UNIVERSITI SAINS MALAYSIA

Semester I Examination
Academic Session 2003/2004

September/October 2003

EEE 503 – STOCHASTIC PROCESS

Time : 3 hours

INSTRUCTION TO CANDIDATE:

Please ensure that this examination paper contains **SEVEN (7)** printed pages and **SIX (6)** questions before answering.

Answer **FIVE (5)** questions.

Distribution of marks for each question is given accordingly.

All questions must be answered in English.

...2/-

1. With reference to Figure 1 the input signal is a Gaussian-Markov Process with power spectral density $S_s(s) = \frac{2\sigma^2\beta}{s^2 + \beta^2}$ and the corrupting noise is white noise with power spectral density $S_n(s) = A$. The optimum non-causal filter that minimizes the mean square error in the estimation of s(t) is given by

$$G(s) = \frac{S_{s+n,s}(s)e^{\alpha s}}{S_{s+n}(s)}$$

The symbols have their usual meanings.

Determine the weighting function of the filter if the signal and noise are uncorrelated and $\sigma^2 = \beta = A = 1$. Justify any assumption made.

(100%)

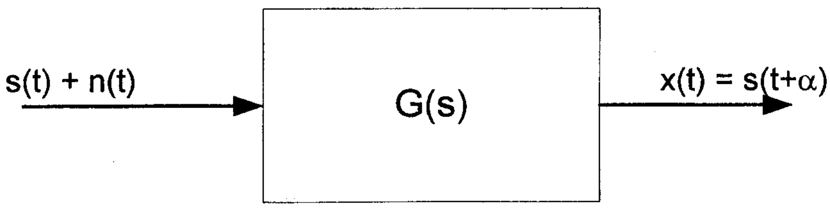

Figure 1

2. Figure 2 shows combination of two independent noisy measurements of the same signal s(t). Using Weiner Filtering Theory determine $G_1(s)$ and $G_2(s)$ that minimize the mean square error in the estimation of s(t) subject to the condition that the resulting structure does not introduce any delay or distortion in s(t).

(100%)

...3/-

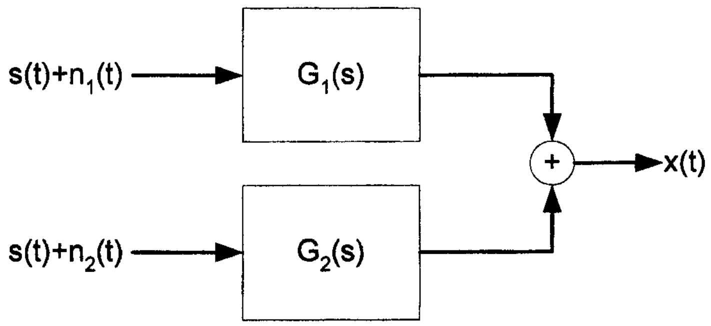

Figure 2

3. Discrete Kalman filter is described by the following recursive relations.

$$\hat{X}_k = \hat{X}_K^- + K_k(Z_K - H_K\hat{X}_K^-)$$
$$K_K = P_K^- H_K^T (H_K P_K^- H_K^T + R_K)^{-1}$$
$$P_K = (I - K_K H_K)P_K^-$$
$$\hat{X}_{K+1}^- = \Phi_K \hat{X}_K$$
$$P_{K+1}^- = \Phi_K P_K \Phi_K^T + Q_K$$

The symbols have their usual meanings.

Assume that we have a sequence of independent noisy measurements taken at unit intervals as shown in Figure 3. Let the standard deviation of the measurement error be 1/2. Using the discrete Kalman filter determine the optimal estimate of the process at the sampling instants t=0 and t=1. Clearly specify any assumption made.

(100%)

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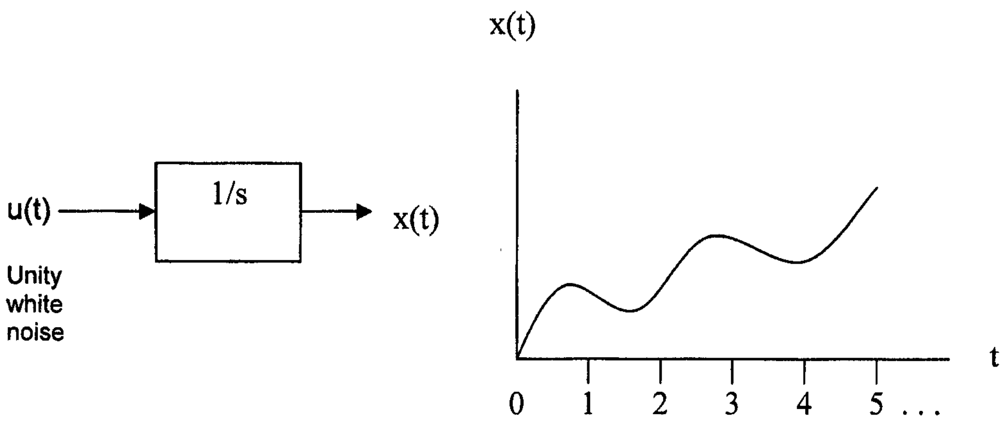

Figure 3

4. (a) Random variables X and Y have a joint PMF described by the following table 1:

Table 1

$P_{X,Y}(x,y)$	$y = -1$	$y = 0$	$y = 1$
$x = -1$	$3/16$	$1/16$	0
$X = 0$	$1/6$	$1/6$	$1/6$
$x = 1$	0	$1/8$	$1/8$

(i) Are X and Y independent ?

