UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2010/2011

Jun 2011

MSS 212 - Further Linear Algebra
[Aljabar Linear Lanjutan]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of SIX pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi ENAM muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all six** [6] questions.

*[**Arahan:** Jawab **semua enam** [6] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Find $\det\begin{pmatrix} a & a & a & a \\ -a & a & a & a \\ -a & -a & a & a \\ -a & -a & -a & a \end{pmatrix}$.

[30 marks]

(b) Using Cramer's rule, solve the following system of linear equations

$$2x + y - z = 1$$
$$-x - y + z = \frac{1}{2}$$
$$4x + y - 2z = 2$$

[70 marks]

1. *(a)* *Cari* $\det\begin{pmatrix} a & a & a & a \\ -a & a & a & a \\ -a & -a & a & a \\ -a & -a & -a & a \end{pmatrix}$.

[30 markah]

(b) *Dengan menggunakan petua Cramer, selesaikan system persamaan linear yang berikut:*

$$2x + y - z = 1$$
$$-x - y + z = \frac{1}{2}$$
$$4x + y - 2z = 2$$

[70 markah]

2. (a) Let $M_{2x2}(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a,b,c,d \in \mathbb{R} \right\}$ a vector space over $\mathbb{R}$ and

$$\alpha = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\} \subseteq M_{2\times 2}(\mathbb{R})$$

(i) Is α linearly independent over $\mathbb{R}$?

[15 marks]

(ii) Find a basis β of $M_{2x2}(\mathbb{R})$ such that $|\beta \cap \alpha| = 2$.

[50 marks]

(b) Let $P_2(\mathbb{R}) = \{ a_0 + a_1x + a_2x^2 \mid a_0, a_1, a_2 \in \mathbb{R} \}$ a vector space over $\mathbb{R}$.

Let $W = \{ p(x) \in P_2(\mathbb{R}) \mid p(0) = 1 \}$. Is W a subspace of $P_2(\mathbb{R})$?

[35 marks]

2. (a) *Biar* $M_{2x2}(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| a,b,c,d \in \mathbb{R} \right\}$ *suatu ruang vektor atas* $\mathbb{R}$ *dan*

$$\alpha = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\} \subseteq M_{2\times 2}(\mathbb{R})$$

(i) Adakah α *tak bersandar linear atas* $\mathbb{R}$ *?*

[15 markah]

(ii) Cari suatu asas β *bagi* $M_{2x2}(\mathbb{R})$ *sedemikian hingga* $|\beta \cap \alpha| = 2$.

[50 markah]

(b) Biar $P_2(\mathbb{R}) = \{ a_0 + a_1x + a_2x^2 \mid a_0, a_1, a_2 \in \mathbb{R} \}$ *suatu ruang vektor atas* $\mathbb{R}$.

Biar $W = \{ p(x) \in P_2(\mathbb{R}) \mid p(0) = 1 \}$. *Adakah* W *suatu subruang bagi* $P_2(\mathbb{R})$?

[35 markah]

3. Let T be a linear transformation over $\mathbb{R}$ from $M_{2x2}(\mathbb{R})$ to $\mathbb{R}^3$ having

$$T_{\alpha,\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

where $\alpha = e_{11}, e_{12}, e_{22}, e_{21}$ and $\beta = e_1, e_1 + e_2, e_3$.

(a) Find the definition of T.

[50 marks]

(b) Find another matrix A that representing T.

[50 marks]

3. Biar T suatu transformasi linear atas $\mathbb{R}$ dari $M_{2x2}(\mathbb{R})$ ke $\mathbb{R}^3$ dengan

$$T_{\alpha,\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix},$$

$\alpha = e_{11}, e_{12}, e_{22}, e_{21}$ *dan* $\beta = e_1, e_1 + e_2, e_3$.

(a) Cari definasi T.

[50 markah]

(b) Cari satu lagi matrix berlainan A yang juga mewakili T.

[50 markah]

4. Find $\begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}^{100}$.

[100 marks]

4. Cari $\begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}^{100}$.

[100 markah]

5. Let A be a 3×3 matrix in M_{2x2} □

(i) Suppose λ_1 and λ_2 are two distinct eigen values of A with the algebraic multiplicity of λ_1 is greater than the algebraic multiplicity of λ_2.
Give all possibilities of $J\ A$. Justify your answer.

[30 marks]

(ii) Suppose λ_1, λ_2 and λ_3 are three distinct eigen values of *A*. Give all possibilities of $J\ A$. Justify your answer.

[20 marks]

(iii) Suppose λ is the only eigen value of *A*. Give all possibilities of $J\ A$. Justify your answer.

[50 marks]

5. Biar A ialah suatu matrik 3×3 dalam M_{2x2} □

(i) Katakan λ_1 dan λ_2 adalah dua nilai eigen yang berlainan bagi A dengan pekali aljabar λ_1 adalah lebih besar daripada pekali aljabar λ_2. Berikan semua kemungkinan $J\ A$. Justifikasilan jawapan anda.

[30 markah]

(ii) Katakan λ_1, λ_2 dan λ_3 adalah tiga nilai eigen yang berlainan bagi A. Berikan semua kemungkinan $J\ A$. Justifikasilan jawapan anda.

[20 markah]

(iii) Katakan λ adalah nilai eigen tunggal bagi A. Berikan semua kemungkinan $J\ A$. Justifikasilan jawapan anda.

[50 markah]

6. Let $\mathbb{R}^3$ be an inner product space with inner product $\langle (a_1, a_2, a_3), (b_1, b_2, b_3) \rangle = \sum_{i=1}^{3} a_i b_i$.

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation such that

$$(x, y, z)T = (y + z, x + z, x + y).$$

(i) Show that T is self-adjoint

[30 marks]

(ii) Find an orthonormal basis α of $\mathbb{R}^3$ such that $T_{\alpha,\alpha}$ is a diagonal matrix.

[70 marks]

6. Biar $\mathbb{R}^3$ ialah suatu ruang hasil darab terkedalaman dengan hasil darab terkedalaman $\langle (a_1, a_2, a_3), (b_1, b_2, b_3) \rangle = \sum_{i=1}^{3} a_i b_i$. Biar $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ ialah suatu transformasi linear sedemikian hingga

$$(x, y, z)T = (y + z, x + z, x + y).$$

(i) Tunjukkan T adalah swaadjoin.

[30 markah]

(ii) Cari suatu asas ortonormal α bagi $\mathbb{R}^3$ sedemikian hingga $T_{\alpha,\alpha}$ adalah suatu matrik pepenjuru.

[70 markah]

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