UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Kedua
Sidang Akademik 2004/2005

Mac 2005

EEE 223 – TEORI ELEKTROMAGNET

Masa : 3 jam

ARAHAN KEPADA CALON:

Sila pastikan bahawa kertas peperiksaan ini mengandungi **SEMBILAN (9)** muka surat beserta **Lampiran (2 mukasurat)** bercetak dan **ENAM (6)** soalan sebelum anda memulakan peperiksaan ini.

Jawab **LIMA (5)** soalan.

Agihan markah bagi soalan diberikan disudut sebelah kanan soalan berkenaan.

Jawab semua soalan di dalam Bahasa Malaysia.

Simbol mempunyai makna yang biasa.
Vektor diwakili oleh huruf `Bold Face'.
Guna sistem unit SI.
Guna ε_0 = 8.85 x 10^{-12} F/m, μ_0 = 4 x 10^{-7} H/m.
Anggap data bersesuaian jika tidak diberi.

1. (a) Diberi ketumpatan fluks elektrik.
Given the electric flux density.

$\mathbf{D} = xy^2 z^3 \mathbf{a}_x$ [C/m^2],

Tentukan
Determine

(i) Ketumpatan cas, ρ_v.
The charge density, ρ_v.

(ii) Jumlah cas Q yang terkandung dalam sebuah kiub 2[m] pada setiap sisi, terletak dalam sukuan pertama dengan tiga sisinya pada paksi-x, paksi-y dan paksi-z dan satu pepenjurunya pada asalan.

The total charge Q enclosed in a cube 2[m] on a side, located in the first octant with three of its sides coincident with the x, y and z-axes and one of its corners at the origin.

(10%)

(b) Kawasan di antara kelompang konduktor sfera sepusat r = 0.5[m] dan r = 1[m] adalah bebas-cas. Jika V (r = 0.5) = -50 [v] dan V(r = 1) = 50[v], tentukan taburan upaya dalam kawasan antara kelompang.

The region between concentric spherical conducting shells r = 0.5[m] and r = 1[m] is charge-free. If V (r = 0.5) = -50 [v] and V(r = 1) = 50[v], determine the potential distribution in the region between the shells.

(10%)

2. (a) Tukarkan titik P (0, 0, 3) daripada koordinat cartesian ke koordinat silinder dan sfera.

Convert point P (0, 0, 3) from Cartesian to cylindrical and spherical coordinates.

(5%)

(b) Medan elektrostatik ialah sebuah medan konservatif. Tunjukkan bahawa persamaan Maxwell dalam bentuk pembezaan untuk medan elektrik statik ialah $\nabla \times \mathbf{E} = 0$.

An electrostatic field is a conservative field. Show that the differential form of Maxwell's equation for static electric field is $\nabla \times \mathbf{E} = 0$.

(5%)

(c) Guna hukum Gauss untuk mendapatkan satu ungkapan untuk keamatan medan elektrik, **E** dalam ruang bebas yang disebabkan oleh sekeping cas planar infiniti dengan ketumpatan cas permukaan seragam ρ_s [C/m^2] yang terletak pada satah z = 0. Dengan ini, apa yang kamu boleh ulaskan mengenai **E** oleh kepingan cas infiniti tersebut pada mana-mana titik yang berada di atas dan di bawah satah x-y?

Use Gauss's law to obtain an expression for the electric field intensity, **E** *in free space due to an infinite planar charge with a uniform surface charge density* ρ_s *[C/m^2] lying on the z = 0 plane. Hence, what can you say about* **E** *of the infinite sheet of charge at any point located above and below the x-y plane?*

(10%)

3. (a) Satah x = 2 dan satah y = -3 masing-masing membawa cas 10 [nC/m^2] dan 15 [nC/m^2]. Jika garis x = 0, z = 2 membawa cas 10π [nC/m], kira **E** pada (1, 1, -1) yang disebabkan oleh ketiga-tiga taburan cas tersebut. Biarkan jawapan anda dalam sebutan π.

*Planes x = 2 and y = -3 respectively carry charges 10 [nC/m^2] and 15 [nC/m^2]. If the line x = 0, z = 2 carries charge 10π [nC/m], calculate **E** at (1, 1, -1) due to the three charge distribution.*

Ambil

Take

(i) $$\epsilon_0 = \frac{10^{-9}}{36\pi} \text{ [F/m]}$$

(ii) Untuk suatu cas garis infinit dengan ketumpatan cas ρ_L,

For an infinite line charge with charge density ρ_L,

$$E = \frac{\rho_L}{2\pi \epsilon_0 R} \mathbf{a_R} \text{ [V/m]}$$

(iii) Untuk sekeping cas infinit dengan ketumpatan cas ρ_s,

For an infinite sheet of charge with charge density ρ_s,

$$\mathbf{E} = \frac{\rho_s}{2\epsilon_0} \mathbf{a_n} \text{ [V/m]}$$

yang mana $\mathbf{a}_n$ ialah vektor unit normal kepada kepingan cas.

where $\mathbf{a}_n$ is a unit vector normal to the sheet.

