UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2010/2011

Jun 2011

MAT 222 – Differential Equations II
[Persamaan Pembezaan II]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FOUR pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi EMPAT muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

*[**Arahan:** Jawab **semua empat** [4] soalan.]*

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. Consider the system of equations

$$\left.\begin{aligned} \frac{dx}{dt} &= a_1(t)x + b_1(t)y \\ \frac{dy}{dt} &= a_2(t)x + b_2(t)y \end{aligned}\right\} \quad *$$

Prove the following theorem:

If the system $*$ has the two solutions

$$\begin{Bmatrix} x = x_1(t) \\ y = y_1(t) \end{Bmatrix} \text{ and } \begin{Bmatrix} x = x_2(t) \\ y = y_2(t) \end{Bmatrix}$$

in the interval a,b , then

$$\begin{Bmatrix} x = c_1 x_1(t) + c_2 x_2(t) \\ y = c_1 y_1(t) + c_2 y_2(t) \end{Bmatrix}$$

is also a solution for $*$ in a,b for any constants c_1 and c_2.

[25 marks]

1. Pertimbangkan sistem persamaan

$$\left.\begin{aligned} \frac{dx}{dt} &= a_1(t)x + b_1(t)y \\ \frac{dy}{dt} &= a_2(t)x + b_2(t)y \end{aligned}\right\} \quad *$$

Buktikan teorem berikut:

Jika sistem $*$ *mempunyai dua penyelesaian*

$$\begin{Bmatrix} x = x_1(t) \\ y = y_1(t) \end{Bmatrix} \textit{ dan } \begin{Bmatrix} x = x_2(t) \\ y = y_2(t) \end{Bmatrix}$$

pada selang a,b *, maka*

$$\begin{Bmatrix} x = c_1 x_1(t) + c_2 x_2(t) \\ y = c_1 y_1(t) + c_2 y_2(t) \end{Bmatrix}$$

juga merupakan suatu penyelesaian bagi $*$ *pada* a,b *bagi sebarang pemalar* c_1 *dan* c_2.

[25 markah]

2. Consider the first order partial differential equation $u_x - u_y = \dfrac{x+y}{y}$ where $u = u(x, y)$.

(a) Suppose $\hat{u}(x, y)$ is a particular solution for the equation above. Consider now the two possible choices for $\hat{u}_y$ below and obtain a suitable expression for $\hat{u}_x$ in each case. Determine which choice produces a particular solution for the differential equation above.

(i) $\hat{u}_y = -\dfrac{x+y}{y}$

(ii) $\hat{u}_y = \ln x$

(b) Hence, solve the given partial differential equation using the initial condition $u(x, 0) = x^2 + 3$.

[30 marks]

2. Pertimbangkan persamaan pembezaan separa peringkat pertama $u_x - u_y = \dfrac{x+y}{y}$ *di mana* $u = u(x, y)$.

(a) Katakan $\hat{u}(x, y)$ *merupakan suatu penyelesaian khusus bagi persamaan di atas. Pertimbangkan kedua-dua pilihan yang mungkin bagi* $\hat{u}_y$ *seperti di bawah dan dapatkan suatu ungkapan yang mungkin untuk* $\hat{u}_x$ *dalam setiap kes. Tentukan pilihan yang mana dapat menghasilkan suatu penyelesaian khusus bagi persamaan pembezaan di atas.*

(i) $\hat{u}_y = -\dfrac{x+y}{y}$

(ii) $\hat{u}_y = \ln x$

(b) Dengan itu, selesaikan persamaan pembezaan separa di atas dengan menggunakan syarat awal $u(x, 0) = x^2 + 3$.

[30 markah]

3. Consider the function below:

$$f(x) = \frac{1}{2}\ x + |x| \quad \text{for } a < x < \pi \text{ with } f(x + 2\pi) = f(x).$$

(a) Given $a = -\pi$, find the Fourier series for $f(x)$.

(b) Given $a = 0$, find the Fourier Cosine series for $f(x)$.

[20 marks]

3. *Pertimbangkan fungsi berikut:*

$$f(x)=\frac{1}{2}\ x+|x| \quad \textit{bagi} \ a<x<\pi \ \textit{ dengan } \ f(x+2\pi)=f(x).$$

(a) Diberi $a=-\pi$ *, cari siri Fourier bagi* $f(x)$.

(b) Diberi $a=0$*, cari siri Kosinus Fourier bagi* $f(x)$.

[20 markah]

4. Consider the following one-dimensional wave equation for $u=u(x,t)$:

$$u_{xx}=c^{-2}u_{tt},\ 0<x<\pi,\ t>0,$$

with the initial conditions

$$u(0,t)=u(\pi,t)=0,\ t>0, \text{ and}$$

$$u(x,0)=f(x),\ u_t(x,0)=0,\ 0<x<\pi.$$

(i) Solve the given problem using the method of separation of variables.

(ii) Hence, simplify your solution given that

$$f(x)=\begin{cases}\frac{1}{4}x, & 0<x\le\frac{\pi}{2}\\ \frac{1}{4}(\pi-x), & \frac{\pi}{2}<x<\pi\end{cases}.$$

[25 marks]

4. Pertimbangkan persamaan gelombang satu dimensi bagi $u=u(x,t)$ *yang berikut:*

$$u_{xx}=c^{-2}u_{tt},\ 0<x<\pi,\ t>0,$$

dengan syarat awal

$$u(0,t)=u(\pi,t)=0,\ t>0, \textit{ dan}$$

$$u(x,0)=f(x),\ u_t(x,0)=0,\ 0<x<\pi.$$

(i) Selesaikan masalah di atas dengan menggunakan kaedah pemisahan pembolehubah.

(ii) Dengan demikian, ringkaskan penyelesaian anda diberikan

$$f(x)=\begin{cases}\frac{1}{4}x, & 0<x\le\frac{\pi}{2}\\ \frac{1}{4}(\pi-x), & \frac{\pi}{2}<x<\pi\end{cases}.$$

[25 markah]

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