
UNIVERSITI SAINS MALAYSIA

Peperiksaan Semester Pertama
Sidang Akademik 2011/2012

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EUM222- KEBARANGKALIAN DAN STATISTIK GUNAAN

Masa : 3 jam

ARAHAN KEPADA CALON:

Sila pastikan bahawa kertas peperiksaan ini mengandungi TIGA BELAS muka surat bercetak sebelum anda memulakan peperiksaan ini.

Kertas soalan ini mengandungi ENAM soalan.

Jawab **EMPAT** soalan.

Mulakan jawapan anda untuk setiap soalan pada muka surat yang baru.

Gunakan $\alpha = 0.05$ jika nilai α tidak diberikan dalam soalan.

Agihan markah bagi soalan diberikan disudut sebelah kanan soalan berkenaan.

Jawab semua soalan di dalam Bahasa Malaysia atau Bahasa Inggeris atau kombinasi kedua-duanya.

“Sekiranya terdapat sebarang percanggahan pada soalan peperiksaan, versi Bahasa Inggeris hendaklah diguna pakai.”

“In the event of any discrepancies, the English version shall be used.”

1. (a) Dua tolok yang mengukur tahap kehausan tayar dibanding. Sepuluh lokasi dipilih di atas permukaan tayar menggunakan kedua-dua tolok. Keputusannya di dalam mm diberi dalam jadual di bawah.

Two gauges that measure tire tread depth are being compared. Ten different locations on a tire are measured once by each gauge. The results, in mm, are presented in the following table.

| Locations | Gauges 1 | Gauges 2 | Difference |
|-----------|----------|----------|------------|
| 1 | 3.95 | 3.89 | 0.15 |
| 2 | 3.23 | 3.30 | -0.07 |
| 3 | 3.60 | 3.59 | 0.01 |
| 4 | 3.48 | 3.61 | -0.13 |
| 5 | 3.89 | 3.88 | 0.01 |
| 6 | 3.76 | 3.73 | 0.03 |
| 7 | 3.45 | 3.56 | -0.11 |
| 8 | 3.01 | 3.02 | -0.01 |
| 9 | 3.82 | 3.77 | 0.05 |
| 10 | 3.44 | 3.49 | -0.05 |

Dengan mengandaikan perbezaan adalah hampir tertabur secara simetri dengan nilai min μ . Dengan menggunakan Ujian Tandaan Berpangkat Wilcoxon uji bahawa $H_0: \mu_D = 0$ melawan $H_1: \mu_D \neq 0$. Guna $\alpha = 0.05$

Assume the differences are a sample from an approximately symmetric population with mean μ . Use the Wilcoxon signed-rank test to test $H_0: \mu_D = 0$ versus $H_1: \mu_D \neq 0$. Use $\alpha = 0.05$

(8 markah/marks)

- (b) Rintangan dalam $m\Omega$, diukur untuk 5 wayar jenis pertama dan 6 untuk jenis kedua. Keputusan seperti di dalam jadual di bawah:

Resistance in $m\Omega$, are measured for five wires of one type and six wires of another type. The results are as follows:

| | | | | | |
|---|----|----|----|----|----|
| X | 36 | 28 | 29 | 20 | 38 |
| Y | 34 | 41 | 35 | 47 | 46 |

Guna Ujian Hasil Tambah Berpangkat Wilcoxon untuk menguji

Use Wilcoxon rank-sum test to test

$$H_0 : \mu_X \geq \mu_Y$$

$$H_1 : \mu_X < \mu_Y$$

(8 markah/marks)

- (c) Satu rawatan baru selepas pembedahan ingin dibuat perbandingan dengan rawatan piawai. 7 orang pesakit menerima rawatan baru manakala 7 yang lain menerima rawatan piawai. Masa untuk pulih di dalam hari adalah seperti berikut,

A new postsurgical treatment is being compared with a standard treatment. Seven subjects receive the new treatment, while seven others receive the standard treatment. The recovery times, in days, are as follows:

| | | | | | | | |
|---------------|----|----|----|----|----|----|----|
| Treatment (X) | 12 | 13 | 15 | 19 | 20 | 21 | 27 |
| Control (Y) | 18 | 23 | 24 | 30 | 32 | 35 | 40 |

Dengan menggunakan ujian hasil tambah berpangkat Wilcoxon, bolehkah anda membuat kesimpulan bahawa nilai min berbeza bagi kedua-dua rawatan.

Using Wilcoxon rank-sum test, can you conclude that the mean rate differs between the treatment and control?

