UNIVERSITI SAINS MALAYSIA

First Semester Examination
2010/2011 Academic Session

November 2010

MAT 203 – Vector Calculus
[Kalkulus Vektor]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of <u>SEVEN</u> pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi <u>TUJUH</u> muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

<u>Instructions</u>: Answer **<u>all five</u>** [5] questions.

*[**<u>Arahan:</u>** Jawab **<u>semua lima</u>** [5] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Find c such that the planes $2cx + 3y + z = 1$ and $x - cy + 3z = 5$ are orthogonal.

(b) Find the equation of the line in symmetric form passing through $P(3, 4, -1)$ and parallel to the line of intersection of the planes $x + 2y + 2z + 5 = 0$ and $2x + y - 3z - 6 = 0$.

(c) Find the tangential and normal components of the acceleration of an object that moves along the parabolic curve formed by $\mathbf{r}(t) = \langle t,\ 4t^2 \rangle$ at a speed $ds/dt = 20$.

[20 marks]

2. (a) Let $\hat{T}$ be the unit tangent vector, $\hat{N}$ the unit normal vector and $\hat{B}$ the unit binormal vector to a given curve L and s the length of the path from a fix point to any other (moving) point on the curve. If $\hat{B} = \hat{T} \times \hat{N}$, show that

$$\frac{d\hat{B}}{ds} = \hat{T} \times \frac{d\hat{N}}{ds}$$

(b) Let C be the curve given by

$$\mathbf{R}(t) = (\sin t)\ \mathbf{i} + (\cos t)\ \mathbf{j} + t\ \mathbf{k}.$$

Find
(i) the unit tangent vector to C.
(ii) the length of the curve from $t = 0$ to $t = \pi$.
(iii) the curvature at $t = \pi$.

[19 marks]

1. (a) *Dapatkan c supaya satah-satah* $2cx + 3y + z = 1$ *dan* $x - cy + 3z = 5$ *ortogonal.*

   (b) *Dapatkan persamaan garis dalam bentuk simetrik yang melalui titik P* $(3,4,-1)$ *dan selari dengan garis persilangan satah-satah* $x + 2y + 2z + 5 = 0$ *dan* $2x + y - 3z - 6 = 0$.

   (c) *Dapatkan komponen tangent dan normal bagi pecutan suatu objek yang bergerak sepanjang lengkung parabolik yang dibentuk oleh* $\mathbf{r}(\mathrm{t}) = \left\langle t,\ 4t^2 \right\rangle$ *pada laju* $ds/dt = 20$.

[20 markah]

2. (a) *Biarkan* $\hat{\mathrm{T}}$ *vektor unit tangent,* $\hat{\mathrm{N}}$ *vektor unit normal and* $\hat{\mathrm{B}}$ *vektor unit binormal pada lengkung L dan s panjang laluan dari suatu titik tetap ke suatu titik (bergerak) lain kepada lengkung tersebut. Jika* $\hat{\mathrm{B}}=\hat{\mathrm{T}}\times\hat{\mathrm{N}}$ *, tunjukkan bahawa*

$$\frac{d\hat{\mathrm{B}}}{ds} = \hat{\mathrm{T}} \times \frac{d\hat{\mathrm{N}}}{ds}$$

   (b) *Biarkan C lengkung yang diberi oleh*

$$\mathbf{R}(\mathrm{t}) = (\sin t)\ \mathbf{i} + (\cos t)\ \mathbf{j} + t\ \mathbf{k}.$$

Dapatkan

(i) *vektor unit tangent kepada C.*

(ii) *panjang lengkung dari* $t = 0$ ke $t = \pi$.

(iii) *kelengkungan pada* $t = \pi$.

[19 markah]

3. (a) Find parametric equation of the tangent line to the curve of intersection of paraboloid $z = x^2 + y^2$ and ellipsoid $3x^2 + 2y^2 + z^2 = 9$ at the point $(1,1,2)$.

(b) Find the directional derivative of the scalar field

$$f = x^2 + y^2 + z^2$$

in the direction of the outward normal to the surface

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad a, b \text{ and } c \text{ are constants},$$

at any point on the surface.

(c) A vector field **F** satisfies
$\nabla \times \mathbf{F} = f\ \mathbf{F}$, where f is the scalar function.
If $\mathbf{F} = A\sin(y^3)\,\mathbf{i} + A\cos(y^3)\,\mathbf{k}$, A is a constant, find the appropriate *f*.

