
UNIVERSITI SAINS MALAYSIA

Peperiksaan Kursus Semasa Cuti Panjang
Sidang Akademik 2010/2011

Jun 2011

MAT 122 – Differential Equations I
[Persamaan Pembezaan I]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed material before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all four** [4] questions.

Arahan: Jawab **semua empat** [4] soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai].

1. (a) Find an integrating factor for the equation

$$2x^3y^2 + 4x^2y + 2xy^2 + xy^4 + 2y \, dx + 2y^3 + x^2y + x \, dy = 0.$$

Hence, solve the differential equation.

[30 marks]

- (b) (i) Show that both the parabola $y = x^2$ and line $y = 2x - 1$ are solutions of the equation

$$y' = 2x - 2\sqrt{x^2 - y}$$

with $y(1) = 1$.

- (ii) Does this fact contradict the Existence and Uniqueness theorem? Justify your answer.

- (iii) Determine a function $M(x, y)$ such that the differential equation

$$M(x, y) \, dx + \left(xe^{xy} + 2xy + \frac{1}{x} \right) dy = 0$$

is exact.

[40 marks]

- (c) Solve the differential equation

$$x \frac{dy}{dx} + 6y = 3xy^{4/3}.$$

[30 marks]

1. (a) Dapatkan suatu faktor pengamir bagi persamaan

$$2x^3y^2 + 4x^2y + 2xy^2 + xy^4 + 2y \, dx + 2y^3 + x^2y + x \, dy = 0.$$

Seterusnya, selesaikan persamaan pembezaan tersebut.

[30 markah]

- (b) (i) Tunjukkan bahawa parabola $y = x^2$ dan garis $y = 2x - 1$ yang kedua-duanya adalah penyelesaian bagi persamaan

$$y' = 2x - 2\sqrt{x^2 - y}$$

dengan $y(1) = 1$.

- (ii) Adakah hal ini bercanggah dengan Teorem Kewujudan dan Keunikan? Jelaskan jawapan anda.

- (iii) Tentukan suatu fungsi $M(x, y)$ supaya persamaan pembezaan

$$M(x, y) \, dx + \left(xe^{xy} + 2xy + \frac{1}{x} \right) dy = 0$$

adalah tepat.

[40 markah]

- (c) Selesaikan persamaan pembezaan

$$x \frac{dy}{dx} + 6y = 3xy^{4/3}.$$

[30 markah]

2. (a) Show that the equation

$$yF_{xy} dx + xG_{xy} dy = 0$$

can be solved by using the transformation $u = xy$.

Hence, solve the differential equation

$$x^2y^3 + 2xy^2 + y dx + x^3y^2 - 2x^2y + x dy = 0.$$

[60 marks]

- (b) Solve the differential equation

$$\frac{xdx + ydy}{x^2 + y^2} + \frac{ydx - xdy}{x^2 + y^2} = 0.$$

[40 marks]

2. (a) *Tunjukkan bahawa persamaan*

$$yF_{xy} dx + xG_{xy} dy = 0$$

boleh diselesaikan dengan menggunakan transformasi $u = xy$.

Dengan ini, selesaikan persamaan pembezaan

$$x^2y^3 + 2xy^2 + y dx + x^3y^2 - 2x^2y + x dy = 0.$$

[60 markah]

- (b) *Selesaikan persamaan pembezaan*

$$\frac{xdx + ydy}{x^2 + y^2} + \frac{ydx - xdy}{x^2 + y^2} = 0.$$

[40 markah]

3. (a) Solve the following differential equations:

(i) $y'' - 3y' + 2 = \frac{1}{1 + e^{-x}},$

(ii) $y'' + y' - 2 = 2x - 40\cos 2x.$

[60 marks]

- (b) Solve the differential equation

$$1 - x^2 y'' - 6xy' - 4y = 0$$

about the ordinary point $x=0$. Determine the interval where the solution is valid.

[40 marks]

3. (a) *Selesaikan persamaan pembezaan berikut:*

(i) $y'' - 3y' + 2 = \frac{1}{1 + e^{-x}},$

(ii) $y'' + y' - 2 = 2x - 40\cos 2x.$

[60 markah]

(b) *Selesaikan persamaan pembezaan*

$$1 - x^2 y'' - 6xy' - 4y = 0$$

sekitar titik biasa $x=0$. Tentukan selang yang penyelesaian tersebut adalah sah.

[40 markah]

4. (a) By using the matrix method, solve the system

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 4 & -1 \\ -4 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

[40 marks]

(b) A model to predict consumer behaviour towards a product A is given by

$$X' = a(Y - 2X),$$

$$Y' = b(X - 2Y) + I_0,$$

where

$X(t)$ represents the purchasing level for A,

$Y(t)$ represents consumer attitude towards A,

I_0 is the advertisement effect which is considered constant,

and a and b are positive numbers.

(i) Show that

$$x = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \frac{I_0}{3b}$$

where

$$\lambda_1 = -(a+b) - (a^2 + b^2 - ab)^{1/2},$$

$$\lambda_2 = -(a+b) + (a^2 + b^2 - ab)^{1/2},$$

and c_1, c_2 are arbitrary constants.

(ii) Deduce that

$$x \rightarrow \frac{I_0}{3b} \quad \text{when } t \rightarrow \infty.$$

[60 marks]

4. (a) Dengan menggunakan kaedah matriks, selesaikan sistem

$$\begin{pmatrix} x \\ y \end{pmatrix}' = \begin{pmatrix} 4 & -1 \\ -4 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.$$

[40 markah]

- (b) Suatu model untuk meramalkan kelakuan pengguna terhadap barangan A diberikan oleh

$$X' = a(Y - 2X),$$

$$Y' = b(X - 2Y) + I_0,$$

dengan

$X(t)$ mewakili paras pembelian bagi A,

$Y(t)$ mewakili sikap pengguna terhadap A,

I_0 ialah kesan iklan yang dianggap malar, di mana a dan b adalah nombor-nombor positif

- (i) Tunjukkan bahawa

$$x = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \frac{I_0}{3b}$$

dengan

$$\lambda_1 = -(a+b) - (a^2 + b^2 - ab)^{1/2},$$

$$\lambda_2 = -(a+b) + (a^2 + b^2 - ab)^{1/2},$$

dan c_1, c_2 adalah pemalar sebarang.

- (ii) Tunjukkan bahawa

$$x \rightarrow \frac{I_0}{3b} \text{ bila } t \rightarrow \infty.$$

[60 markah]