(9 markah/marks)

2. (a) Diberi X mewakili bilangan kereta dan Y mewakili bilangan trak yang melalui plaza tol dalam selang masa seminit. Fungsi ketumpatan kebarangkalian bercantum bagi X dan Y adalah seperti berikut.

Let X represent the number of cars and Y the number of trucks that pass through a certain toll booth in a one-minute time interval. The joint probability mass function of X and Y is given in the following table.

| | y | | |
|-----|------|------|------|
| x | 0 | 1 | 2 |
| 0 | 0.10 | 0.05 | 0.05 |
| 1 | 0.10 | 0.10 | 0.05 |
| 2 | 0.05 | 0.20 | 0.10 |
| 3 | 0.05 | 0.05 | 0.10 |

- (i) Cari fungsi ketumpatan kebarangkalian marginal $P_x(x)$.
Find the marginal probability mass function $P_x(x)$.

- (ii) Cari fungsi ketumpatan kebarangkalian marginal $P_y(y)$.
Find the marginal probability mass function $P_y(y)$.

- (iii) Cari μ_x
Find μ_x

- (iv) Cari μ_y
Find μ_y

- (v) Cari σ_x
Find σ_x
- (vi) Cari σ_y
Find σ_y
- (vii) Cari $Cov(X, Y)$
Find $Cov(X, Y)$
- (viii) Cari $\rho_{x,y}$
Find $\rho_{x,y}$

(12 markah/marks)

- (b) Nilai tinggi H dan radius R (in cm) bagi satu tin silinder adalah secara rawak dengan fungsi ketumpatan kebarangkalian bercantum,

The height H and radius R (in cm) of a cylindrical can are random with joint probability density function

$$f(h, r) = \begin{cases} 3(h - 20)^2(r - 5) & 19 < h < 21 \\ & \text{and} \\ & 5 < r < 6 \\ 0 & \text{otherwise} \end{cases}$$

Isipadu tin diberi sebagai $V = \pi R^2 H$. Cari μ_V .

The volume of a can is $V = \pi R^2 H$. Find μ_V .

(6 markah/marks)

- (c) Bagi sebuah mesin fotokopi, diberi pembolehubah rawak X , adalah masa kerosakan bagi komponen masa dalam jam dan, Y adalah masa kerosakan komponen simpanan dalam jam. Fungsi ketumpatan kebarangkalian bercantum bagi X dan Y ialah

For a copier, let the random variable X , denote the time until failure of age component in hours, Y represent the time until failure of a spare component in hours. The joint probability density function of X and Y is

$$f(x, y) = 2 \times 10^{-6} e^{-0.001x - 0.002y} \text{ for } x \geq 0 \text{ and } y \geq 0$$

Tunjukkan bahawa X dan Y adalah merdeka.

Show that X and Y are independent.

(7 markah/marks)

3. (a) Seorang jurutera elektrik ingin membuat perbandingan min jangka hayat bagi 2 jenis transistor dalam aplikasi prestasi suhu tinggi. Sebanyak 60 sampel transistor dari jenis A telah diuji dan didapati mempunyai min jangka hayat 1827 jam dengan sisihan piawai 168 jam. Sebanyak 180 transistor jenis B telah diuji dan didapati mempunyai min jangka hayat 1658 jam dengan sisihan piawai 225 jam. Dapatkan 95% selang keyakinan bagi perbezaan antara min jangka hayat bagi kedua-dua jenis transistor.

An electrical engineer wishes to compare the mean lifetimes of two types of transistors in an application involving high-temperature performance. A sample of 60 transistors of type A were tested and were found to have a mean lifetime to 1827 hours and a standard deviation of 168 hours. A sample of 180 transistors of types B were tested and were found to have a mean lifetime of 1658 hours and a standard deviation of 225 hours. Find a 95% confidence interval for the difference between the mean lifetimes of the two types of transistors.

(5 markah/marks)

- (b) Seorang kerani sistem komputer mendapati bahawa komputer yang menggunakan sesuatu sistem operasi akan kerap mengalami gangguan seiring dengan usia pemasangan sistem operasi tersebut. Dia mengukur masa (dalam minit) sebelum gangguan bagi 7 komputer sebulan selepas pemasangan dan 9 komputer untuk tujuh bulan selepas pemasangan. Keputusannya seperti berikut :

...7/-

A computer system administrator notices that computers running a particular operating system seem to freeze up more often as the installation of the operating system ages. She measures the time (in minutes) before freeze-up for seven computers one month after installation, and for nine computers seven months after installation. The results are as follows:

| | | | | | | | | | | |
|-------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|--|
| One month after install: | 207.4 | 233.1 | 215.9 | 235.1 | 225.6 | 244.4 | 245.3 | | | |
| Seven months after install | 84.3 | 53.2 | 127.3 | 201.3 | 174.2 | 246.2 | 149.4 | 156.4 | 103.3 | |

Cari 95% selang keyakinan bagi perbezaan min gangguan antara bulan pertama dan bulan ketujuh.

