
UNIVERSITI SAINS MALAYSIA

Supplementary Semester Examination
2010/2011 Academic Session

June 2011

MAT 101 – Calculus
[Kalkulus]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of FIVE pages of printed materials before you begin the examination.

[Sila pastikan bahawa kertas peperiksaan ini mengandungi LIMA muka surat yang bercetak sebelum anda memulakan peperiksaan ini.]

Instructions: Answer **all ten** [10] questions.

Arahan: Jawab **semua sepuluh** [10] soalan.]

In the event of any discrepancies, the English version shall be used.

[Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.]

1. Find $\frac{dy}{dx}$ for the following y :

(a) $y = \sqrt{x} + \frac{1}{e^{\sqrt{x}}}$

(b) $y = x^3 + \int_1^x 1 + \sqrt{t}^3 dt$

(c) $y = 1 + \sin^{-1} x^2$

[7 marks]

1. Cari $\frac{dy}{dx}$ untuk y yang berikut:

(a) $y = \sqrt{x} + \frac{1}{e^{\sqrt{x}}}$

(b) $y = x^3 + \int_1^x 1 + \sqrt{t}^3 dt$

(c) $y = 1 + \sin^{-1} x^2$

[7 markah]

2. Evaluate the following limits:

(a) $\lim_{x \rightarrow 0} \frac{x^2}{\tan 2x}$

(b) $\lim_{x \rightarrow \infty} \frac{1 - \sqrt{3x}}{\sqrt{3x+1}}$

(c) $\lim_{x \rightarrow 1} \frac{x^2 - 7x + 6}{x - 1}$

[7 marks]

2. Nilaiikan had berikut:

(a) $\lim_{x \rightarrow 0} \frac{x^2}{\tan 2x}$

(b) $\lim_{x \rightarrow \infty} \frac{1 - \sqrt{3x}}{\sqrt{3x+1}}$

(c) $\lim_{x \rightarrow 1} \frac{x^2 - 7x + 6}{x - 1}$

[7 markah]

3. (a) If $f^{-1} = A$, then express f^{-1} in term of A , given

$$f^{-1}(x) = \frac{1}{\sqrt{x}} \int_0^x \cos \frac{\pi}{1+t^2} dt.$$

- (b) Find all solutions to the equation $|x-1|+|x+3|+x=8$

[7 marks]

3. (a) Jika $f^{-1} = A$, maka ungkapkan f^{-1} dalam sebutan A , diberikan

$$f^{-1}(x) = \frac{1}{\sqrt{x}} \int_0^x \cos \frac{\pi}{1+t^2} dt.$$

- (b) Cari semua penyelesaian kepada persamaan $|x-1|+|x+3|+x=8$

[7 markah]

4. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by

$$f(x) = \begin{cases} h & , x=1 \\ \frac{1-\sqrt{x}}{1-x} & , x \neq 1 \end{cases}$$

- (a) Determine the value of h so that f is continuous at $x=1$.
(b) Express f^{-1} in terms of the definition of derivative, and then evaluate its value.

[7 marks]

4. Biar $f : \mathbb{R} \rightarrow \mathbb{R}$ diberi sebagai

$$f(x) = \begin{cases} h & , x=1 \\ \frac{1-\sqrt{x}}{1-x} & , x \neq 1 \end{cases}$$

- (a) Tentukan nilai h supaya f adalah selanjut pada $x=1$.
(b) Ungkapkan f^{-1} dalam sebutan pembezaan, dan seterusnya nilaikan nilainya.

[7 markah]

5. (a) Show that on the interval $0,1$, $f(x) = \sqrt{3x-x^3}$ is increasing.
(b) Find the absolute maximum and minimum values of $f(x) = \sqrt{3x-x^3}$ on the interval $0,1$.
- [7 marks]

5. (a) Tunjukkan pada selang $0,1$, $f(x) = \sqrt{3x-x^3}$ adalah menaik.
(b) Cari nilai maksimum dan minimum mutlak $f(x) = \sqrt{3x-x^3}$ pada selang $0,1$.
- [7 markah]

6. (a) Evaluate $\int \frac{\sin^2\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)}{x^2} dx$
(b) Differentiate $\frac{2+x}{x}$ with respect to x . Hence determine $\int \left(\frac{2+x}{x^5}\right)^{\frac{1}{2}} dx$
- [7 marks]

6. (a) Nilaikan $\int \frac{\sin^2\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)}{x^2} dx$
(b) Bezakan $\frac{2+x}{x}$ terhadap x . Seterusnya dapatkan $\int \left(\frac{2+x}{x^5}\right)^{\frac{1}{2}} dx$
- [7 markah]

7. (a) Using $\varepsilon - \delta$ approach show that $\lim_{x \rightarrow 1} 2x + 1 = 3$.
(b) Using the definition of first derivative find $\lim_{x \rightarrow 1} \frac{e^{x-1} - 1}{x - 1}$.
- [7 marks]

7. (a) Dengan kaedah $\varepsilon - \delta$ tunjukkan bahawa $\lim_{x \rightarrow 1} 2x + 1 = 3$.
(b) Dengan menggunakan takrif terbitan pertama cari $\lim_{x \rightarrow 1} \frac{e^{x-1} - 1}{x - 1}$.
- [7 markah]

8. Evaluate the followings:

(a) $\int t \ln t \, dt$

(b) $\int_0^1 \frac{2x}{1+x^2} dx$

(c) $\int \sin x \cos^2 2x \, dx$

[7 marks]

8. *Nilaiikan berikut:*

(a) $\int t \ln t \, dt$

(b) $\int_0^1 \frac{2x}{1+x^2} dx$

(c) $\int \sin x \cos^2 2x \, dx$

[7 markah]

9. A line L is a tangent to the graph $f(x) = x^3$ at $x=1$. The line L intersects the y -axis at $(0, p)$.

(a) Find the value of p .

(b) The region bounded by the line L , y -axis and the graph f is rotated about the x -axis. find the volume of the rotation.

[7 marks]

9. *Suatu garis L menyetuh graf $f(x) = x^3$ pada $x=1$. Garis L menyilang paksi- y pada $(0, p)$.*

(a) *Cari nilai p .*

(b) *Rantau yang dibatasi oleh garis L , paksi- y dan graf f dikisarkan pada paksi- x . Cari isipadu kisanan.*

[7 markah]

10. On the interval $0 \leq x \leq 1$ show that $2 \leq 1 + e^{x^2} \leq 1 + e^x$

Hence show that $2 \leq \int_0^1 1 + e^{x^2} \, dx \leq e$

[7 marks]

10. *Pada selang $0 \leq x \leq 1$ tunjukkan $2 \leq 1 + e^{x^2} \leq 1 + e^x$*

Seterusnya tunjukkan bahawa $2 \leq \int_0^1 1 + e^{x^2} \, dx \leq e$

[7 markah]