UNIVERSITI SAINS MALAYSIA

First Semester Examination

Academic Session 2003/2004

September/October 2003

EEE 501 – ADVANCED ENGINEERING MATHEMATICS

Time : 3 Hours

INSTRUCTION TO CANDIDATE:-

Please ensure that this examination paper contains **SIX** (6) printed pages and **SIX** (6) question before answering.

Answer **FIVE** (5) questions.

Distribution of marks for each question is given accordingly.

All questions must be answered in English.

...2/-

1. Elastic Deformation

An elastic membrane in the x_1, x_2 - plane with boundary circle $x_1^2 + x_2^2 = 1$ is stretched so that a point P : (x_1, x_2) goes over into the point Q : (y_1, y_2) given by

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = AX = \begin{bmatrix} 4 & \sqrt{8} \\ \sqrt{8} & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ : in components}$$

$$y_1 = 4x_1 + \sqrt{8}\, x_2$$

$$y_2 = 3x_1 + 5\, x_2$$

Find the principle direction, that is, the direction of the position vector x of P for which the direction of the position of rector y of Q is the same or exactly opposite. What shape does the boundary circle take under this deformation.

(100 marks)

2. (a) Determine the orthogonal trajectories of the given family of curves. Plot or sketch the curves and their trajectories on common axes. Show each step of your calculation.

(50 marks)

$$y = x^2 + c$$

(b) If in a reactor, uranium ${}_{92}U^{237}$ Joses 10% of its weight within 1 day, what is its half-life? How long would it take for 99% of the original amount to disappear?

(50 marks)

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3. A long rectangular plate Γ of width b cm with insulated surface has its temperature $u\,(x, y)$ equal to zero on both the long unbounded sides as shown in Figure 3.

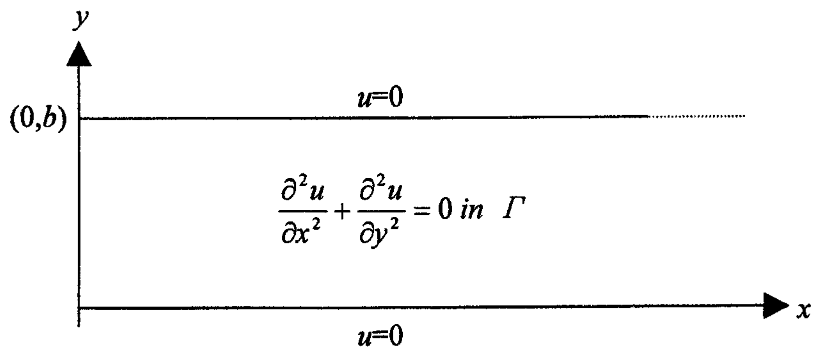

Figure 3

The temperature of one of its short side is maintained at kx. Hence the boundary problem can be expressed as the two-dimensional Laplace equation such as:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \ in \ \text{in } \Gamma$$
$$u(x,0) = u(x,b) = 0 \text{ on } \Gamma$$
$$u(0,y) = g(y) \text{ on } \Gamma$$
$$u\,(x,y) \text{ is bounded as } x \to \infty$$

Show that the steady state temperature within the plate is given by:

$$u(x,y) = \frac{2bk}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} e^{\frac{-n\pi x}{b}} \sin\left(\frac{n\pi}{b}\right)$$

Given:

(1) General solutions of

$$Y'' + \lambda Y = 0$$
$$X'' - \lambda X = 0$$

are

$$Y = C_1 \cos py + C_2 \sin py$$
$X = C_3\, e^{px} + C_4\, e^{-px}$ where P, C_1, C_2, C_3 and C_4 are constants.

(2) $$\int x \sin ax\, dx = \frac{\sin ax}{a^2} - \frac{x \cos a}{a}$$

(100 marks)

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4. The distribution of an electrical potential ϕ in a cylinder of radius r is given by:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

Perform conformal transformation on ϕ using Joukowski's operator. Hence, plot the equipotential lines in z and w planes.

(100 marks)

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5. (a) Using the L U Factorization matrix, solve this linear equation system bellow :

$$\begin{aligned} 2x_1 + 6x_2 - x_3 + 12 &= 0 \\ 5x_1 + x_2 + 2x_3 - 29 &= 0 \\ -3x_1 - 4x_2 + 5x_3 - 5 &= 0 \end{aligned}$$

(50 marks)

(b) From the values in the table below, find value of current (approximation) for voltage is 0.50 volt, use the Lagrange interpolation.

Voltage [volt]	0.10	0.30	0.50	0.70	0.90
Current [ampere]	0.01	0.05	.....	3.11	7.13

Lagrange Formula's :

$$P_n(x) = f(x) = \sum_{k=0}^{n} L_k(x)\, f(k) = \sum_{k=0}^{n} \frac{l_k(x)}{l_k(x_k)} f(k)$$

where :

$$l_0 = (x - x_1)(x - x_2)\ldots(x - x_n)$$

$$l_k(x) = (x - x_0)\ldots(x - x_{k-1})(x - x_{k+1})\ldots(x - x_n) \qquad 0 < k < n$$

$$l_n(x) = (x - x_0)(x - x_1)\ldots(x - x_{n-1})$$

(50 marks)

6. (a) Compute C in the integral below by using Simpson's rule :
Get n = 4.

$$C = \int_{t_1=0}^{t_2=2.0} \left(1 - \sin t^2\right)^3 dt$$

Simpson's rule for evaluate of integral is :

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$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f_0 + 4f_1 + 2f_2 + 4f_3 + \ldots + 2f_{2m-2} + 4f_{2m-1} + f_{2m}\right]$$

(50 marks)

(b) Apply the Runge-Kutta method to find $y=f(x)$ of

$$\frac{dy}{dx} = \frac{2\sqrt{y-\ln x}}{x} + \frac{1}{x}$$

in interval $1.0 \leq x \leq 1.4$, where the initial value $y(0)=0$ and $h = 0.1$

(50 marks)

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