Find a 95% confidence interval for the mean difference in time to freeze-up between the first month and the seventh.

(10 markah/marks)

- (c) Satu kumpulan yang terdiri dari 5 orang dengan tekanan darah tinggi diberi ubat baru untuk mengurangkan tekanan darah. Tekanan darah Sistolik dikira sebelum dan selepas pengambilan ubat baru dan keputusan seperti berikut.

A group of five individuals with high blood pressure were given a new drug that was designed to lower blood pressure. Systolic blood pressure was measured before and after treatment for each individual, with the following results:

| Subject | Before | After |
|---------|--------|-------|
| 1 | 170 | 145 |
| 2 | 164 | 132 |
| 3 | 168 | 129 |
| 4 | 158 | 135 |
| 5 | 183 | 145 |

Cari 90% selang keyakinan bagi purata pengurangan tekanan darah sistolik.
Find a 90% confidence for the mean reduction in systolic blood pressure.

(10 markah/marks)

4. (a) Di dalam sebuah pasaraya, sampel 60 pelanggan yang menggunakan sistem layan diri mengambil masa purata 5.2 minit untuk keluar, dengan sisihan piawai 3.1 minit. Sampel 72 pelanggan yang menggunakan khidmat juruwang mengambil masa purata 6.1 minit untuk keluar, dengan sisihan piawai 2.8 minit.

In a certain supermarket, a sample of 60 customers who use a self-service checkout lane averaged 5.2 minutes of checkout time, with a standard deviation of 3.1 minutes. A sample of 72 customers who use a cashier averaged 6.1 minutes with a standard deviation of 2.8 minutes

- (i) Buktikan bahawa masa purata untuk keluar untuk pelanggan layan diri adalah lebih kecil berbanding pelanggan menggunakan juruwang. Guna $\alpha = 0.05$.

Can you conclude that the mean checkout times is less for people who use the self-service lane? Use $\alpha = 0.05$.

- (ii) Buktikan bahawa jika semua pelanggan menggunakan khidmat layan diri, adakah nilai min keluar pasaraya akan berkurang? Ambil kira bilangan barangan yang dibeli dalam mengira jawapan. Guna $\alpha = 0.05$.

Can you conclude that if everyone used the self-service lane, that the mean checkout time would decrease? Consider the number of items checked out when formulating your answer. Use $\alpha = 0.05$.

(8 markah/marks)

- (b) Diberi keputusan MINITAB bagi ujian hipotesis $\mu_X - \mu_Y$. Terdapat nilai yang hilang.

The following MINITAB output presents the results of a hypothesis test for the difference $\mu_X - \mu_Y$ between two population means. Some of the numbers are missing.

| Two-samples T for X vs Y | | | | |
|--------------------------|----|-------|-------|---------|
| | N | Mean | StDev | SE Mean |
| X | 78 | 23.3 | A | 1.26 |
| Y | 63 | 20.63 | 3.02 | B |

Difference = mu (X) – mu (Y)
 Estimate for difference: 2.670
 95% CI for the difference: (0.05472, 5.2853)
 T-Test of difference = 0 (vs not =) : T- Value = 2.03
 P-Value = 0.045 DF = 90

- (i) Dapatkan nilai A dan B.
Find the values of A and B.
- (ii) Keputusan memberi nilai ujian Student's t . Cari nilai P menggunakan ujian z . Adakah keputusannya sama?

The output presents a Student's t test. Compute the P -value using a z test. Are the two results similar?

- (iii) Cari 98% selang keyakinan bagi $\mu_X - \mu_Y$ menggunakan statistik z .

Use the output and an appropriate table to compute a 98% confidence interval for $\mu_X - \mu_Y$ based on the z statistic.

(7 markah/ marks)

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- (c) Dua mikropemproses dibanding berdasarkan 6 sampel kod untuk mengenalpasti perbezaan dalam kelajuannya. Masa dalam saat bagi kedua-dua mikropemproses diberi seperti berikut,

Two microprocessors are compared on a sample of six benchmark codes to determine whether there is a difference in speed. The times (in seconds) used by each processor on each code are given in the following table.

| | Code | | | | | |
|-------------|------|------|------|------|------|------|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| Processor A | 27.2 | 18.1 | 27.2 | 19.7 | 24.5 | 22.1 |
| Processor B | 24.1 | 19.3 | 26.8 | 20.1 | 27.6 | 29.8 |

Bolehkah disimpulkan bahawa nilai min bagi kelajuan adalah berbeza bagi kedua-dua mikropemproses? Guna $\alpha = 0.05$.

