendahulu	Jangkamasa (hari)
A	-	6
B	-	8
C	-	7
D	B	5
E	A	4
F	A	7
G	C	6
H	A	3
I	G, H	2
J	D, E, F	4
K	E	5

- (i) Lakarkan gambarajah aliran projek ini.
(ii) Tunjukkan lintasan genting projek ini dan berikan masa terpendek menyiapkannya.
(iii) Senaraikan kegiatan-kegiatan genting.

[10 markah]

8. Gambarajah rangkaian berikut mewakili suatu projek:



The normal and crash durations, and the normal and crash costs are given as follows:

Activity	Duration (days)		Direct Cost (RM)	
	Normal	Crash	Normal	Crash
A	6	4	100	150
B	4	3	90	120
C	5	4	150	180
D	4	3	180	250
E	6	4	200	300
F	7	5	180	240
G	8	6	120	200
H	10	7	100	180
I	7	5	200	300
J	5	4	140	170
K	6	5	170	180
L	5	3	150	170

The indirect cost per day is RM150. The contract agreement states that there is a penalty cost of RM100 per day if the project is completed later than 25 days and there is a bonus of RM120 per day if it is completed earlier than 25 days.

- (i) Determine all the possible critical paths.
- (ii) List all of the critical activities.
- (iii) Determine the minimum total cost for the project.
- (iv) Perform **two** iterations to crash the project. Show the total cost for each iterations.

[10 marks]

Jangkamasa biasa dan nahas, serta kos biasa dan nahas bagi setiap kegiatan diberikan seperti berikut:

<i>Kegiatan</i>	<i>Jangkamasa (hari)</i>		<i>Kos Langsung (RM)</i>	
	<i>Biasa</i>	<i>Nahas</i>	<i>Biasa</i>	<i>Nahas</i>
<i>A</i>	6	4	100	150
<i>B</i>	4	3	90	120
<i>C</i>	5	4	150	180
<i>D</i>	4	3	180	250
<i>E</i>	6	4	200	300
<i>F</i>	7	5	180	240
<i>G</i>	8	6	120	200
<i>H</i>	10	7	100	180
<i>I</i>	7	5	200	300
<i>J</i>	5	4	140	170
<i>K</i>	6	5	170	180
<i>L</i>	5	3	150	170

Kos tak langsung projek ialah RM150 sehari. Tercatat di dalam perjanjian kontrak bahawa denda sebanyak RM100 sehari dikenakan jika projek ini siap lewat daripada 25 hari dan ditawarkan bonus sebanyak RM120 sehari jika ia siap awal daripada 25 hari.

- (i) Tentukan semua lintasan genting yang mungkin.*
- (ii) Senaraikan kesemua kegiatan genting.*
- (iii) Tentukan jumlah kos minimum projek ini.*
- (iv) Lakukan **dua** lelaran pemampatan bagi projek ini. Tunjukkan jumlah kos projek bagi setiap lelaran.*

[10 markah]