[19 marks]

4. (a) Consider a rigid body that is rotating about the *z*-axis with constant angular velocity $\omega = a\,\mathbf{i} + b\,\mathbf{j} + c\,\mathbf{k}$, a, b and c are constants. If P is a point in the body located at $\mathbf{R}(t) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ and the velocity at P is given by the vector field $\mathbf{V} = \omega \times \mathbf{R}$.
(i) Find the div **V**.
(ii) Find the curl **V**.

(b) A certain closed path C on the plane $2x + 2y + z = 1$ is known to project onto the unit circle $x^2 + y^2 = 1$ in the *xy*-plane. Let α be a constant and $\mathbf{R}(t) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$. Use Stoke's Theorem to evaluate

$$\int_C \alpha\mathbf{k} \times \mathbf{R} \cdot d\mathbf{R}$$

[20 marks]

3. (a) *Dapatkan persamaan parametrik garis tangent kepada lengkung persilangan paraboloid* $z = x^2 + y^2$ *dan ellipsoid* $3x^2 + 2y^2 + z^2 = 9$ *pada titik* $(1,1,2)$.

(b) *Dapatkan terbitan berarah suatu medan skalar*

$$f = x^2 + y^2 + z^2$$

dalam arah keluar normal kepada permukaan

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad a, b \text{ dan } c \text{ adalah pemalar },$$

pada sebarang titik atas permukaan tersebut.

(c) *Suatu medan vektor* $\mathbf{F}$ *memenuhi* $\nabla \times \mathbf{F} = f\ \mathbf{F}$ *, yangmana f ialah fungsi skalar.*
Jika $\mathbf{F} = A\ \sin(y^3)\ \mathbf{i} + A\ kos(y^3)\ \mathbf{k}$ *, A suatu pemalar, dapatkan f yang sepadan.* .

[19 markah]

4. (a) *Pertimbangkan suatu jasad tegar yang berkisar terhadap paksi-z dengan halaju angular malar* $\omega = a\ \mathbf{i} + b\ \mathbf{j} + c\ \mathbf{k}$ *, a,b dan c adalah malar. Jika P suatu titik terletak dalam jasad pada* $\mathbf{R}(\mathrm{t}) = x\ \mathbf{i} + y\ \mathbf{j} + z\ \mathbf{k}$ *dan halaju pada P diberi oleh medan vektor* $\mathbf{V} = \omega \times \mathbf{R}$.

(i) *Dapatkan* div $\mathbf{V}$.

(ii) *Dapatkan* curl $\mathbf{V}$.

(b) *Suatu laluan tertutup atas satah* $2x + 2y + z = 1$ *diketahui terunjur kepada unit bulatan* $x^2 + y^2 = 1$ *dalam satah-xy. Biarkan* α *suatu malar dan* $\mathbf{R}(\mathrm{t}) = x\ \mathbf{i} + y\ \mathbf{j} + z\ \mathbf{k}$. *Gunakan Teorem Stoke untuk menilai*

$$\int_C \alpha \mathbf{k} \times \mathbf{R} \cdot d\mathbf{R}$$

[20 markah]

5. (a) Given that a vector field $\mathbf{F} = \left\langle y^2 \cos x, x^2 + 2y \sin x \right\rangle$ is on a region D enclosed by a triangle from (0,0) to (2,6) to (2,0) to (0,0). Sketch the region D. Hence, by using Green's Theorem evaluate the line integral of **F** around the unit triangle.

(b) Let $\mathbf{F} = x^2\ \mathbf{i} + y^2\ \mathbf{j} + z^2\ \mathbf{k}$ and S the surface of the sphere $x^2 + y^2 + z^2 = 4$. Evaluate $\iint_S \mathbf{F} \cdot \mathbf{N}\, dS$, **N** is the outward unit normal vector.

[22 marks]

5. (a) *Diberi bahawa suatu medan vektor* $\mathbf{F} = \left\langle y^2 \cos x, x^2 + 2y \sin x \right\rangle$ *berada dalam kawasan D yang dibatasi oleh segitiga dari (0,0) kepada (2,6) kepada (2,0) kepada (0,0). Lakarkan kawasan D. Seterusnya, gunakan Teorem Green untuk menilaikan kamiran garis untuk* **F** *mengelilingi segitiga satu unit.*

(b) *Biarkan* $\mathbf{F} = x^2\ \mathbf{i} + y^2\ \mathbf{j} + z^2\ \mathbf{k}$ *dan S permukaan sfera* $x^2 + y^2 + z^2 = 4$.
Nilaikan $\iint_S \mathbf{F} \cdot \mathbf{N}\ dS$, ***N*** *ialah vektor unit normal arah keluar.*

[22 markah]

- ooo O ooo -