UNIVERSITI SAINS MALAYSIA

First Semester Examination
2012/2013 Academic Session

January 2013

EAS 661/4 – Advanced Structural Mechanics

Duration : 3 hours

Please check that this examination paper consists of **TEN (10)** pages of printed material before you begin the examination.

Instructions : This paper contains **SIX (6)** questions. Answer **FIVE (5)** questions.

All questions must be answered in English.

Each question **MUST BE** answered on a new page.

1. (a) Figure 1 shows an infinitesimal volume taken from an interior of an elastic body under the action of external load. Show all the stress components (in Cartesian coordinate system) acting on the infinitesimal volume.

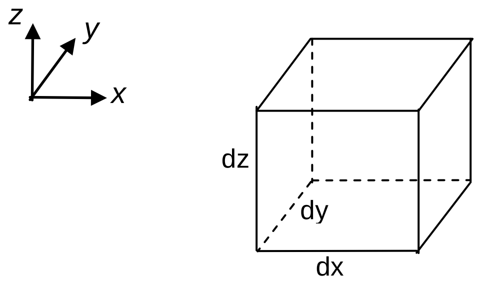

Figure 1

Derive the equilibrium equation in *z*-direction for the infinitesimal volume shown in Figure 1 above. It is given that body forces B_x, B_y and B_z act on the infinitesimal volume in *x*, *y* and *z*-direction respectively.

Making use of the derived equilibrium equation with proper specialization, obtain the governing equation in terms of *u* for a prismatic bar subjected to a uniformly distributed load *w* as shown in Figure 2. State clearly the specialization made in the process of obtaining the governing equation. It is given that the material of the bar is linearly elastic with modulus of elasticity *E*.

[10 marks]

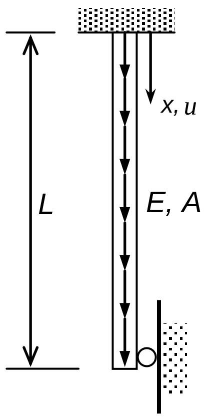

Figure 2

(b) Figure 3 shows a thin wall structure loaded with a uniformly distributed load *w* in *y*-direction. Justify why this problem can be solved as a plane stress problem.

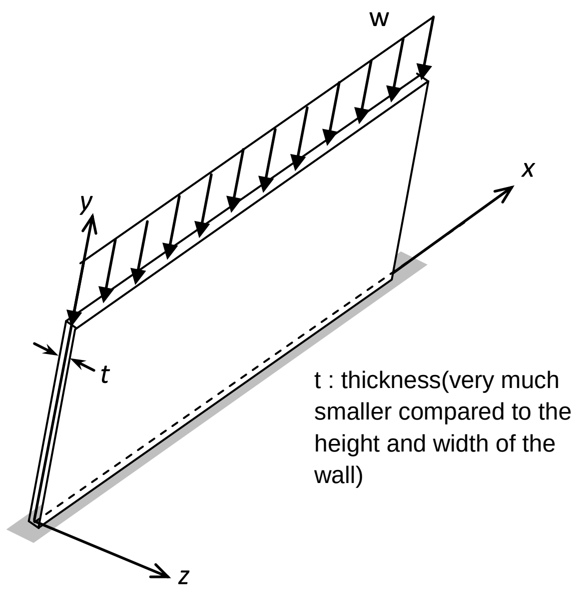

Figure 3

Figure 4 shows an infinitesimal volume taken from the interior of the wall shown in Figure 3. Indicate on the infinitesimal volume the non-zero stress components.

