UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 2003/2004

September – Oktober 2003

ZCT 211E - Analisis Vektor

Masa : 2 jam

Sila pastikan bahawa kertas peperiksaan ini mengandungi **ENAM** muka surat yang bercetak sebelum anda memulakan peperiksaan ini.

Jawab kesemua LIMA soalan. Pelajar dibenarkan menjawab semua soalan dalam Bahasa Inggeris ATAU Bahasa Malaysia ATAU kombinasi kedua-duanya.

1. (a) Dua zarah yang terpancar daripada suatu sumber mempunyai sesaran $\mathbf{r}_1 = 4\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ dan $\mathbf{r}_2 = 2\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}$ pada semua masa. Carikan sesaran zarah kedua relatif kepada yang pertama.

(3/15)

(b) Buktikan bahawa $(\mathbf{a} \times \mathbf{a}') + (\mathbf{b} + \mathbf{b}') + (\mathbf{c} \times \mathbf{c}') = O$, di mana $\mathbf{a}, \mathbf{b}, \mathbf{c}$ adalah vektor dan $\mathbf{a}', \mathbf{b}', \mathbf{c}'$ adalah songsangnya.

(6/15)

(c) Suatu zarah bergerak dalam satu orbit bulatan berjejari 10 cm. Jika frekuensi pergerakannya adalah 60 putaran/s, carikan kala masa, halaju dan pecutan zarah tersebut.

(6/15)

2. (a) Jika $\vec{\mathbf{r}} = t^2\vec{\mathbf{i}} - t^2\vec{\mathbf{j}} + (2t+1)\vec{\mathbf{k}}$. Carikan nilai-nilai bagi

$$\frac{d\mathbf{r}}{dt}, \frac{d^2\mathbf{r}}{dt^2}, \left|\frac{d\mathbf{r}}{dt}\right|, \left|\frac{d^2\mathbf{r}}{dt^2}\right| \text{ pada } t = 0 .$$

(3/15)

(b) Jika $\mathbf{r} = \mathbf{a}e^{\omega t} - \mathbf{b}e^{-\omega t}$, tunjukkan bahawa $\frac{d^2\mathbf{r}}{dt^2} - \omega^2\mathbf{r} = 0$; $\mathbf{a}, \mathbf{b}$ adalah vektor malar dan ω adalah malar.

(6/15)

(c) Suatu zarah bergerak disepanjang suatu lengkungan yang mempunyai persamaan parametrik $x = e^{-t}$, $y = 2 \cos 3t$, $z = 2 \sin 3t$, di mana t adalah masa..

(i) Tentukan halaju dan pecutan zarah tersebut pada sebarang masa.
(ii) Carikan magnitud halaju dan pecutan zarah pada t = 0.

(6/15)

3. (a) Jika $\mathbf{A} = x^2yz\mathbf{i} - 2xz^3\mathbf{j} + xz^3\mathbf{k}$, $\mathbf{B} = 2z\mathbf{i} + y\mathbf{j} - x^2\mathbf{k}$ carikan $\frac{\partial^2}{\partial x \partial y}(\mathbf{A} \times \mathbf{B})$ pada (1, 0, -2).

(5/20)

(b) Jika **r** ialah vektor kedudukan bagi satu titik, simpulkan nilai bagi kecerunan $\left(\frac{1}{r}\right)$.

(5/20)

(c) Jika $\mathbf{V} = x^2z\mathbf{i} - 2y^3z^2\,\mathbf{j} + xy^2z\mathbf{k}$

(i) Cari $\nabla.\mathbf{V}$ pada titik (1, -1, 1).

(ii) Jika $\mathbf{V} = \frac{x\mathbf{i} + y\mathbf{j}}{x + y}$ cari $\nabla.\mathbf{V}$

(iii) Jika $\mathbf{V} = x\cos z\,\mathbf{i} + y\log x\,\mathbf{j} - z^2\mathbf{k}$, nilaikan $\nabla.\mathbf{V}$

(10/20)

4. Buktikan yang berikut:

(i) div grad $\phi = \nabla^2\phi$ (6/25)

(ii) curl grad $\phi = \nabla \times (\nabla\phi) = 0$ (6/25)

(iii) div curl $\mathbf{f} = \nabla.(\nabla \times \mathbf{f}) = 0$ (6/25)

(iv) curl curl $\mathbf{f} = \nabla \times (\nabla \times \mathbf{f})$ = grad div $\mathbf{f}$ - $\nabla^2\mathbf{f}$ (7/25)

...3/-

5. (a) Diberikan $\mathbf{r}(t) = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ apabila $t = 2$ dan $\mathbf{r}(t) = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ apabila $t = 3$.