(12%)

(b) Kabel sepaksi RG-58U mempunyai konduktor dalaman pejal 20-AWG dengan jejari 0.406 [mm], dilingkungi oleh dielektrik polithelene pejal dan konduktor luaran yang berjejari 1.553 [mm]. Cari nilai kapasitan per meter. (Pemalar dielektrik polithelene ialah $\epsilon_r = 2.26$)

RG-58U coaxial cable has a 20-AWG solid inner conductor with a 0.406 [mm] radius, surrounded by a solid polyethelene dielectric and a outer braided conductor of radius 1.553 [mm]. Find its capacitance per meter. (The dielectric constant of polyethylene is $\epsilon_r = 2.26$).

(8%)

4. (a) Persamaan am untuk talian penghantaran hilangan yang diuja oleh masukan sinus diberikan oleh

The general equation for a lossy transmission line excited by sinusoidal inputs is given by

$$\frac{d^2V(z)}{dz^2} = \gamma^2 V(z) \qquad ; \qquad \frac{d^2I(z)}{dz^2} = \gamma^2 I(z)$$

Yang mana $\gamma = \alpha + j\beta$ ialah pemalar perambatan dan α dan β ialah pelemahan dan pemalar fasa, masing-masing. Penyelesaian untuk persamaan di atas ialah

Where $\gamma = \alpha + j\beta$ is the propagation constant and α and β are the attenuation and phase constants, respectively. The solution to the above equations is

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{-\gamma z} \qquad ; \qquad I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{-\gamma z}$$

Yang mana simbol-simbol mempunyai makna biasa. Jika talian ialah kurang-hilangan dengan panjang 80 sentimeter, impedan cirian ialah 50 Ohm, kapasitan ialah 100 pF/meter dan beroperasi pada 600 MHz, tentukan

Where the symbols have their usual meanings. If the line is loss-less has a length of 80 centimeters, a characteristic impedance of 50 Ohms, 100 pF/meter capacitance and is operating at 600 MHz, determine

(10%)

(i) Jumlah induktan talian
Total inductance of the line

(ii) Pemalar fasa talian
Phase constant of the line

(iii) Halaju fasa talian
Phase velocity on the line

(iv) Impedan masukan talian apabila ia litar-pintas pada satu hujung.
The input impedance of the line when it is short circuited at one end.

(v) Impedan masukan apabila talian di litar pintas dan mempunyai panjang.
The input impedance when the line is short circuited and has a length of exactly equal to $\lambda/4$ where λ is the wavelength.

(b) Vektor potential magnetic **A** dalam sebuah kawasan ruang bebas tertentu diberikan sebagai **A** = 50 ρ^2 a_z Wb/m. Gunakan hubungan bahawa ikalan suatu vektor **A** bersamaan dengan ketumpatan fluks magnet **B** dan ikalan vektor **H** bersamaan dengan ketumpatan arus **J**, tentukan **B**, **H** dan **J**. Ikalan suatu vektor **F** dalam koordinat silinder (ρ, φ, z) diberikan oleh

*The vector magnetic potential **A** in a certain region of free space is given as **A** = 50 ρ^2 a_z Wb/m. Using the relation that the Curl of **A** is equal to the magnetic flux density **B** and curl of **H** is equal to the current density **J**, determine **B**, **H** and **J**. Curl of a vector **F** in cylindrical* (ρ, φ, z) *coordinates is given by*

$$\nabla \times \mathbf{F} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\mathbf{a}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\mathbf{a}_\varphi + \frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \varphi}\right)\mathbf{a}_z$$

(10%)

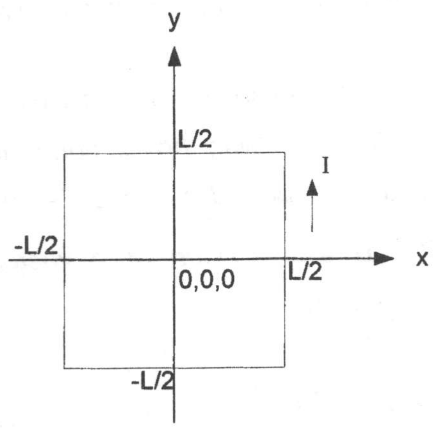
5. (a) Rajah 5 menunjukkan satu gelung arus segiempat bersisi L dengan tengahnya berada pada asalan dalam sistem koordinat Cartesian. Menggunakan hukum Biot-Savart tentukan keamatan medan magnet **H** pada pusat tengah gelung tersebut. Hukum Biot-Savart diungkapkan dalam matematik sebagai