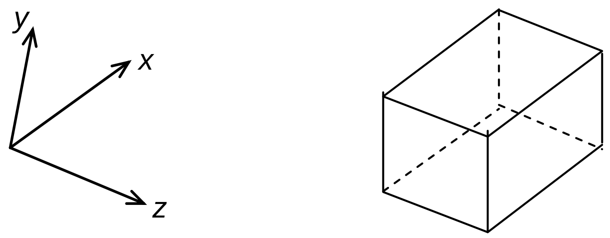

Figure 4

Using the following general stress-strain relation, derive the stress-strain relation for plane stress problem.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

[10 marks]

2. (a) The strain energy U_p stored in an elastic body can be obtained using the following equation:

$$U_P = \int_{Vol} v_p dvol$$

where v_p is strain energy density and *vol* is volume of the elastic body. Using the above equation, show that the strain energy stored in an elastic bar with elastic modulus *E* of length *L* with arbitrary variation of cross-section *A* can be represented using the following equation:

$$U_P = \frac{1}{2} E \int_0^L A \left(\frac{du}{dx} \right)^2 dx$$

where *u* : axial displacement of the bar.

Specialize the above derived equation of U_p to the case of the prismatic elastic bar as shown in Figure 5 below.

[8 marks]

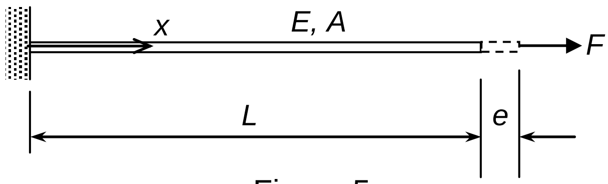

Figure 5

(b) Figure 6 shows a cantilever column with height *H* subjected to a linearly distributed load from *W* at bottom end until *0* at the upper end. One linear spring with spring constant *k* is located at mid-height of the column. The following expression for lateral displacement field *v* has been suggested:

$$v = A(1 - \cos(\pi / 2H))$$

where *A* is a constant. Show that the above displacement field is kinematically admissible. Next, solve for constant *A* by applying the principle of minimum potential energy (PMPE). Flexural rigidity of the column is *EI.*

[12 marks]

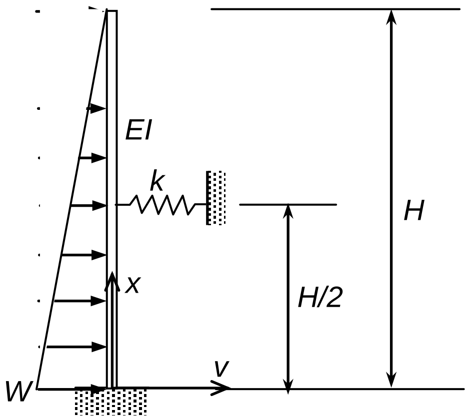

Figure 6

3. (a) State the three basic relations that a structural mechanics problem must satisfy in order that an exact solution is obtained. Then, write down the corresponding three basic relations for the case of a simple 1D linearly elastic prismatic bar subjected to end force P as shown in Figure 7, where Δ : elongation of bar due to force P, u : axial displacement in the direction of bar axis x, E : elastic modulus of material of bar, A : cross-sectional area and L : original length of bar.

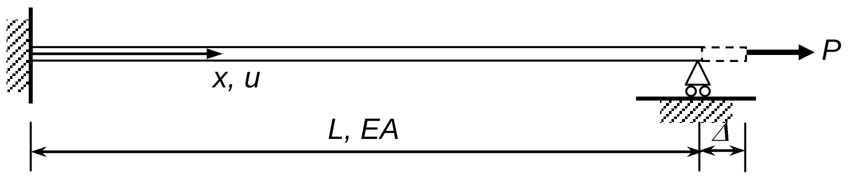

Figure 7

[5 marks]

(b) Figure 8 shows a prismatic bar which is fixed at both ends. Length of the bar is L, elastic modulus is E and cross-sectional area is A. The bar is subjected to a uniformly distributed load w per unit length and two concentrated loads P and Q at points $L/3$ and $2L/3$ from the upper end, respectively. Using piece-wise Rayleigh-Ritz method, derive the expression for axial displacement u. Divide the bar into three equal portions and assume linear displacement field for each portion. Express u in terms of point displacements at the ends of each portion. Plot the distribution of u and axial stress σ along the bar.

