UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2011/2012

January 2012

EEE 512 – ADVANCED DIGITAL SIGNAL AND IMAGE

Time : 3 hours

INSTRUCTION TO CANDIDATE:

Please ensure that this examination paper contains **NINE** printed pages and **SIX** questions before answering.

Answer **FIVE** questions.

Answer to any question must start on a new page.

Distribution of marks for each question is given accordingly.

All questions must be answered in English.

1. Sampled-data sequences $v_d[n]$ and $w_d[n]$ are created by uniformly sampling two independent sequences, $v_a(t)$ and $w_a(t)$. The sampling period for creating $v_d[n]$ from $v_a(t)$ is 0.01s, while the sampling period for creating $w_d[n]$ from $w_a(t)$ is 0.02s. The waveforms of $v_a(t)$ and $w_a(t)$ are shown in Figure 1, and given by the following equations:

$$v_a(t) = 6\cos(60\pi t) + 3\sin(300\pi t) + 2\cos(340\pi t) + 4\cos(500\pi t) + 10\sin(660\pi t)$$
$$w_a(t) = 2\cos(60\pi t) + 4\cos(100\pi t) - 10\sin(260\pi t) + 6\cos(460\pi t) + 3\sin(700\pi t)$$

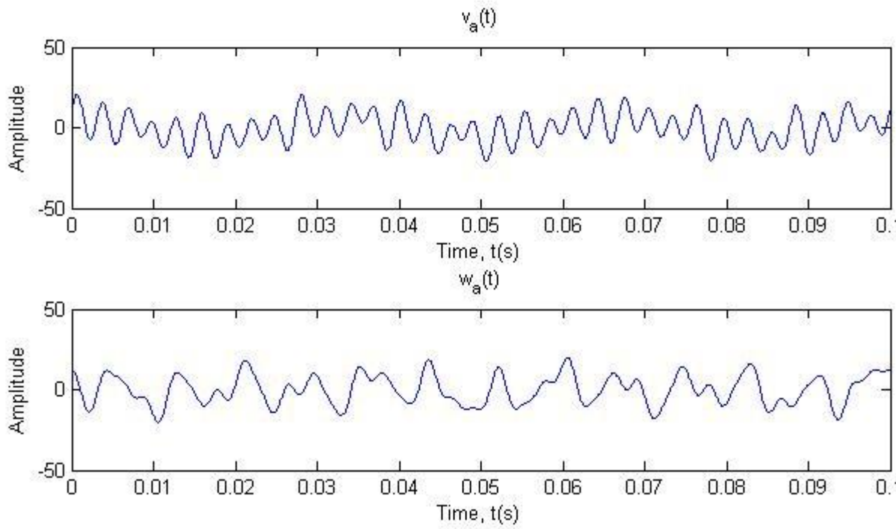


Figure 1

The digital signals are obtained by quantizing these sampled-value sequences by rounding them to the nearest integer values. These two digital signals, $x[n]$ and $y[n]$, are defined as:

$$x[n] = v_d[n](u[n] - u[n-5])$$

$$y[n] = w_d[n](u[n] - u[n-5])$$

From above information:

(a) Find the period N for the discrete-time sequence $v_d[n]$. (15 marks)

(b) Plot $x[n]$. (15 marks)

(c) Plot $y[n]$. (15 marks)

(d) Find the L_1–norm of $x[n]$. (10 marks)

(e) Find the L_2–norm of $x[n]$. (10 marks)

(f) Find the L_∞–norm of $y[n]$. (10 marks)

(g) Find the relative error between $x[n]$ and $y[n]$, with respect to $x[n]$. (15 marks)

(h) Find the energy contained in $x[n]$. (10 marks)

Hint:

L_p-norm of a sequence $\{a[n]\}$

$$\|a\|_p = \left(\sum_{n=-\infty}^{\infty} |a[n]|^p \right)^{1/p}$$

2. (a) Find the total solution for $n \geq 0$ of a discrete-time system characterized by the following difference equation:

$$y[n] + 2y[n-1] - 8y[n-2] = x[n]$$

for a step input $x[n] = 5u[n]$, and with initial conditions $y[-1] = -1$ and $y[-2] = 2$.

