UNIVERSITI SAINS MALAYSIA

Second Semester Examination
2010/2011 Academic Session

November 2010

MAT 111 – Linear Algebra
[Aljabar Linear]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SEVEN pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi TUJUH muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all seven** [7] questions.

*[**Arahan:** Jawab **semua tujuh** [7] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Consider the set $A = \{(1, 1, 0), (0, 1, 0), (2, 0, 0)\}$.

(a) Is A linearly independent? Justify your answer.

(b) Can A generate $\mathbb{R}^3$? Give your reason.

(c) Find a basis for $\mathbb{R}^3$ using A. Justify your answer.

[18 marks]

2. Consider the subspace $W = \{(a, b, c) \in \mathbb{R}^3 \mid c = 0\}$.

(a) Find a subset S of $\mathbb{R}^3$ such that $W = L(S)$, the linear span of S.

(b) What is the dimension of W? Give your reason.

(c) Find the orthogonal complement $W^{\perp}$ of W.

(d) What is the dimension of $W^{\perp}$?

[16 marks]

1. *Pertimbangkan set* $A = \{(1, 1, 0), (0, 1, 0), (2, 0, 0)\}$.

(a) *Adakah* A *tak bersandar linear? Jelaskan jawapan anda.*

(b) *Bolehkah* A *menjana* $\mathbb{R}^3$*? Beri alasan anda.*

(c) *Cari asas bagi* $\mathbb{R}^3$ *dengan menggunakan* A. *Jelaskan jawapan anda.*

[18 markah]

2. *Pertimbangkan subruang* $W = \{(a, b, c) \in \mathbb{R}^3 \mid c = 0\}$.

(a) *Cari subset* S *untuk* $\mathbb{R}^3$ *sedemikian* $W = L(S)$, *rentangan linear untuk* S.

(b) *Apakah dimensi untuk* W*? Beri alasan anda.*

(c) *Cari pelengkap ortogon* $W^{\perp}$ *untuk* W.

(d) *Apakah dimensi untuk* $W^{\perp}$*?*

[16 markah]

3. Consider the following system of linear equations:

$$\begin{aligned} x+2y+3z &= -1 \\ x-2y-z &= -1 \\ 3x+y+z &= 3 \end{aligned}$$

(a) Write the coefficient matrix A for the above system.

(b) Find the null space $N(A)$ of A. Then determine the rank $\rho(A)$ of A.

(c) Write the definition of the linear transformation T that has matrix A obtained in (a) as its standard matrix. What are the domain and codomain of T?

(d) Is T one to one? Give your reason.

(e) Is T onto?

(f) Is the system consistent? Give your reason.

(g) Does the constant vector $-1,\ -1,\ 3$ lie in the image $\mathrm{Im}T$ of T? Give your reason.

[30 marks]

4. (a) Write the augmented matrix for the system

$$\begin{aligned} x+y &= 3 \\ -2x+3y &= 1 \\ 2x-y &= 2 \end{aligned}$$

(b) By using the Gaussian elimination method, show that the system in (a) has no solution.

(c) Find the best approximate solution to the system in (a).

[21 marks]

3. *Pertimbangkan system persamaan linear berikut:*

$$x+2y+3z=-1$$
$$x-2y-z=-1$$
$$3x+y+z=3$$

(a) *Tuliskan matriks pekali* A *bagi system di atas.*

(b) *Cari ruang nol* $N(A)$ *untuk* A. *Kemudian tentukan pangkat* $\rho(A)$ *untuk* A.

(c) *Tuliskan definisi untuk transformasi linear* T *yang mempunyai matriks* A *diperoleh di (a) sebagai matriks piawai. Apakah domain dan kodomain untuk* T*?*

(d) *Adakah* T *satu ke satu? Beri alasan anda.*

(e) *Adakah* T *keseluruh?*

(f) *Adakah sistem ini konsisten? Beri alasan anda.*

(g) *Adakah vektor pemalar* $-1, \ -1, \ 3$ *terletak pada imej* $\mathrm{Im}T$ *untuk* T*? Beri alasan anda.*

[30 markah]

4. (a) *Tuliskan matriks imbuhan bagi sistem*

$$x+y=3$$
$$-2x+3y=1$$
$$2x-y=2$$

(b) *Dengan menggunakan kaedah penghapusan Gaussian, tunjukkan bahawa system di (a) tidak mempunyai penyelesaian.*

(c) *Cari penyelesaian hampiran terbaik untuk sistem di (a).*

[21 markah]

5. Let $S = \{(1, -1, 0), (1, 0, -1)\}$ be a basis for a subspace W of $\mathbb{R}^3$.

   (a) Use the Gram-Schmidt process to obtain an orthonormal basis $B = \{v_1, v_2\}$ for W.

   (b) Find the scalar product of the vector $u = (5, -2, -3)$ with each of the orthonormal vectors (v_1 and v_2) obtained in part (a).

   (c) Find $(u \cdot v_1)v_1 + (u \cdot v_2)v_2$.

[16 marks]

6. Consider the matrix $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$.

   (a) Find the eigenvalues of A.

   (b) Find all the eigenvectors of A.

   (c) What are the algebraic and geometric multiplicities of each eigenvalue?

   (d) Is A diagonalizable? Give your reason. If yes, find the 2×2 matrix C such that CAC^{-1} is diagonal and write the matrix CAC^{-1}.

[30 marks]

7. Let $T : \mathbb{R}^4 \to \mathbb{R}^5$ be a linear transformation defined by

$$(x_1, x_2, x_3, x_4, x_5)T = (x_1, x_2, x_3, x_4, x_5)\begin{bmatrix} 1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

   (a) Find a basis for the image $\operatorname{Im}T$ of T.

   (b) What are the nullity $n(T)$ and the rank $\rho(T)$ of T?

[8 marks]

5. *Andaikan* $S = \{(1, -1, 0), (1, 0, -1)\}$ *asas bagi suatu subruang* W *untuk* $\mathbb{R}^3$.

(a) *Guna proses Gram-Schmidt untuk memperoleh satu asas ortonormal* $B = \{v_1, v_2\}$ *bagi* W.

(b) *Cari hasil darab scalar antara vektor* $u = (5, -2, -3)$ *dengan setiap vektor ortonormal (*v_1 *and* v_2*) diperoleh dalam bahagian (a).*

(c) *Cari* $(u \cdot v_1) v_1 + (u \cdot v_2) v_2$.

[16 markah]

6. *Pertimbangkan matriks* $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$.

(a) *Cari nilai eigen untuk* A.

(b) *Cari semua vektor eigen untuk* A.

(c) *Apakah kegandaan algebra dan geometri untuk setiap nilai eigen?*

(c) *Adakah* A *terperpenjurukan? Jika ya, cari matriks* 2×2 C *sedemikian* CAC^{-1} *adalah terperpenjurukan dan tuliskan matriks* CAC^{-1}.

[30 marks]

7. *Andaikan* $T : \mathbb{R}^4 \to \mathbb{R}^5$ *satu transformasi linear ditakrif oleh*

$$(x_1, \; x_2, \; x_3, \; x_4, \; x_5)\,T = (x_1, \; x_2, \; x_3, \; x_4, \; x_5) \begin{bmatrix} 1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) *Cari satu asas bagi imej* $\mathrm{Im}T$ *untuk* T.

(b) *Apakah kenolan* $n(T)$ *dan pangkat* $\rho(T)$ *untuk* T *?*

[8 markah]

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