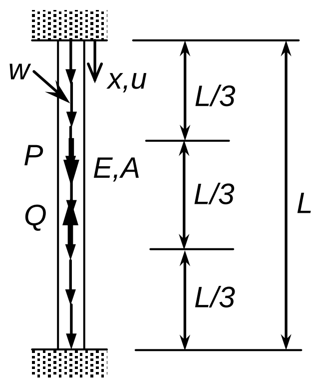

Figure 8

[15 marks]

4. (a) Write down the element stiffness matrices and global matrix for the three bars assembly (as shown in Figure 9a) which is loaded with force P, and constrained at the two ends in terms of E, A and L

[5 marks]

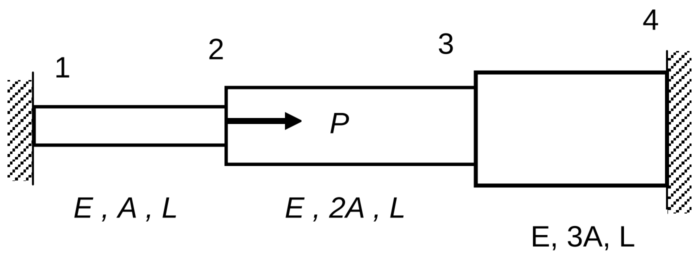

Figure 9a

(b) Write down the element stiffness matrices and global matrix for the two bars assembly which is loaded with force 10P at node 2 as shown in Figure 9b. End bars are constrained at end B and free at end A with a gap of Δ at end A. Given the value of P = 60kN, E = 20 kN/mm^2, L = 200 mm, A= 250 mm^2 and Δ = 1.2 mm, determine :

i. the displacements at node 1, 2 and 3

ii. the support reaction forces at A

[15 marks]

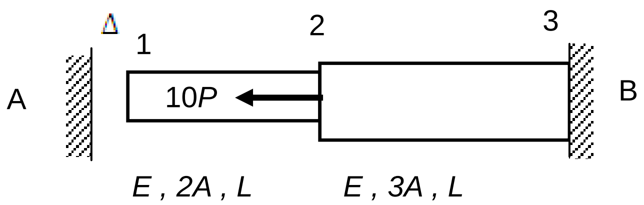

Figure 9b

5. (a) Two plates shown in Figure 10a and 10b, shall be analysed as a plane strain problem. Both plates are divided into 9 elements. Each nodes has been labelled accordingly. Calculate the bandwidth, B = (R+1) NDOF for the plate assuming two degrees of freedom at each node.

[4 marks]

(b) Rearrange the node labeling in such a way that a minimum value of R is obtained.

[4 marks]

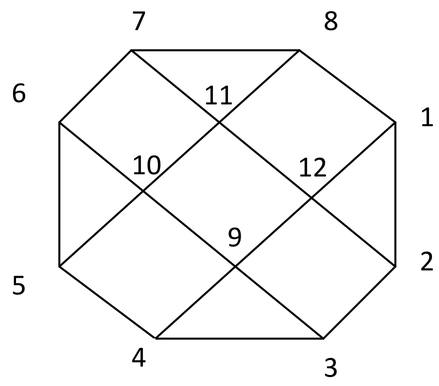

Figure 10a

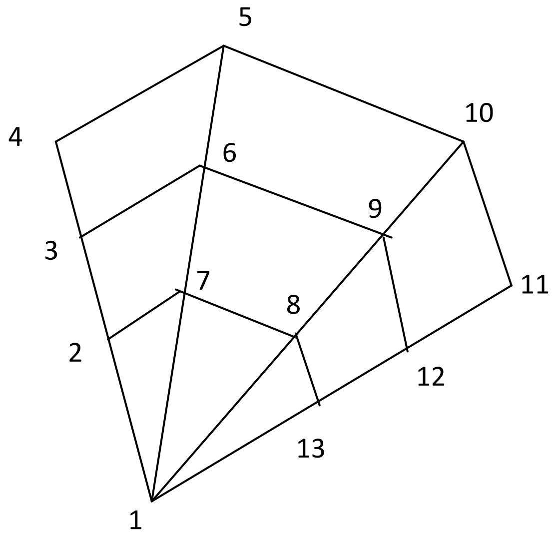

Figure 10b

(c) Figure 10c shows a frame structure with nodes labeled as 1, 2 and 3. The frame is subjected to a nodal forces of P = 20 kN at node 2 and uniformly distributed load of 6 kN/m along member 2-3 . The frame is fixed at node 1 and pinned at 3. Given the value of E = 207 GPa, I = 3×10^{-5} m^4 and A = $0.005 m^2$.

