UNIVERSITI SAINS MALAYSIA

First Semester Examination

Academic Session 2003/2004

September/October 2003

EEE 504 - NONLINEAR DYNAMIC SYSTEMS

Time : 3 Hours

INSTRUCTION TO CANDIDATE:-

Please ensure that this examination paper contains **SEVEN** (7) printed pages with **2 Appendix** and **SEVEN** (7) question before answering.

Answer **FIVE** (5) questions.

Distribution of marks for each question is given accordingly.

All questions must be answered in English.

...2/-

1. (a) The circuit shown in Figure 1.1 has the input, $u = i_s$ and the output, $y = i_{L1}$. Select v_{C1}, v_{C2}, i_{L1} *and* i_{L2} as the state variables and obtain the state equations.

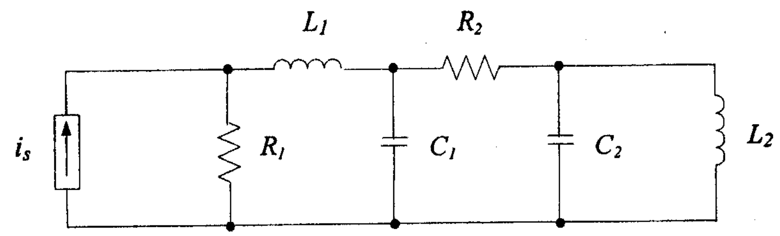

Figure 1.1

(40%)

(b) Determine $X(t)$ and the output, $y(t)$ for a time-invariant system represented by the following state equations:

$$\dot{x}_1 = -2x_1 + x_2 + 3u$$

$$\dot{x}_2 = -x_1 + u$$

$$y = x_1 + x_2$$

where $x_1(0) = 10, x_2(0) = 1$ *and* $u(t) = e^{2t}$

(60%)

2. (a) Determine the equilibrium points and phase portraits of the following non-linear system:

$$\ddot{y} - 2(1 - y^2)\dot{y} + |y| = 0$$

(35%)

(b) Determine the piecewise linear isocline equations for the non-linear system shown in Figure 2.1 with unit step input. Assume, the system constants as T = 1 and K = 4, $e_o = 0.2$, $M_o = 0.2$.

...3/-

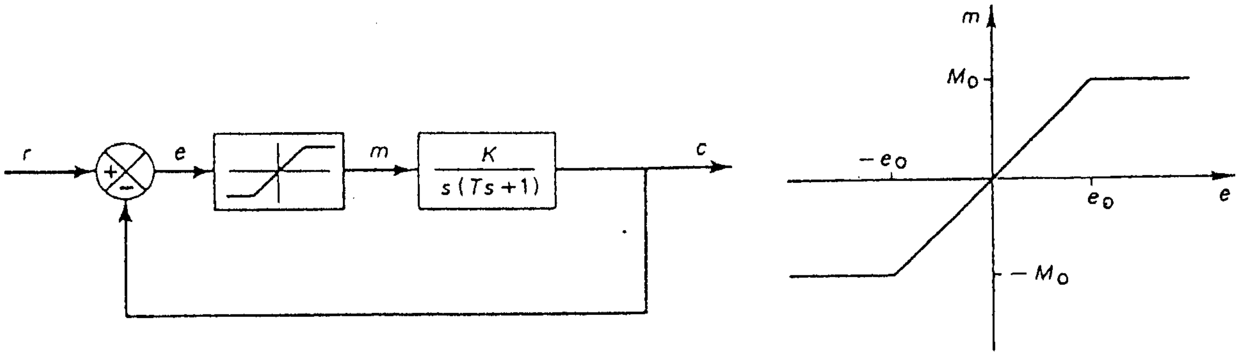

Figure 2.1

Draw the sketch of approximate trajectory for linear operation

(65%)

3. (a) Derive and show that the Describing Function of saturation non linearity as:

$$N = \frac{2k}{\pi}\left[\sin^{-1}\frac{s}{X} + \frac{s}{X}\sqrt{1-\left(\frac{s}{X}\right)^2}\right]$$

where k = slope and s = input at saturation

X = amplitude of the input sinusoidal signal.

(50%)

(b) A nonlinear system is shown in Figure 3.1 with r = 0 and $f(u) = 2u^3$. Determine the condition for limit cycles when the input to the non-linear element, $u(t) = U_0 + U_1 \sin \omega t$.