Can you conclude that the mean speeds of the two processors differ ?

Use $\alpha = 0.05$.

(10 markah/marks)

5. (a) Keputusan MINITAB adalah bagi regresi berganda. Dapatkan nilai A, B, C, D, E, F, G dan H.

The following MINITAB output is for a multiple regression. Find the values of A, B, C, D, E, F, G and H.

| Predictor | Coef | SE Coef | T | P |
|-----------|----------|---------|-------|-------|
| Constant | -0.58762 | 0.2873 | (A) | 0.086 |
| X1 | 1.5102 | (B) | 4.30 | 0.005 |
| X2 | (C) | 0.3944 | -0.62 | 0.560 |
| X3 | 1.8233 | 0.3867 | (D) | 0.003 |

S = 0.869 R-Sq = 90.2% R-Sq(adj) = 85.3%

Analysis of Variance

| Source | DF | SS | MS | F | P |
|----------------|-----|-------|------|-----|-------|
| Regression | 3 | 41.76 | (E) | (F) | 0.000 |
| Residual Error | 6 | (G) | 0.76 | | |
| Total | (H) | 46.30 | | | |

(10 markah/marks)

- (b) Data adalah bagi ujikaji pertalian antara kehausan (mg) dan tekanan (KPa)

The following data were collected in a experiment to study the relationship between extrusion pressure (in KPa) and wear (in mg).

| | | | | | | |
|---|------|------|------|------|------|------|
| x | 150 | 175 | 200 | 225 | 250 | 275 |
| y | 10.4 | 12.4 | 14.9 | 15.0 | 13.9 | 11.9 |

Model kuasa dua terkecil ialah is $y = -32.445714 + 0.43154286x - 0.000982857x^2$.

The least-squares quadratic model is $y = -32.445714 + 0.43154286x - 0.000982857x^2$.

- (i) Dapatkan nilai reja bagi model tersebut.

Find the residuals for this model.

- (ii) Cari nilai ralat hasil tambah kuasa dua dan jumlah ralat hasil tambah kuasa dua.

Compute the error sum of squares SSE and the total sum of squares SST.

- (iii) Cari nilai penganggar ralat varians s^2 .

Compute the error variance estimate s^2 .

- (iv) Cari nilai pekali penentu R^2 .

Compute the coefficient of determination R^2 .

- (v) Cari F statistik bagi hipotesis $H_0 : \beta_1 = \beta_2 = 0$

Compute the value of the F statistic for the hypothesis $H_0 : \beta_1 = \beta_2 = 0$

(15 markah/marks)

6. (a) Diberi X dan Y adalah pembolehubah rawak, dan a dan b merupakan pemalar.

Let X and Y be random variables, and a and b be constants.

- (i) Buktikan bahawa $\text{Cov}(aX, bY) = ab \text{Cov}(X, Y)$.

Prove that $\text{Cov}(aX, bY) = ab \text{Cov}(X, Y)$.

- (ii) Buktikan jika $a < 0$ dan $b > 0$, maka $\rho_{aX,bY} = \rho_{X,Y}$.

Prove that if $a < 0$ and $b > 0$, then $\rho_{aX,bY} = \rho_{X,Y}$. Conclude that the correlation coefficient is unaffected by changes in units.

(9 markah/marks)

- (b) Buktikan bahawa $\sigma_{aX+b}^2 = a^2 \sigma_X^2$

Prove that $\sigma_{aX+b}^2 = a^2 \sigma_X^2$

(7 markah/marks)

- (c) Di dalam ujian redaman sambungan elektrik, 100 sambungan elektrik diuji di bawah keadaan redam dan 150 diuji dalam keadaan kering. 20 daripada sambungan redam gagal dan 10 daripada sambungan kering gagal. Dapatkan 90% selang keyakinan bagi perbezaan nisbah sambungan gagal bagi kedua-dua keadaan.

In a test of the effect of dampness on electric connections, 100 electric connections were tested under damp conditions and 150 were tested under dry conditions. 20 of the damp connections failed and only 10 of the dry ones failed. Find a 90% confidence interval for the difference between the proportions of connections that fail when damp as opposed to dry.

(9 markah/marks)

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