(60 marks)

(b) Draw a canonic cascade form realization for the following third-order IIR transfer function given below:

$$H(z) = \frac{0.287z^2 + 0.355z + 0.02}{(z^2 + 0.7z + 0.5)(z - 0.4)}$$

(40 marks)

3. (a) Determine the output from a linear convolution defined as $y[n] = x[n]*h[n]$. Given:

$$x[n] = \{9,0,4,5,2,1\}$$
$$h[n] = \{1,2,3\}$$

(20 marks)

(b) Determine the output from a circular convolution, defined as $y[n] = x[n] ⑥ h[n]$, using the same input sequences as defined in part (a).

(20 marks)

(c) We want to design an FIR low-pass filter with the shortest length as possible. The specifications of the filter are; pass-band edge ω_p=0.25π, stop-band edge ω_s=0.45π, and minimum attenuation α_s=40dB. Properties of some window functions are given in Table 1. Base on this table, which window function will be used? What is the window length required by this choice?

Table 1: Properties of some fixed window functions

Type of Window	Main Lobe Width Δ_{ML}	Relative Sidelobe Level A_{sl}	Minimum Stopband Attentuation	Transition Bandwidth, $\Delta\omega$ $\Delta\omega \approx \frac{c}{M}$
Rectangular	$4\pi/(2M+1)$	13.3dB	20.9dB	$0.92\pi/M$
Hann	$8\pi/(2M+1)$	31.5dB	43.9dB	$3.11\pi/M$
Hamming	$8\pi/(2M+1)$	42.7dB	54.5dB	$3.32\pi/M$
Blackman	$12\pi/(2M+1)$	58.1dB	75.3dB	$5.56\pi/M$

(60 marks)

4. (a) What is homomorphic filtering? Theoretically show how homormorphic filtering can improve an image which suffers from variable illumination.

(40 marks)

(b) The 1-D ideal low pass filter is defined as

$$H(u) = \begin{cases} 1; & D(u) \leq D_0 \\ 0; & D(u) > D_0 \end{cases}$$

where, D_0 is the cut-off frequency and $D(u)$ is the distance from the center point.

(i) derive the impulse response of the above filter. (30 marks)

(ii) from 4(b)(i) or otherwise, plot the filter function in the time domain.
(10 marks)

(iii) using suitable example, explain why the use of such a filter will result in a ringing artifact.

(20 marks)

Given

The Fourier transform pair

$$H(u) = \int_{-\infty}^{+\infty} h(x)e^{-jux}dx$$

$$h(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(u)e^{jux}du$$

The Euler identities

$$2\cos(x) = e^{jx} + e^{-jx}$$

$$2j\sin(x) = e^{jx} - e^{-jx}$$

Indefinite integral

$$\int e^{ax}dx = \frac{e^{ax}}{a}$$

Sinc function

$$Sinc\ A = \frac{Sin\ A}{A}$$

5. (a) Explain TWO principal reasons why the discrete Fourier transform is a preferred choice than the Haar and Walsh transforms in many transformation applications. Hence, state its main drawback.

(40 marks)

(b) Consider the 4 x 4 image as follows

$$f = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(i) perform Fourier transformation of *f* using unitary matrix.

(20 marks)

(ii) reconstruct *f* using the first three basis or elementary images.

(20 marks)

(iii) repeat 5(b)(ii) using the expansion of vector outer product.