*Figure 5 shows a square current loop of side L located with its centre at the origin in a Cartesian coordinate system. Using Biot-Savart law determine the magnetic field intensity **H** at the centre of the loop. The Biot-Savart law is mathematically expressed as*

$$\mathbf{dH} = \frac{\mathbf{I\,dL} \times \mathbf{a_R}}{4R^2}$$

yang mana simbol-simbol mempunyai makna mereka seperti biasa
where the symbols have their usual meanings

(10%)



Rajah 5
Figure 5

(b) Satu filamen arus pada paksi-z membawa arus 7 mA dalam arah $\mathbf{a_z}$, dan kepingan-kepingan arus 0.5 $\mathbf{a_z}$ A/m dan -0.2 $\mathbf{a_z}$ A/m berada pada ρ = 1 cm dan ρ = 0.5 cm dalam sistem koordinat silinder, masing-masing. Menggunakan hukum Ampere, kira keamatan medan magnet pada ρ = 0.5 cm dan ρ = 1.5 cm. Hukum Ampere menyatakan bahawa kamiran garis keamatan medan magnet **H** pada mana-mana laluan tertutup adalah sama dengan arus terus yang dilitupi oleh laluan tersebut. Secara matematik

A current filament on the z-axis carries a current of 7 mA in the $\mathbf{a_z}$ *direction, and current sheets of 0.5* $\mathbf{a_z}$ *A/m and -0.2* $\mathbf{a_z}$ *A/m are located at* ρ *= 1 cm and* ρ *= 0.5 cm in cylindrical coordinate system, respectively. Using Ampere's law calculate the magnetic field intensity at* ρ *= 0.5 cm and* ρ *= 1.5 cm. The Ampere's law states that the line integral of the magnetic field intensity* ***H*** *about any closed path is exactly equal to the direct current enclosed by the path. Mathematically*

$$\oint \mathbf{H} \cdot d\mathbf{L} = I$$

yang mana simbol-simbol mempunyai makna mereka yang biasa.
where the symbols have their usual meaning.

(10%)

6. Satu talian penghantaran tanpa hilangan dengan panjang 0.434λ dan impedan ciri 100 Ω ditamatkan dalam satu impedan 260 + j180 Ω. Menggunakan carta Smith, tentukan

A lossless transmission line of length 0.434λ and characteristic impedance 100 Ω is terminated in an impedance 260 + j180 Ω. Using Smiths chart determine

(a) Pekali pantulan voltan
The voltage reflection coefficient

(b) Nisbah gelombang pegun
The standing wave ratio

(c) Impedan masukan
The input impedance

(d) Lokasi maksima voltan pada talian
The location of voltage maximum on the line

(20%)

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VECTOR DERIVATIVES (cont.)

Spherical Coordinates (r, θ, ϕ)

$$\mathbf{A} = A_r \mathbf{a}_r + A_\theta \mathbf{a}_\theta + A_\phi \mathbf{a}_\phi$$

$$\nabla V = \frac{\partial V}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r \sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r \sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \mathbf{a}_r & r\mathbf{a}_\theta & (r\sin\theta)\,\mathbf{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & (r\sin\theta)\,A_\phi \end{vmatrix}$$

$$= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right]\mathbf{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\mathbf{a}_\theta$$

$$+ \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right]\mathbf{a}_\phi$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

VECTOR DERIVATIVES

Cartesian Coordinates (x, y, z)

$$\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$$

$$\nabla V = \frac{\partial V}{\partial x}\mathbf{a}_x + \frac{\partial V}{\partial y}\mathbf{a}_y + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\mathbf{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\mathbf{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\mathbf{a}_z$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical Coordinates (ρ, ϕ, z)

$$\mathbf{A} = A_\rho\mathbf{a}_\rho + A_\phi\mathbf{a}_\phi + A_z\mathbf{a}_z$$

$$\nabla \mathbf{V} = \frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \frac{1}{\rho}\begin{vmatrix} \mathbf{a}_\rho & \rho\mathbf{a}_\phi & \mathbf{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$= \left[\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right]\mathbf{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right]\mathbf{a}_\phi + \frac{1}{\rho}\left[\frac{\partial}{\partial \rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial \phi}\right]\mathbf{a}_z$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$