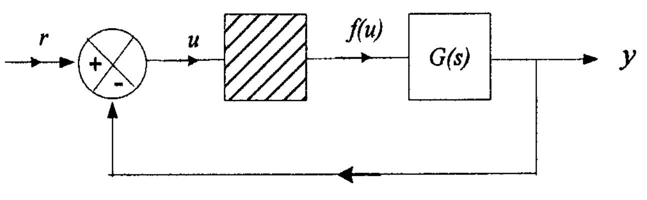

Figure 3.1

(50%)

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4. Using Single Input Describing Function Technique, determine the amplitude and frequency of the limit cycle, if any, for the system shown in Figure 4.1.

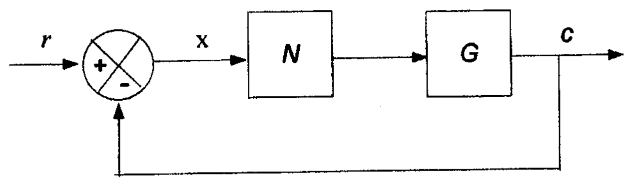

Figure 4.1

$$x = X \sin \omega t$$

$$\text{Where } N = \frac{4}{\pi X} \angle -\sin^{-1}\frac{0.2}{X}$$

$$G(s) = \frac{2.5}{s(s+1)^2}$$

(100%)

5. (a) Explain with suitable diagrams the meaning of stability, asymptotic stability and asymptotic stability in the large of an equilibrium state of a system in the sense of Lyapunov.

(25%)

(b) For the following system:

$$\dot{x}_1 = -x_1 - \frac{3}{16}x_2$$

$$\dot{x}_2 = x_1$$

Determine (i) the matrix P which verifies the Lyapunov function

(ii) the Lyapunov function

(iii) the stability of equilibrium state of the system.

(50%)

...5/-

(c) Investigate the stability of the following non-linear system by the method of Lyapunov:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -f(x_1) - g(x_2)$$

A possible Lyapunov function, $V(X) = \frac{1}{2}x_2^{\,2} + \int_0^{x_1} f(\sigma)d\sigma$

(25%)

6. (a) Determine the state transition matrix from the recurrence relations:

$$x_1(k+1) = x_2(k)$$
$$x_2(k+1) = -x_1(k)$$

(20%)

(b) For a linear time-invariant discrete-time system, $X(k+1) = A\,X(k)$, select a positive definite Lyapunov function in quadratic form and prove that the necessary and sufficient conditions for the equilibrium state of the system to be asymptotically stable.

(30%)

(c) Investigate the stability of the following system by Lyapunov method:

$$x_1(k+1) = \frac{1}{2}x_2(k) + x_1^{\,2}(k)\ x_2(k)$$

$$x_2(k+1) = -\frac{1}{2}x_1(k) + x_1(k)\ x_2^{\,2}(k)$$

(50%)

7. Explain briefly any **three** of the following with suitable diagrams:

(a) Equilibrium points and phase-plane portraits.

(b) Variable structure systems.

(c) Feedback system stability.

(d) Adaptive control

(100%)

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Table 1 : Laplace Transform Pairs

	$f(t)$	$F(s)$
1	unit impulse $\delta(t)$	1
2	unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	e^{-at}	$\frac{1}{s+a}$
5	te^{-at}	$\frac{1}{(s+a)^2}$
6	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
7	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
8	$t^n \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{s^{n+1}}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
11	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
12	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$
13	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
14	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
15	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$

Table 1.A : A Table of Z Transforms

	$X(s)$	$x(t)$ or $x(k)$	$X(z)$
1	1	$\delta(t)$	1
2	e^{-kTs}	$\delta(t-kT)$	z^{-k}
3	$\frac{1}{s}$	$1(t)$	$\frac{z}{z-1}$
4	$\frac{1}{s^2}$	t	$\frac{Tz}{(z-1)^2}$
5	$\frac{1}{s+a}$	e^{-at}	$\frac{z}{z-e^{-aT}}$
6	$\frac{a}{s(s+a)}$	$1-e^{-at}$	$\frac{(1-e^{-aT})z}{(z-1)(z-e^{-aT})}$
7	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$
8	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\frac{z(z-\cos \omega T)}{z^2 - 2z \cos \omega T + 1}$
9	$\frac{1}{(s+a)^2}$	te^{-at}	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
10	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at} \sin \omega t$	$\frac{ze^{-aT} \sin \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$
11	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at} \cos \omega t$	$\frac{z^2 - ze^{-aT} \cos \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$
12	$\frac{2}{s^3}$	t^2	$\frac{T^2 z(z+1)}{(z-1)^3}$
13		a^k	$\frac{z}{z-a}$
14		$a^k \cos k\pi$	$\frac{z}{z+a}$