i. Derive the global stiffness matrix for the frame.

ii. Determine the deflection u_2 , v_2 , θ_2 and u_3 , v_3 , θ_3 in unit metre and rad, respectively.

[12 marks]

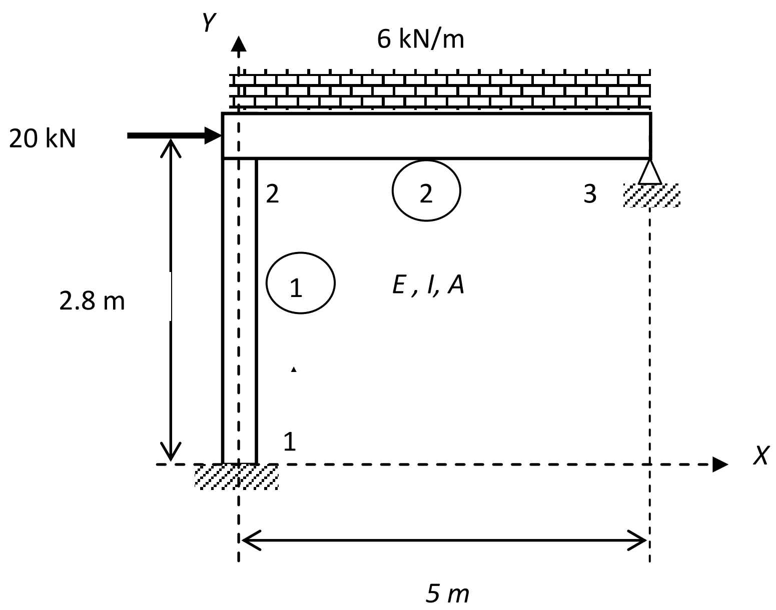

Figure 10c

Given the stiffness of the beam element in two dimensional space :

$$k = \frac{EI}{L^3}\begin{array}{c} \begin{array}{cccc} v_i & \theta_i & v_j & \theta_j \end{array} \\ \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \end{array}$$ for a beam element

$$k = \begin{array}{c} \begin{array}{cc} u_i & u_j \end{array} \\ \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \end{array}$$ for a spring element

6. (a) Explain the importance of model validity and accuracy of the following factors in the modeling procedures for Finite Element Method.

[6 marks]

i. geometry
ii. material properties
iii. loading conditions

(b) Show clearly in a step by step manner the development process of a stiffness matrix, $[K]^e$, for a triangular element in a state of plane stress as shown in Figure 11. Then calculate the displacements at node 3 and horizontal displacement at node 2. Given E = 200 GN/m^2, ν = 0.3 and t = 1 cm.

[14 marks]

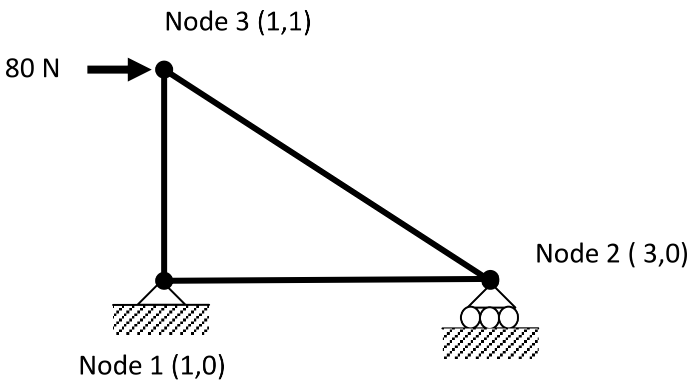

Figure 11

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