(ii) In fact, the experiment from which X and Y are derived is performed sequentially. First, X is found, then Y is found. In this context, please label the conditional branch probabilities of the following tree (Figure 4):

(50%)

...5/-

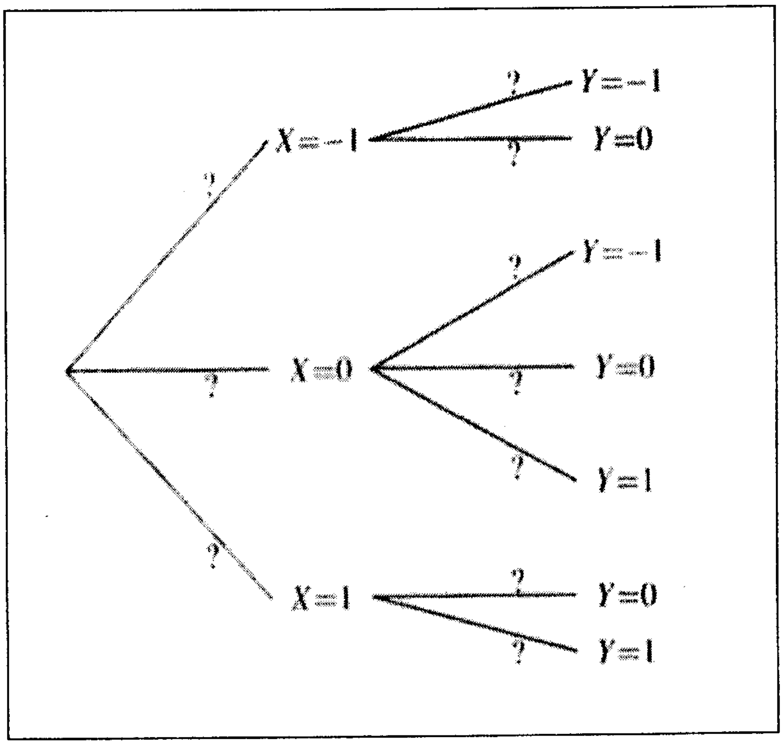

Figure 4

(b) The voltage V at the output of a microphone is a uniform random variable with limits -1 volt and 1 volt. The microphone voltage is processed by a hard limiter with cut-off points -0.5 volt and 0.5 volt. The magnitude of the limiter output L is a random variable such that

$$L = \begin{cases} V & |V| \leq 0.5 \\ 0.5 & \text{otherwise} \end{cases}$$

(i) What is $P[L = 0.5]$?

(ii) What is $F_L(l)$?

(iii) What is $E[L]$?

(50%)

...6/-

5. (a) Let X denote the position of the pointer after a spin on a wheel of circumference 1. For that same spin, let Y denote the area within the arc defined by the stopping position of the pointer (see Figure 5):

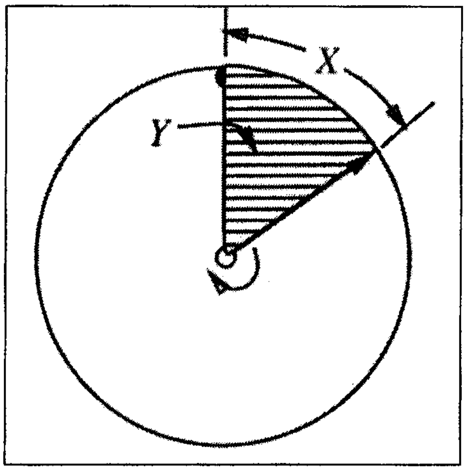

Figure 5

(i) What is the relationship between X and Y ?

(ii) What is $F_Y(y)$?

(iii) What is $f_Y(y)$?

(iv) What is $E[Y]$?

(40%)

(b) The wide sense stationary process $X(t)$ with autocorrelation function $R_X(\tau)$ and power spectral density $S_X(f)$ is the input to a tapped delay line filter

$$H(f) = a_1 e^{-j2\pi f t_1} + a_2 e^{-j2\pi f t_2}$$

Find the output power spectral density $S_Y(f)$ and the output autocorrelation $R_Y(\tau)$.

(60%)

...7/-

6. The input to a digital filter is a sequence of random variables ...., $X_{-1}, X_0, X_1, \ldots$.. The output is also a sequence of random variables ...,$W_{-1}, W_0, W_1, \ldots$. The relationship between the input and output is

$$W_n = \frac{1}{2}(X_n + X_{n-1})$$

Let the input be a sequence of ***iid*** random variables with E[Xi] = 0 and Var[Xi] =1. Find the following properties of the output sequence:

(a) $E[W_i]$

(b) $Var[W_i]$

(c) $Cov[W_{i+1}, W_i]$

(d) ρ_{W_{i+1}, W_i}

(100%)

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