UNIVERSITI SAINS MALAYSIA

First Semester Examination
2010/2011 Academic Session

November 2010

MAA 101 – Calculus for Science Students I
[Kalkulus untuk Pelajar Sains I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

*[**Arahan:*** *Jawab **semua sepuluh** [10] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Solve $|x-1| \geq 5+|x-3|$.

[9 marks]

2. Determine whether $f(x) = \frac{x+1}{2x+1}$, $x \neq -\frac{1}{2}$ a one-to-one function. If yes, find its inverse.

[6 marks]

3. At what value(s) of x is the function

$$f(x) = \begin{cases} |x+1|-1 & , \quad x < 0 \\ x^2 + x & , \quad 0 \leq x < 1 \\ 3-x & , \quad x \geq 1 \end{cases} \quad \text{discontinuous?}$$

[6 marks]

4. Find $\frac{dy}{dx}$.

(a) $xy^4 + x^2 y = x + 3y$

(b) $y = \tan^2 \sin x$

[8 marks]

5. Suppose that f is continuous on $[0,4]$, $f(0) = 1$ and $2 \leq f'(x) \leq 5$ for all x in $(0,4)$. Show that $9 \leq f(4) \leq 21$.

[4 marks]

6. Evaluate, if it exists.

(a) $\lim_{x \to \infty} x^3 e^{-x}$

(b) $\lim_{x \to 0} \dfrac{\frac{1}{4+x} - \frac{1}{4}}{x}$

(c) $\lim_{x \to 3} f(x)$ where $f(x) = \begin{cases} 6x-4 & , \quad x < 3 \\ 2 & , \quad x = 3 \\ x^2 & , \quad x > 3 \end{cases}$

[10 marks]

1. *Selesaikan* $|x-1| \geq 5+|x-3|$.

[9 markah]

2. *Tentukan sama ada* $f(x) = \frac{x+1}{2x+1}, x \neq -\frac{1}{2}$ *adalah fungsi satu-ke-satu. Jika ya, dapatkan songsangannya.*

[6 markah]

3. *Apakah nilai-nilai* x *yang fungsinya*

$$f(x) = \begin{cases} |x+1|-1 & , \quad x < 0 \\ x^2 + x & , \quad 0 \leq x < 1 \\ 3-x & , \quad x \geq 1 \end{cases} \quad \textit{tak selanjar}?$$

[6 markah]

4. *Dapatkan* $\frac{dy}{dx}$.

(a) $xy^4 + x^2 y = x + 3y$

(b) $y = \tan^2 \sin x$

[8 markah]

5. *Katakan* f *selanjar pada* $0,4$, $f(0) = 1$ *dan* $2 \leq f'(x) \leq 5$ *bagi semua* x *dalam selang* $0,4$. *Tunjukkan bahawa* $9 \leq f(4) \leq 21$.

[4 markah]

6. *Nilaikan, jika ianya wujud.*

(a) $\lim_{x \to \infty} x^3 e^{-x}$

(b) $\lim_{x \to 0} \dfrac{\frac{1}{4+x} - \frac{1}{4}}{x}$

(c) $\lim_{x \to 3} f(x)$ *yang mana* $f(x) = \begin{cases} 6x-4 & , \quad x < 3 \\ 2 & , \quad x = 3 \\ x^2 & , \quad x > 3 \end{cases}$

[10 markah]

7. Find the slope of the tangent to the curve $y = \int_1^{\sin x} \frac{1-t^2}{1+t^4}\,dt$ at the point where $x = 0$.

[5 marks]

8. Let $f(x) = \frac{x-1}{x^2}$. Find

(a) the domain and all the asymptotes.
(b) the intervals on which f is increasing or decreasing.
(c) the local maximum and minimum value of f (if any).
(d) the intervals of concavity and the inflection points (if any).

Use the information from parts (a) – (d) to sketch the graph $f(x)$.

[20 marks]

9. Evaluate the integral.

(a) $\int \frac{x+2}{\sqrt{x^2+4x}}\,dx$

(b) $\int_1^4 x^{3/2} \ln x\,dx$

(c) $\int \frac{x^2-x+6}{x^3+3x}\,dx$.

[20 marks]

10. Set up, but do not evaluate the integral for

(a) the volume of the solid obtained by rotating the region bounded by $y = x^2 + 1$ and $y = 9 - x^2$ about $y = -1$.

(b) the area of the surface obtained by rotating the curve $y = \frac{x^4}{16} + \frac{1}{2x^2}$, $1 \le x \le 2$ about the $y-$axis.

(c) the length of the curve $y = \frac{1}{6}(x^2+4)^{3/2}$, $0 \le x \le 3$.

[12 marks]

7. *Dapatkan kecerunan tangen pada lengkung* $y = \int_{1}^{\sin x} \frac{1-t^2}{1+t^4}\,dt$ *pada titik yang mana* $x = 0.$

[5 markah]

8. *Biar* $f(x) = \frac{x-1}{x^2}$. *Dapatkan*

(a) domain dan semua asimptot.
(b) selang yang mana f *adalah menokok atau menyusut.*
(c) nilai maksimun dan minimum tempatan bagi f *(jika ada).*
(d) selang kecekungan dan titik lengkok balas (jika ada).

Gunakan maklumat daripada bahagian (a) – (d), untuk melakarkan graf $f(x)$.

[20 markah]

9. *Nilaikan kamiran.*

(a) $\int \frac{x+2}{\sqrt{x^2+4x}}\,dx$

(b) $\int_{1}^{4} x^{3/2} \ln x\,dx$

(c) $\int \frac{x^2-x+6}{x^3+3x}\,dx$

[20 markah]

10. *Nyatakan kamiran, tanpa menilaikannya bagi*

(a) isipadu pepejal yang dihasilkan dengan memutarkan rantau yang dibatasi oleh $y = x^2 + 1$ *dan* $y = 9 - x^2$ *sekitar* $y = -1$.

(b) luas permukaan yang dihasilkan dengan memutar lengkung $y = \frac{x^4}{16} + \frac{1}{2x^2}$ *yang mana* $1 \le x \le 2$ *sekitar paksi* $-y$.

(c) panjang lengkung $y = \frac{1}{6}\left(x^2+4\right)^{3/2}$, $0 \le x \le 3$.

[12 markah]

- ooo O ooo -