(20 marks)

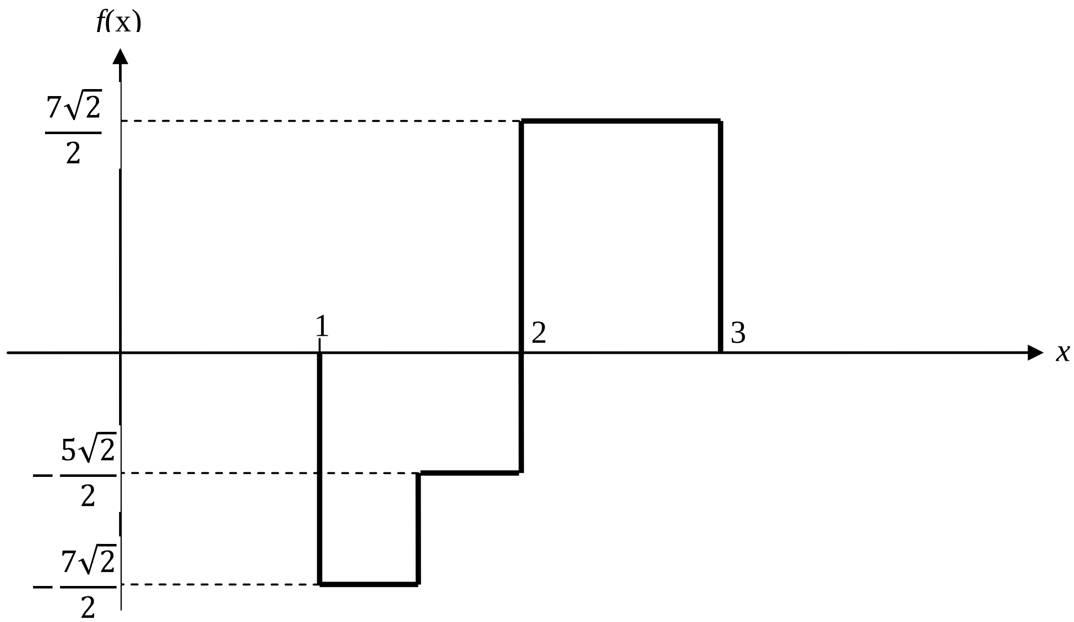
<u>Given</u>

The unitary matrix is defined as

$$U_{xy} = \frac{1}{N} e^{-j\frac{2\pi xy}{N}}$$

where, $x, y = 0,1, \dots, N-1$

6. (a) Consider a function $f(x)$ as shown below



Construct $f(x)$ using Haar scaling function $\varphi(x)$ and Haar wavelet function $\psi(x)$.

(40 marks)

(b) Consider the 8×8 image as follows

$$f(x,y)=\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 10 & 10 & 10 & 10 & 2 & 2 \\ 2 & 2 & 10 & 10 & 10 & 10 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{bmatrix}$$

(i) draw the require filter bank to implement a first-scale two-dimensional FWT of $f(x, y)$. Label all inputs and outputs with the proper arrays.

(30 marks)

(ii) use the result from 6(b)(i) to draw the require filter bank to implement the two-dimensional inverse FWT. Label all inputs and outputs with the proper arrays.

(30 marks)

Given:

The wavelet functions are defined as:

$$\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi\left(2^{j}x - k\right)$$

$$\psi(x) = \sum_{n} h_{\psi}(n)\sqrt{2}\varphi(2x - n)$$

$$\psi(x) = \begin{cases} 1 & ; \quad 0 \leq x < 0.5 \\ -1 & ; \quad 0.5 \leq x < 1.0 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

The Haar scaling functions are defined as :

$$\varphi(x) = \begin{cases} 1 & ; \quad 0 \leq x < 1 \\ 0 & ; \quad \text{elsewhere} \end{cases}$$

$$\varphi_{j,k}(x) = 2^{\frac{j}{2}}\varphi\left(2^{j}x - k\right)$$

The scaling function coefficients for the Haar function are given by:

$$h_{\varphi}(n) = \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\} \quad \text{for} \quad n = 0,1$$

The scaling function coefficients for the Haar wavelet are given by:

$$h_{\psi}(n) = \left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right\} \quad \text{for} \quad n = 0,1$$

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