Tunjukkan $\int_2^3 \mathbf{r}.\frac{d\mathbf{r}}{dt}\, dt = 10$

(10/25)

(b) Biarkan $\phi = 45\, x^2 y$ dan V menandakan kawasan tertutup yang disempadan oleh satah-satah.

$$4x + 2y + z = 8,\ x = 0,\ y = 0,\ z = 0$$

nilaikan $\iiint_V \phi\, dv$.

(15/25)

UNIVERSITI SAINS MALAYSIA

First Semester Examination
2003/2004 Academic Session

September - October 2003

ZCT 211E - Vector Analysis

Time : 2 hours

Please check that the examination paper consists of **SIX** printed pages before you commence this examination.

Answer all FIVE questions. Students are allowed to answer all questions in English OR Bahasa Malaysia OR combinations of both.

1. (a) Two particles emitting from a source have displacements $\mathbf{r}_1 = 4\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ and $\mathbf{r}_2 = 2\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}$ at any time. Find the displacement of second particle relative to first.

(3/15)

(b) Prove that $(\mathbf{a} \times \mathbf{a}') + (\mathbf{b} + \mathbf{b}') + (\mathbf{c} \times \mathbf{c}') = O$, where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors and $\mathbf{a}', \mathbf{b}', \mathbf{c}'$ their reciprocals.

(6/15)

(c) A particle is moving in a circular orbit of radius 10 cm. If its frequency of motion is 60 cycles/sec., find the time period, velocity and acceleration of the particle.

(6/15)

2. (a) If $\vec{r} = t^2\vec{i} - t^2\vec{j} + (2t+1)\vec{k}$. Find the value of

$$\frac{dr}{dt}, \frac{d^2r}{dt^2}, \left|\frac{dr}{dt}\right|, \left|\frac{d^2r}{dt^2}\right| \text{ at } t = 0 .$$

(3/15)

(b) If $\mathbf{r} = \mathbf{a}e^{\omega t} - \mathbf{b}e^{-\omega t}$, show that $\frac{d^2\mathbf{r}}{dt^2} - \omega^2\mathbf{r} = 0$; **a, b** are constant vectors and ω being a constant.

(6/15)

(c) A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2 \cos 3t$, $z = 2 \sin 3t$, where t is the time

(i) Determine its velocity and acceleration at any time.
(ii) Find the magnitudes of velocity and acceleration at $t = 0$.

(6/15)

3. (a) If $\mathbf{A} = x^2yz\mathbf{i} - 2xz^3\mathbf{j} + xz^3\mathbf{k}$, $\mathbf{B} = 2z\mathbf{i} + y\mathbf{j} - x^2\mathbf{k}$ find $\frac{\partial^2}{\partial x \partial y}(\mathbf{A} \times \mathbf{B})$ at (1, 0, -2).

(5/20)

(b) If **r** is the position vector of a point, deduce the value of grad $\left(\frac{1}{r}\right)$.

(5/20)

(c) If $\mathbf{V} = x^2z\mathbf{i} - 2y^3z^2\mathbf{j} + xy^2z\mathbf{k}$

(i) find $\nabla.\mathbf{V}$ at the point (1, -1, 1).

(ii) If $\mathbf{V} = \frac{x\mathbf{i} + y\mathbf{j}}{x + y}$ find $\nabla.\mathbf{V}$

(iii) If $\mathbf{V} = x\cos z\,\mathbf{i} + y\log x\,\mathbf{j} - z^2\mathbf{k}$, evaluate $\nabla.\mathbf{V}$

(10/20)

4. Prove the following:

(i) div grad $\phi = \nabla^2\phi$ (6/25)

(ii) curl grad $\phi = \nabla \times (\nabla\phi) = 0$ (6/25)

(iii) div curl $\mathbf{f} = \nabla.(\nabla \times \mathbf{f}) = 0$ (6/25)

(iv) curl curl $\mathbf{f} = \nabla \times (\nabla \times \mathbf{f})$ = grad div $\mathbf{f}$ - $\nabla^2\mathbf{f}$ (7/25)

...6/-

5. (a) Given that $\mathbf{r}(t) = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ when t = 2 and $\mathbf{r}(t) = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ when $t = 3$.

Show that $\int_2^3 \mathbf{r} . \frac{d\mathbf{r}}{dt} dt = 10$

(10/25)

(b) Let $\phi = 45\, x^2 y$ and let V denote the closed region bounded by the planes

$$4x + 2y + z = 8, \; x = 0, \; y = 0, \; z = 0$$

evaluate $\iiint_V \phi \, dv$.

(15/25)

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